Phase Noise Analysis of SAW Oscillators

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Abstract Surface acoustic wave (SAW) oscillator is a kind of typical feedback oscillators with feedback elements. Its phase noise characteristics is analyzed in the context of Leeson's model of feedback oscillator phase noise and particular attention is given to the cases of different feedback SAW elements, since different feedback elements ordinarily denote different phase noise characteristics in the view of our points in this paper. Approaches and methods for reducing phase noise levels are discussed along with an analysis of reported attempts to obtain improved spectral purity.

Key words surface acoustic wave; oscillator; phase noise; delay line

SAW oscillators are the feedback oscillators with feedback elements, such as SAW resonators, SAW delay lines, etc. As they are shown by Parker and Montress^[1-3], great efforts have been made to improve oscillator performance, especially the temperature and phase noise, longterm stability and vibration sensitivity. The theory of SAW oscillators has, however, received little attention so far. Many schorlars try to search for solutions in crystal oscillators. Although many problems can be solved in such a way, some different aspects have been noticed between SAW oscillators and crystal oscillators^[4], e. g. 1) physical essence of surface wave and bulk wave; 2) many SAW devices can be used as frequency control elements; 3) performance of SAW oscillators would be changed by different delay time, etc. Not only would the high Q factor SAW devices, such as SAW resonators be provided, but also other SAW devices with arbitary performance would be used to act as feedback elements discussed in this paper.

1 SAW Oscillator Phase Noise Basics

Leeson's model^[5] for the phase noise spectrum of an oscillator is widely used by designers and users of communication circuits, in relating effects of internal noise perturbations. His model is a heuristic one; derived for the case of a single RIC (resistance, inductance and capacitance) resonator in the oscillator feedback loop, as depicted in Fig. 1. Fig. 2 illustrates the general form of the oscillator phase noise spectrum, as derive from his model. Sauvage^[6] has given a mathematical analysis of the phase noise response analytical method was based on s-plane input-output autocorrelation and cross-correlation techniques such as used to derive the power spectrum of an optimum linear one, and thereby obtained the output noise power spectrum Q_{00} (jk) relationship in the form

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$$Q_0^{(s)}|_{s=jk} = Q(s)Q(-s)Q_n(s)|_{s=jk}$$
(1)

where $\Omega_n(j\mathbf{k})$ represents the internal or input noise spectral density, and $\Omega(j\mathbf{k})$ is the oscillator closed-loop voltage response transfer function. In Fig. 2, f^{T} , f^{U} and f^{f} present the corresponding breakpoint frequencies of fliker frequency, white frequency, fliker phase and white phase respectively.



Fig. 1 Schematic block diagram of a feedback oscillator

Fig. 2 Leeson's model form of phase noise response of an oscillator due to flicker and white phase noise perturbations

If SAW resonators are used as frequency control elements, SAW oscillators have the same phase noise features with traditional BAW oscillators. But if one broadband SAW delay line is used, its derivation is different from the derivation of SAW resonator oscillators. Its analysis mathematical diagram block is redrawn in Fig. 3.



(a) Block diagram of an uniform SAW oscillator



(b) Basic SAW delay line with narrow-band uniform input interdigital transducers (IDT) and wideband output IDT.

Fig. 3 An uniform SAW oscillator and its relevant frequency control element

To simplify the analysis we assume that this consists of a narrow-band input IDT with N fingers in conjunction with a wide-band output IDT; with response transfer functions $H_1(\mathbf{k})$ and $H_2(\mathbf{k})$, respectively. Thus the overall delay line response function $H(\mathbf{k})$ may be approximated as $H(\mathbf{k}) = |H(\mathbf{k})| e^{ip} = H_1(\mathbf{k}) H_2(\mathbf{k}) e^{ipr} \approx H_1(\mathbf{k}) e^{ipr}$. Here, we set f as the SAW delay time between intermost ends of the IDT's and note that an additional phase delay term will be included in the $H_1(\mathbf{k})$ term. If the voltage gain of the amplifier is $K_1($ and is assumed to be constant over the oscillator response), the closed loop gain Q(s) for positive feedback is

$$Q(s) = \frac{K_1}{1 - K_1 H(s)}$$
 (2)

We obtain that

$$Q(s)Q(-s)|_{s=jk} = \frac{K_1^2}{1 - K_1 [H(s) + H(-s)] + K_1^2 H(s) H(-s)}|_{s=jk} = \frac{K_1^2}{1 - 2K_1 |H(k)| \cos\theta + K_1^2 |H(k)|^2}$$
(3)

It is further recognized that the term $K_1||H(\mathbf{k})| \approx 1$ at small offset from nominal center frequency k_0 , so that

$$(Q_{jk})Q_{-jk} \approx \frac{K_{1}^{2}}{2(1-\cos\theta)} = \frac{K_{1}^{2}}{4\sin^{2}(\theta/2)}$$
 (4)

We can then set the spectral density of input phase noise perturbations $Q_m(jk)$ as

$$Q_m(jk) = (FkT^0 + \frac{T}{k_F})$$
⁽⁵⁾

per unit bandwidth, where F= amplifier noise figure, k= Boltsmann' s constant, T^0 = temperature in K in the first term for white phase noise, T= experimentally determined constant for the given circuit and k_F is the Fourier frequency in the flicker noise term. Substituting of these relationships in Eq. (1), then the result in the output power spectral density is

$$Q_0(jk) = \frac{K_1^2}{4\sin^2(\theta/2)} (FkT^0 + \frac{T}{k_F})$$
(6)

When normalized with respect to oscillator average output power Po, we obtain

$$\frac{Q_0(j\mathbf{k})}{P_0} = 10 \lg \left[\frac{K_1^2}{4\sin^2(\theta/2)} \left(Fk T^0 + \frac{T}{\mathbf{k}_F} \right) \right]$$
(7)

in units of dBc/Hz, as the general form of the oscillator phase noise spectrum.

2 Results

Case 1 With Basic Uniform IDT

In this case, the oscillator (shown in Fig. 3) spectral response is obtained by Eq. (6). From Eq. (4), we obtain the 3 dB breakpoint above the floor noise level when $\cos\theta = \cos(k - k_0) f = 1/2$. This then gives the angular offset Fourier transition frequency from open-loop to closed-loop response as $(k - k_0) f_{r=} = k_{FC} f \approx 1$, or $k_{FC} \approx k_0 / 2Q_r$; where Q_r is the "effective" Q of the phase shift oscillator circuit. This result is in agreement with that obtained by Lesson in his heuristic approach to the problem^[5].

The right-hand term in Eq. (4) may also be written in the form

$$Q(\mathbf{j}\mathbf{k})Q - \mathbf{j}\mathbf{k}) \approx \frac{K_1^2}{\theta^2} = \frac{K_1^2 \mathbf{k}_0^2}{4Q_\ell^2 \mathbf{k}_F^2}$$
(8)

on substitution of $Q = \frac{k_0 f}{2}$ Substituting of Eq. (8) into Eq. (7), we obtain that $Q_0(ik) = \frac{K_1^2 k_0^2}{2} = \frac{K_1^2 k_0^2}{2}$

$$\frac{Q_{0}(jk)}{P_{0}} = 10 \lg \left[\frac{K_{1}^{2}k_{0}^{2}}{4Q_{\ell}^{2}k_{F}^{2}} \left(FkT^{0} + \frac{T}{k_{F}} \right) + K_{1}^{2} \left(FkT^{0} + \frac{T}{k_{F}} \right) \right]$$
(9)

Case 2 With "Lewis-Type" Delay Line

It is possible for the SAW delay line oscillator to operate spuriously at other than the desired frequency, unless special precautions are taken to ensure proper mode selection. Oscillations can build up at sidelobe response characterisitic of the IDT. The special SAW oscillators are accomplished by setting the length h of the narrow-band input IDT equal to the distance d between IDT phase centers so that $h = d = V^{f_t}$. The denominator terms in Eq. (3) are then all positive at frequencies corresponding to peaks in the IDT sidelobe response, so that oscillation will not be posible, except at the main lobe frequency. The Q factor is the only difference between uniform SAW

Case 3 With Dispersive Delay Line

Dispersive delay lines are used as feedback elements with $\theta = f(\mathbf{k}, t)$ for special usages, such as novel temperature compensating method. For example, $\mathbf{k} = \mathbf{k}_0 + \mathbf{k}_t$, $\theta \leq \mathbf{k} \in T$, for linear chirp filter with _ chirp rate and T dewilling time. We obtain

$$\cos\theta = \cos(k - k_0) \, \sharp = 2\sin^2(\pi_{\pm} \, \sharp t)$$
 (10)

Substituting Eq. (10) into Eq. (4), we obtain that

$$O(jk)O(-jk) \approx \frac{K_1^2}{4\sin^2(\pi_{-} t_{t})}$$
(11)

and

$$\frac{Q_0(jk)}{P_0} = 10 \, \log\left[\frac{K_1^2}{4\sin^2(\pi_- f_t)} \left(Fk \, T^0 + \frac{T}{k_F}\right)\right]$$
(12)

Above analysis shows that the phase noise of such oscillators is a function of time t, so we must design the oscillators carefully in order to obtain good phase noise characteristics and special usages.

Case 4 With SAW Resonator

SAW resonator oscillators have been analyzed in detail by Parker and Motress^[3]. The approximate phase noise of an oscillator, based on the Leeson model also, can be expressed in equation form as show below^[3].

$$\frac{Q_{0}(jk)}{P_{0}} = 10lg \{ [T_{R}F_{0}^{4} + T_{E}(F_{0}/(2Q_{L}))^{2}] / f^{3} + [(2GFkT^{0}/P_{0})(F_{0}/(2Q_{L}))^{2}] / f^{2} + (2T_{R}Q_{L}F_{0}^{3}) / f^{2} + T_{E} / f + 2GFkT^{0}/P_{0} \}$$
(13)

Where Q is the loaded Q of the resonator; $F_0 = \frac{K_0}{2\pi}$; $f = \frac{K_F}{2\pi}$; G is the compressed gain ($\approx K_1$) of the loop amplifier; T_E and T_R are constants. The results are in agreement with case 1, except for the proportionality constant.

3 Discussion

By way of recapitulation, it may be noted that fairly simple IDT geometries have been considered here in order to demonstrate the s-transform method as applied to phase noise determination, as well as to relate the results to those obtained by other analytical. In considering the phase noise spectral density relationships, it was further assumed that the amplifiers in the oscillator feedback circuit were not running heavily into saturation, with waveform clipping. In the above analysis it was also assumed that the oscillator spectral width was sufficiently narrow, so that amplifier gain and IDT insertion loss could be considered as being constant over the Fourier offset frequencies under consideration.

From above analysis of 4 cases, lower noise floors, reduced flicker noise, and increased material Q factor which have been already discussed in Ref. [3], are the commonly used methods for improving oscillator spectral purity. Rarely are there applications where spectral purity is the only consideration, and most of the time many other factors, such as long-term stability, frequency settability, temperature stability, power consumption, size, cost, etc., many also be taken into account. In addition, there are the environmental sensitivity, and temperature stability, which may directly affect the spectral purity, depending on the environment in which the oscillator is intended to operate. The special attention is given to the flexilbe filtering features of feedback elements compared with crystal oscillators, especially their different phase frequency features. Its phase noise characteristics may be a function of different parameters, such as time t, frequency k, and temperature T, etc. So a special method for reducing phase noise levels would be most optimization synthesis mehtods, etc. Such special method is bound to be achieved with available arbitary phase feature of SAW feedback element.

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SAW振荡器的相位噪声分析

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【摘要】 声表面波振荡器是一种典型的反馈振荡器。根据 Leeson反馈振荡器相位噪声模型,分析 了 SAW 振荡器的相位噪声特点,特别是不同的 SAW 反馈元件,分析结果表明不同的 SAW 反馈元件 通常有不同的相位噪声特点。还讨论了降低相位噪声电平,提高振荡器的谱密度的方法和途径。

关键词 声表面波;振荡器;相位噪声;延迟线

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