A New Delayed Feedback Control Approach of Discrete Chaotic Systems

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Abstract Based on the fact that the tracking error asymptotically approaches zero, a new nonlinear delayed feedback control approach of discrete chaotic systems is presented. Such control approach that uses the step-*N* delayed output of a chaotic system to estimate the unstable period-*N* orbit and to be the tracking target can stabilize a controlled chaotic system onto a deterministic point in the phase space as well as a period-*N* orbit. Theory and experiments show that the control approach is robust.

Key words chaos; nonlinear delayed feedback; tracking control; robustness

In the last few years, there has been a great deal of interest in the control of chaotic dynamical systems[1~5]. Its main idea is: 1) to stabilize a controlled chaotic system onto a deterministic point in the phase space[6~8]; 2) to control motion on a chaotic attractor so that it quickly settles down to a desired periodic orbit[9~14]; 3) to synchronize a controlled chaotic system with another system[14]. In a sense, the control of chaotic dynamic systems can be considered as a tracking control problem. The tracking target is a deterministic point or an unstable periodic orbit embedded in a chaotic attractor or the output of another system. The controlling process is to make the tracking error between the tracking target and the output of a controlled chaotic system to approach zero asymptotically and to guarantee the stabilization of controlled systems. So, the problems to be solved in the control of chaotic dynamic systems are to select the tracking target and to design controller in order to guarantee the tracking error to approach zero asymptotically.

In this paper, we select step-*N* delayed output of a discrete chaotic system as the estimation of the unstable period-*N* orbit and the tracking target of the correspondent controlled chaotic system. Continually correcting the estimation of the controller, we can get a stable period-*N* orbit that has been proven by Ref . [1]. We design a nonlinear delayed feedback controller to guarantee the tracking error to approach zero asymptotically. When *N* equals to one, the controlled chaotic system is stabilized to a deterministic point in the phase space. When *N* is more than one, the controlled chaotic system is stabilized to period-*N* orbit.

1 Controlled Discrete Chaotic Systems

The theoretical considerations are based on affined controlled discrete chaotic systems

$$\begin{cases} x_{t+1} = f(x_t, t) + B(t)u_t \\ y_t = g(x_t) \end{cases} \tag{1}$$

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where $x_t \in R^n$ denotes the phase space variable, $t \in R^+$ denotes time, f denotes a $R^n \times R^+ \to R^n$ nonlinear function, R^+ denotes $[0,+\infty)$, $B(t) \in R^{n\times m}$ denotes feedback control matrix, $u_t \in R^m$ denotes control signal, $y_t \in R^m$ denotes the output of the system, and g denotes a $R^n \to R^m$ nonlinear function.

When u_t equals to zero, the system is a chaotic system and its attractor includes infinite unstable periodic orbits. Now we design a controller to guarantee the output of the system to be stabilized onto a period-*N* orbit that is formed by the unstable period-*N* orbit embedded in the chaotic attractor.

Let

$$e_t = y_t - y_{t-N} \tag{2}$$

where $e_t \in R^m$ denotes the tracking error of the controlled system, y_t denotes the output of the controlled system, and y_{t-N} denotes the step-*N* delayed output.

If e_t suffices the condition

$$e_{t+1} - e_t = -\mathrm{diag}(q_1, q_2, \cdots, q_m)\mathrm{sign}e_t - \mathrm{diag}(k_1, k_2, \cdots, k_m)e_t \tag{3}$$

where $\mathrm{diag}(q_1, q_2, \cdots, q_m)$ and $\mathrm{diag}(k_1, k_2, \cdots, k_m)$ are two *m* order diagonally matrices, $q_i (i=1,2,\cdots,m)$ is more than zero but much less than one, $k_i (i=1,2,\cdots,m)$ is more than zero but fewer than one, and sign(•) denotes a sign function. Then, the tracking error will asymptotically approach zero.

Now, we prove the conclusion. From Eq. (3), we get

$$e_{t+1}(i) = (1-k_i)e_t(i) - q_i \mathrm{sign}e\ (i) \qquad i = 1,2,\cdots,m \tag{4}$$

where $e_t(i)$ denotes number i component of e_t. Select a Lyapunov function as

$$V_t = \sum_{i=1}^{m} e_t^2(i) \tag{5}$$

When e_t is nonzero, V_t is more then zero. Then

$$V_{t+1} - V_t = -\sum_{i=1}^{m} (1-(1-k_i)^2)e_t^2(i) - \sum_{i=1}^{m} 2(1-k_i)q_i e_i(i)\mathrm{sign}\ e_t(i) + \sum_{i=1}^{m} q_t^2 (\mathrm{sign}e_t(i))^2 \tag{6}$$

When e_t is nonzero, and q_i and k_i are properly selected, $V_{t+1} - V_t$ is less than zero.

In terms of the Lyapunov stableness theorem, the tracking error e_t is asymptotically stable, i.e., the tracking error e_t will asymptotically approach zero.

From Eq. (3) and the system dynamic Eq.(1), we get

$$g(f(x_t, t) + B(t)u) = y_{t+1-N} + (\boldsymbol{I} - \mathrm{diag}\ (k_1, K_2, \cdots, k_m)e_t - \mathrm{diag}(q_1, q_2, \cdots, q_m)\mathrm{sign}e_t \tag{7}$$

where $\boldsymbol{I}$ denotes the *m* order unit matrix.

From Eq. (7), we can get the control signal as

$$u_t = j\,(x_t, x_{t+1-N}, e_t, t) \tag{8}$$

Because the amplitude of the control signal depends on the tracking error, a large amplitude may be required, which is an undesirable situation in real systems. In order to let the control signal be small, we introduce a limited tracking error

$$u_t = j\,(x_i, x_{t+1-N}, e_t, t) \qquad \text{if } |e_t| < e, \text{or } u_t = 0 \tag{9}$$

When the system motion trajectory enters into the appointed region in the phrase space, the control signal acts on the system; or, the system is kept to be chaotic. Due to the ergodicity of chaotic behavior, its motion trajectory will certainly enter the appointed region in an instantaneous time.

The suggested control approach is robust. From the tracking error Eq.(3), it is known that the tracking error equation is kept to be invariant for the uncertainty of the system structure and the matching external perturbation. If the amplitude of the control signal is limited in the situation of the internal and external perturbation, the time of the transient process will be longer, but the stable result is almost kept invariant.

2 Simulation Experiments

To verify the suggested control approach, Henon chaotic model is used for computer simulation. Controlled Henon chaotic model is

$$x_{n+1} = 1 - a x_n^2 + z_n + a_1 \mathrm{rand}(1,1) + b_1 u_n$$

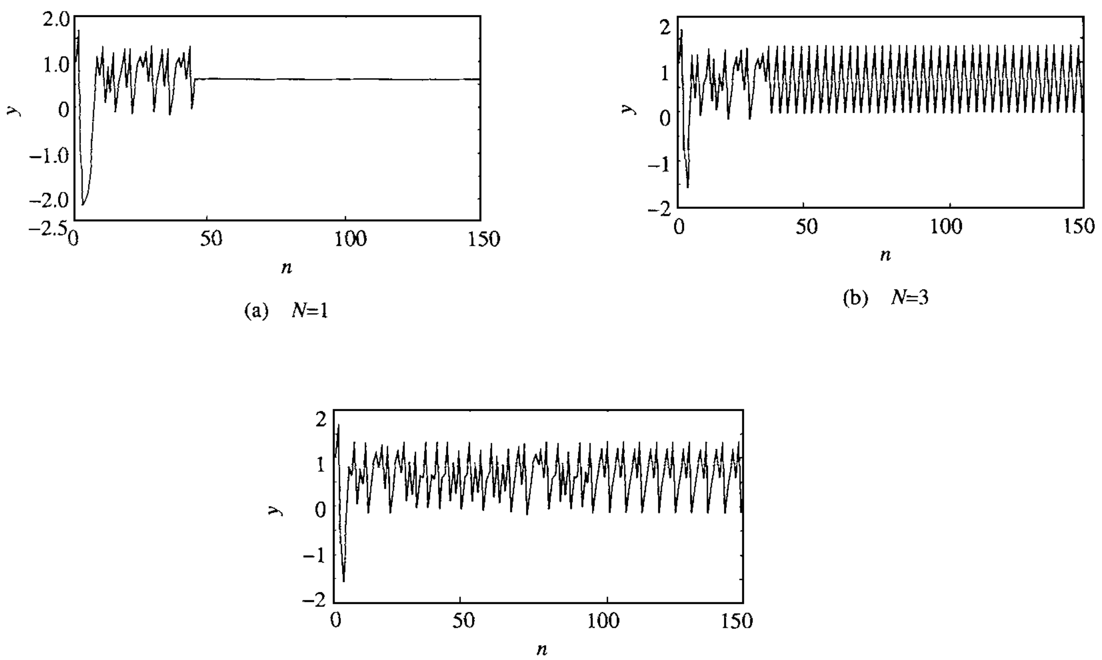
$$z_{n+1} = b x_n + a_2 \mathrm{rand}(1,1) + b_2 u_n$$

$$y_n = c_1 x_n + c_2 z_n$$

$$e_n = y_n - y_{n-N}$$

$$u_n = \begin{cases} (c_1 b_2 + c_2 b_2)^{-1} (c_1 x_{n+1-N} + c_2 z_{n+1-N} + (1-k_1) e_n - q_1 \mathrm{sign} e_n \\ - c_1 + c_1 a x_n^2 - c_1 z_n - c_2 b x_n - c_1 a_1 \mathrm{rand}(1,1)) \quad |e_n| < e \\ 0 \qquad |e_n| \geqslant e \end{cases}$$

where $a_1\mathrm{rand}(1,1)$ and $a_2\mathrm{rand}(1,1)$ denote the uncertain perturbation. Fig.1 shows that the unstable period-N(N=1,3,6) orbits embedded in a chaotic attractor are stabilized without uncertain perturbation. Fig.2 shows that unstable period-N(N=1,3,6) orbits embedded in a chaotic attractor are stabilized with uncertain perturbation. In Fig.1 and Fig.2, the traverse coordinate denotes time n; the longitudinal coordinate does the system output y.



(a) N=1

(b) N=3

(c) N=6

Fig.1 Controlling results of Henon chatic model without uncertain perturbation

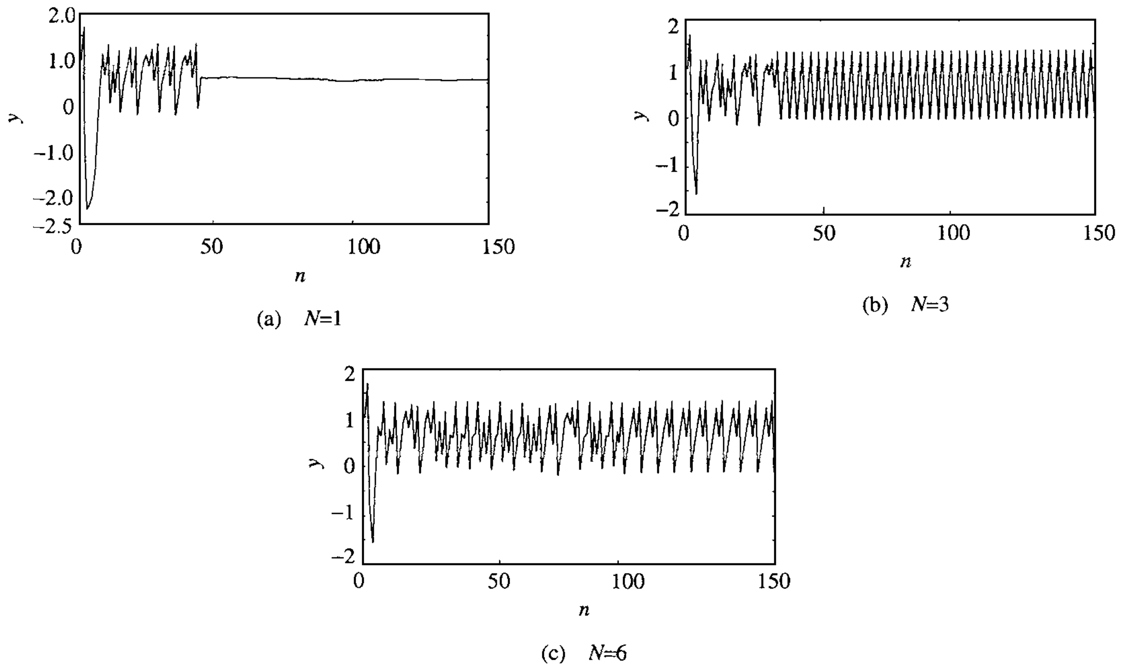


Fig.2 Controlling results of Henon chatic model with uncertain perturbation

3 Conclusion

In conclusion, we have shown that the UPO of a chaotic system can be stabilized onto a stable periodic orbit by a nonlinear delayed feedback controller. When period N equals to one, a controlled chaotic system is stabilized onto an identical output. When period N is more than one, a controlled chaotic system is stabilized onto a periodic orbit. Theory and experiments show this control approach is robust. This control approach has following advantages: first, the tracking error will asymptotically approach zero; second, it does not require the precise system model; third, it does not need to know a UPO extracted from the chaotic attractor; last, it is easy to determine the controller.

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一种新的离散混沌系统的延时反馈控制

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【摘要】以跟踪误差渐进趋近于零为基础，提出了一种新的离散混沌系统延时反馈控制方法。该方法利用离散混沌系统的 N 步延时输出来估计系统的不稳定 N 周期轨道，并作为跟踪目标，使受控混沌系统既能稳定到相空间的确定点，又能稳定到 N 周期轨道。理论分析和实验均表明该控制方法具有鲁棒性。

关　键　词　混沌;　非线性延时反馈;　跟踪控制;　鲁棒性

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