第 34 卷　第 2 期　　电 子 科 技 大 学 学 报　　Vol.34　No.2
2005 年 4 月　　Journal　of　UEST　of　China　　Apr. 2005

Fast MP of Images Based on Atom Energy Property

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Abstract　The computational load in Matching Pursuit (MP) of images is very huge, and this is the main drawback with MP of images. In this paper, a new fast algorithm is presented to speed up MP of images. Energy distribution of atoms used in MP is studied and most of computational loads can therefore be avoided by making use of the atom energy property. Experimental results show that the MP algorithm developed according to this property is many times faster than the conventional MP method and at the same time the quality of reconstructed images by the new algorithm is as good as by the conventional MP method.

Key words　image processing;　matching pursuits;　sparse decomposition;　fast algorithm

基于原子能量特性的快速图像匹配追踪

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【摘要】稀疏分解在图像处理中应用的关键障碍之一是图像稀疏分解速度十分缓慢。针对这一问题，提出了一种新的图像稀疏分解的匹配追踪快速算法。研究了图像稀疏分解中使用的原子的能量分布特性，根据原子能量的分布特性，图像匹配追踪中的绝大部分的计算可以省略，因而极大地提高了图像匹配追踪的计算速度。实验结果表明，新的算法比传统的图像匹配追踪算法速度提高了许多倍，而恢复图像的质量没有任何的降低。

关　键　词　图像处理;　匹配追踪;　稀疏分解;　快速算法

中图分类号　TN911.7;　TP391.4　　　**文献标识码**　A

Sparse representations of images are very suitable for image processing, especially for image coding and denoising. Matching pursuit(MP) is one of the most important ways to decompose image sparsely. MP was originally proposed to decompose one-dimensional signals by Mallat and Zhang[1] and was then extended to image decomposition by Bergeaud and Mallat[2]. As the most commonly used way of decomposing images sparsely, MP of images is drawing more and more people' s attention.

1　MP of Images

MP decomposes any signal into a linear expansion of waveforms(atoms) selected from a redundant

Received data: 2004 – 05 – 09

Foundation item: The project is supported by China Scholarship Council Found (2001-3048) and Applied Foundational Research of Sichuan Province (03JY029 – 048 – 2)

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收稿日期：2004 – 05 – 09

基金项目：国家留学基金资助项目(2001 – 3048)；四川省应用基础研究项目资助(03JY029-048-2)

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dictionary[1,2]. For an image of size $M_1 \times M_2$ to be decomposed, let $D = \{g_\gamma\}_{\gamma\in\Gamma}$ be a dictionary of P atoms with $P >> M_1 \times M_2$ and let $R^n f$ be the residual image of a n term representation of the given image f with $R^0 f = f$. In the process of MP, the main work is to select the best atom g_{γ_n} in the over complete dictionary D so that

$$\left|\left\langle R^n f, g_{\gamma_n}\right\rangle\right| \geq \alpha \sup_{\gamma\in\Gamma}\left|\left\langle R^n f, g_\gamma\right\rangle\right| \tag{1}$$

Where g_{γ_n} is the selected atom from D that matches $R^n f$ at best, $\left\langle R^n f, g_{\gamma_n}\right\rangle$ is the projection of the image residual $R^n f$ on the selected best atom g_{γ_n} and $\alpha \in (0,1]$ is an optimality factor[2]. From equation (1) we can find that the calculation of projections of image residual $R^n f$ on atoms of D has to be carried out for many times and this is why the computational load in MP of images is very huge.

Many researchers have proved that MP is a very useful way to decompose images sparsely. But we can hardly use traditional MP of images in real-time image processing or image coding because of its huge computational burden. To overcome this main drawback with MP of images, it's necessary and urgent to develop fast MP methods. Formula equation (1) shows that, if we want to speed up the MP of images, we have to reduce the calculation of projection of image residual $R^n f$ on atoms of D. According to Refs. [1] and [2], the atoms of D have the same size as the image to be decomposed and to calculate the projection of image residual $R^n f$ on atoms means to calculate the inner product of image residual $R^n f$ and atoms. Because the image size $M_1 \times M_2$ usually is very large, the inner product of image residual $R^n f$ and atoms is an inner product in a very high dimension space that means very huge computation complexity. In order to overcome this key problem in MP, we study the energy property of atoms.

2 Atom Energy Property

Some kinds of atoms used in MP have been presented in [2,3]. For any kind of these atoms, it's true that the energy of one atom is very concentrated in the center area of the atom just as the energy distribution in real physical atoms. As example, we take anisotropic atoms that are considered as one kind of atoms that can represent images more efficiently than other kinds of atoms[3]. In fig.1, fig.1a represents one anisotropic atom and fig.1b represents its energy distribution. As we can see, almost all energy of the atom in fig.1a is concentrated in a small area in the center of the atom, as the white area in fig.1b shows. On the other hand, in the rest area in the atom, as the black part in fig.1b shows, there is almost no energy. Other kinds of atoms have the similar energy distribution property. The reason why atom's energy is so concentrated is that functions, which construct the atoms, usually contain one exponential function that makes energy decreases very fast with the increasing distance away from the center of the atom[1, 3]. In fact, this atoms' energy distribution property is also important and necessary for efficient representation of images because only atoms with concentrated energy can represent image details. We need atoms with disperse energy (which we can call as big atoms) to represent image background, but the number of such atoms is very limited in MP of images.

(a) one atom　　　(b) energy distribution

Fig.1　atom and its energy distribution

3 Fast Algorithms

3.1 Fast Algorithm (1)

The most computational load in MP of images is the calculation of projections of image residual $R^n f$ on atoms, because the projections are inner products in a very high($M_1 \times M_2$) dimension space and calculating them is very time consuming. Since values of one atom are almost zero in most parts of the atom except at its center, the projection of image residual $R^n f$ on this atom can be transformed into projection of image residual $R^n f$ on the center part of this atom. If the center part of the atom is of area of C (usually C is much smaller than $M_1 \times M_2$), calculating inner products in a space of very high ($M_1 \times M_2$) dimension is changed to calculating inner products in a space of much lower (C) dimension. Therefore the calculation of projections of image residual $R^n f$ on atoms can be speeded up many times and, as a result, the MP of images can be speeded up many times too.

3.2 Fast Algorithm (2)

Ref.[4] proves that norm of $R^n f$, $\|R^n f\|$, converges exponentially to 0 when n tends to infinity. This means that the selected best atom will become smaller when n tends to be bigger. Therefore we can preclude bigger atoms from the dictionary D with the MP going on. According to the above fast algorithm (1), the smaller the atom is, the faster the calculation of projection of image residual $R^n f$ on the atom is. So, if we preclude bigger atoms from the dictionary D with the process of MP going on, we can improve the speed of MP a lot. In this way, we can also improve the quality of reconstructed image at the same time, because, when n tends to bigger and larger atoms are precluded, MP program will not waste time on bigger atoms and will have more time to search the best atom in a smaller region in dictionary D where the best atom is most possible to be located. On the other hand, this new method is also better for image compression because it reduces the value region of atoms' scales.

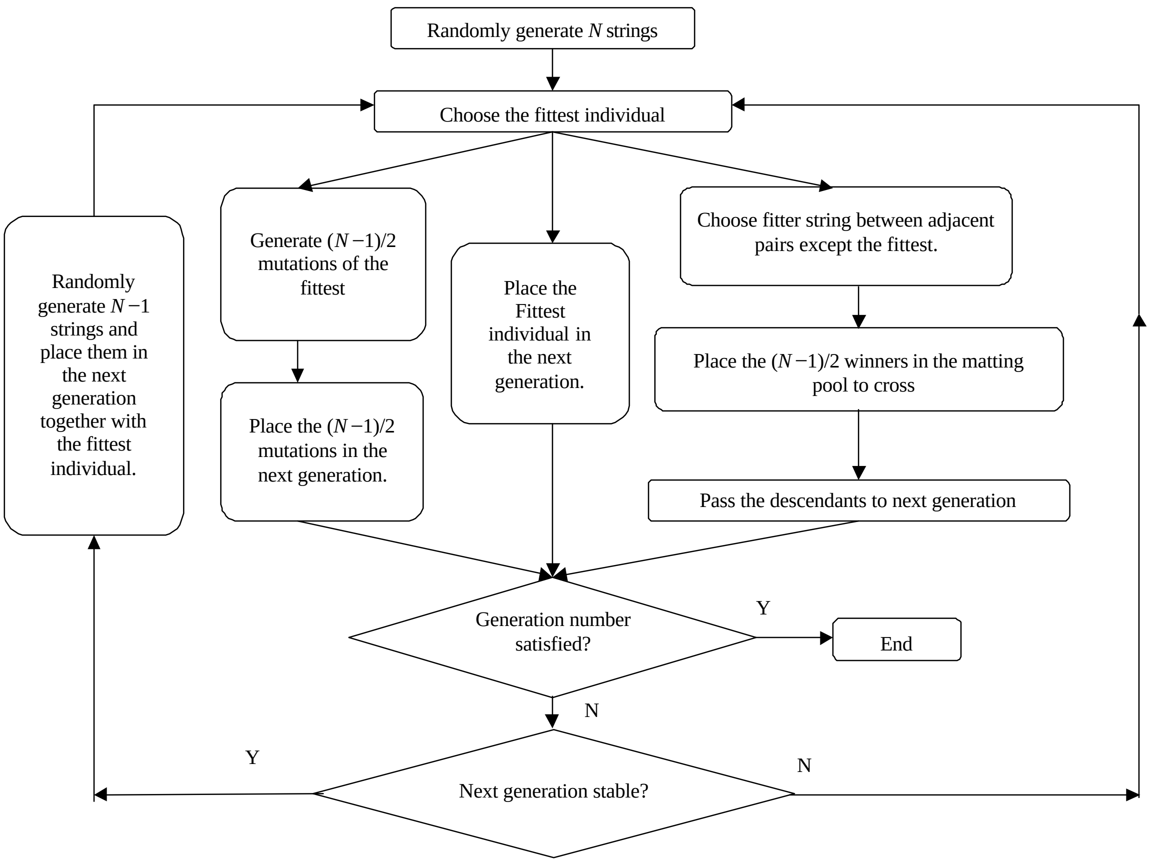

Fig.2 General block diagram of the genetic algorithm

4 Experimental Results

According to above ideas, fast MP programs have been developed. In our MP programs, Genetic Algorithm (GA) is used to search the best atoms in dictionary D, so, our algorithm is called GA based MP (GAMP). In fact, the above ideas are independent from genetic algorithm. Therefore they can be used in many other methods of MP of images. The idea of GAMP was originally presented in Ref.[5] and we have improved GAMP by making use of property of genetic algorithms. The general block diagram of the genetic algorithm is shown in fig.2.

Table 1　Speed-up of the new algorithm

Image size	128×128	256×256	512×512
Speed-up (times)	12	20	31

In our experiment, anisotropic atoms[3] are utilized to decompose Lenna image. In table 1, the first line shows image size, and the second line shows how many times faster our novel fast GAMP algorithm to decompose the corresponding image by making use of the energy property of atoms is than the conventional GAMP algorithm without making use of it. In fig.3a is the Lenna image of size 256×256 reconstructed with 500 anisotropic atoms obtained by conventional GAMP algorithm. Fig.3b is the Lenna image of same size reconstructed with the same number of anisotropic atoms obtained by our new GAMP algorithm. We can see that the new algorithm alsoslightly improves the quality of reconstructed image. By the way, speed-up times in table 1 and PSNR values in fig.3 are average speed-up times and average PSNR values respectively because of the random property of GA. Experimental results also show that this new fast algorithm is useful too for the dictionaries of other kinds of atoms in addition to the dictionary of anisotropic atoms. The speed-up when using other kinds of atoms is very similar to the speed-up when using anisotropic atoms.

(a) by conventional MP, PSNR=26.67 dB.　　(b) by new fast MP, PSNR=26.93 dB.

Fig.3　reconstructed images comparison

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编　辑　徐安玉