# 二维介质散射的T矩阵方法与解析解的一致性分析

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【摘要】基于H波入射，根据二维介质散射的边界条件，利用二维格林函数的展开式和消光定理，求得T矩阵方法构造方程式；在此基础上，对T矩阵方法的极限问题进行了系统的分析，即当散射体的边界趋于理想圆柱边界时，T矩阵方法实现了由数值解到经典解析解的极限过渡。

关　键　词　二维介质散射;　解析解;　T矩阵方法;　一致性

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# Unitary Analysis of T-Matrix Method and Analytic Solutions in 2D Dielectric Scattering

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**Abstract**　The unitary problem between numerical solutions and the analytic solutions is an important issue to value whether or not the physical nature and structure of a numerical method are reasonable in computational electromagnetics. This paper presents the structure functions of T-matrix method based on H-wave incident, boundary conditions of 2D dielectric scattering, and 2D Green's function and extinction theorem. The full analysis of T-matrix's limitation problem shows that when the boundary of a dielectric is limited to the cylindrical one, the limitation transition from the T-matrix solutions to classical ones is obtained.

**Key words**　2D dielectric scattering;　analytic solutions;　T-matrix method;　unitary

T矩阵方法于1969年引入到计算电磁学领域[1]。该方法以消光定理为基础，把边值问题化解为散射体内域和外域两个积分方程，通过边界条件联接表面流及其方向导数。在内域，表面流完全抵消入射场，从而得出边界表面流展开系数与入射场展开系数的关联矩阵；在外域，建立起表面流与散射场之间的关联矩阵，得到入射场与散射场之间的关联。因此，它又被称为扩展边界条件法(extended boundary condition method)。

T矩阵方法是解析解与数值解的一种混合运用，具有以下的显著特点：1) 它以标量格林定理为基础，从而能够消除由谐振腔模式引起的困难[2-3]；2) 它在内外域引用格林函数的解析波函数级数形式来展开表面流，极大地节省了存储[4-5]。为此，T矩阵方法在声散射、光散射以及电磁散射等领域都得到了广泛的应用[6-8]。

然而，目前对T矩阵方法的物理本质方面的分析工作还有待完善[9-10]。特别是对于数值解与解析解的一致性问题，不仅具有重要的理论意义，而且有助于有效地分析和指导数值结果。文献[11-13]完成了E波入射条件下的导体T矩阵极限过渡，但介质散射与导体散射不同。导体散射是面散射，而介质散射是体散射，其边界条件比导体散射复杂[14]。本文首先给出H波条件下介质的电磁散射T矩阵构造，然后在此基础上，分析给出当散射体的边界趋于理想边界时，完成了T矩阵方法到经典解析解的一致性过渡。

## 1　基于H波的介质散射T矩阵构造

入射波为H型平面波对介质的散射问题如图1所示。总场由入射场和散射场的叠加构成。

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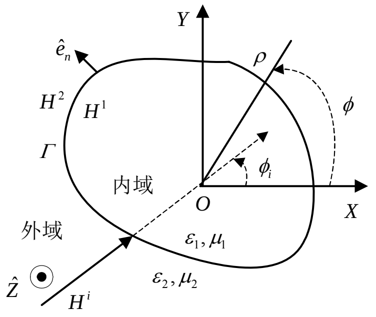


图1 二维H波介质电磁散射结构

$$H = H^i + H^s \tag{1}$$

散射体的边界为$\Gamma$，$\Gamma$内、外附近对应的场量分别为$H^1$和$H^2$，则边界衔接条件为：

$$\begin{cases} \vec{E} = \dfrac{\nabla\times\vec{H}}{\mathrm{j}\omega\varepsilon_0} = -\dfrac{\eta}{\mathrm{j}k_0}\hat{e}_n\times\nabla H \\ \hat{e}_n\times(\vec{E}^i+\vec{E}^s)|_\Gamma = -\dfrac{1}{\mathrm{j}\omega\varepsilon_0}\dfrac{\partial H}{\partial n}\Big|_\Gamma = 0 \end{cases} \tag{2}$$

外域的边界条件可写成：

$$\begin{cases} \dfrac{\partial H^1}{\partial n} = \displaystyle\sum_{m=-\infty}^{\infty}\alpha_m\frac{\partial}{\partial n}[J_m(k_1\rho)\exp(\mathrm{j}m\phi)] \\ \dfrac{\partial H^2}{\partial n} = \dfrac{\varepsilon_2}{\varepsilon_1}\displaystyle\sum_{m=-\infty}^{\infty}\alpha_m\frac{\partial}{\partial n}[J_m(k_1\rho)\exp(\mathrm{j}m\phi)] \end{cases} \tag{3}$$

利用散射体内外域的消光定理[15]：

$$\begin{cases} H^i(\vec{\rho}) + \displaystyle\int_\Gamma\left[H(\vec{\rho}')\frac{\partial G(\vec{\rho},\vec{\rho}')}{\partial n'} - G(\vec{\rho},\vec{\rho}')\frac{\partial H(\vec{\rho}')}{\partial n'}\right]\mathrm{d}l' = 0 \\ \qquad (\rho<\rho') \\ H^s(\vec{\rho}) - \displaystyle\int_\Gamma\left[H(\vec{\rho}')\frac{\partial G(\vec{\rho},\vec{\rho}')}{\partial n'} - G(\vec{\rho},\vec{\rho}')\frac{\partial H(\vec{\rho}')}{\partial n'}\right]\mathrm{d}l' = 0 \\ \qquad (\rho>\rho') \end{cases} \tag{4}$$

根据入射波和散射波的特性，可将它们分别展开成Bessel函数和Hankel函数的级数形式：

$$\begin{cases} H^i(\vec{\rho}) = \displaystyle\sum_{n=-\infty}^{\infty} a_n J_n(k_2\rho)\exp(\mathrm{j}n\phi) \\ H^s(\vec{\rho}) = \displaystyle\sum_{n=-\infty}^{\infty} b_n H_n^{(2)}(k_2\rho)\exp(\mathrm{j}n\phi) \end{cases} \tag{5}$$

利用二维格林函数的展开形式：

$$G(\vec{\rho},\vec{\rho}') = -\frac{\mathrm{j}}{4}\sum_{n=-\infty}^{\infty}\begin{cases} J_n(k\rho)H_n^{(2)}(k\rho')\exp[\mathrm{j}n(\phi-\phi')] \\ \qquad (\rho<\rho') \\ J_n(k\rho')H_n^{(2)}(k\rho)\exp[\mathrm{j}n(\phi-\phi')] \\ \qquad (\rho>\rho') \end{cases} \tag{6}$$

可以得到入射场和散射场的系数与表面量之间的关联为：

$$a_n = \frac{\mathrm{j}}{4}\sum_{m=-\infty}^{\infty}\alpha_{sm}\int_\Gamma\left\{J_m(k_1\rho')\exp(\mathrm{j}m\phi')\times\frac{\partial[H_n^{(2)}(k_2\rho')\exp(-\mathrm{j}n\phi')]}{\partial n'} - \frac{\varepsilon_2}{\varepsilon_1}[H_n^{(2)}(k_2\rho')\exp(-\mathrm{j}n\phi')]\times\frac{\partial[J_m(k_1\rho')\exp(\mathrm{j}m\phi')]}{\partial n'}\right\}\mathrm{d}l' \tag{7a}$$

$$b_n = -\frac{\mathrm{j}}{4}\sum_{m=-\infty}^{\infty}\alpha_{sm}\int_\Gamma\left\{J_m(k_1\rho')\exp(\mathrm{j}m\phi')\times\frac{\partial[J_n(k_2\rho')\exp(-\mathrm{j}n\phi')]}{\partial n'} - \frac{\varepsilon_2}{\varepsilon_1}[J_n(k_2\rho')\exp(-\mathrm{j}n\phi')]\times\frac{\partial[J_m(k_1\rho')\exp(\mathrm{j}m\phi')]}{\partial n'}\right\}\mathrm{d}l' \tag{7b}$$

写成矩阵形式为：

$$\begin{cases} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_\infty \end{bmatrix} = \begin{bmatrix} Q_{11}^- & Q_{12}^- & Q_{1\infty}^- \\ Q_{21}^- & \vdots & \vdots \\ \vdots & & \\ Q_{\infty 1}^- & \cdots & Q_{\infty\infty}^- \end{bmatrix}\begin{bmatrix} \alpha_{s1} \\ \alpha_{s2} \\ \vdots \\ \alpha_{s\infty} \end{bmatrix} \\ \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_\infty \end{bmatrix} = \begin{bmatrix} Q_{11}^+ & Q_{12}^+ & Q_{1\infty}^+ \\ Q_{21}^+ & \vdots & \vdots \\ \vdots & & \\ Q_{\infty 1}^+ & \cdots & Q_{\infty\infty}^+ \end{bmatrix}\begin{bmatrix} \alpha_{s1} \\ \alpha_{s2} \\ \vdots \\ \alpha_{s\infty} \end{bmatrix} \end{cases} \tag{8}$$

矩阵元分别为：

$$Q_{nm}^- = \frac{\mathrm{j}}{4}\int_\Gamma\left\{J_m(k_1\rho')\exp(\mathrm{j}m\phi')\times\frac{\partial[H_n^{(2)}(k_2\rho')\exp(-\mathrm{j}n\phi')]}{\partial n'} - \frac{\varepsilon_2}{\varepsilon_1}[H_n^{(2)}(k_2\rho')\exp(-\mathrm{j}n\phi')]\times\frac{\partial[J_m(k_1\rho')\exp(\mathrm{j}m\phi')]}{\partial n'}\right\}\mathrm{d}l' \tag{9a}$$

$$Q_{nm}^+ = \frac{\mathrm{j}}{4}\int_\Gamma\left\{J_m(k_1\rho')\exp(\mathrm{j}m\phi')\times\frac{\partial[J_n(k_2\rho')\exp(-\mathrm{j}n\phi')]}{\partial n'} - \frac{\varepsilon_2}{\varepsilon_1}[J_n(k_2\rho')\exp(-\mathrm{j}n\phi')]\times\frac{\partial[J_m(k_1\rho')\exp(\mathrm{j}m\phi')]}{\partial n'}\right\}\mathrm{d}l' \tag{9b}$$

根据T矩阵的定义，可以得到：

$$\begin{cases}[T]=-[Q^{+}][Q^{-}]^{-1}\\ T_{nm}=-\sum_{k=-N}^{N}Q_{nk}^{+}(Q_{km}^{-})^{-1}\end{cases} \tag{10}$$

## 2　T矩阵方法到经典解析解得过渡实现

考虑介质圆柱散射的理想情况，散射体边界和柱坐标重合。T矩阵方法是一种处理不规则边界问题的数值方法，它的的出发点不是直接利用电磁场边界条件求解问题。然而，由于它的物理本质是基于解析波函数，对于不规则边界，它可以通过解析延拓的方法来求解。所以，如果散射体横截面趋于圆，那么求解时需要延拓的面积等于零。因此在理论上，对于介质圆柱散射，T矩阵方法所得到的结果应该与经典解析解一致。此时T矩阵方法就过渡到H波入射条件下介质圆柱的解析解。

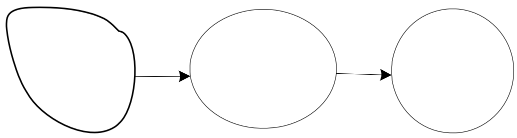


图2　任意边界向规则边界过渡

在运用T矩阵方法进行数值计算时，对于散射体边界趋于半径为 $a$ 的理想圆柱边界的问题，如图2c所示，有 $dl'=ad\phi'$，利用 $\exp(\mathrm{j}n\phi')$ 的正交性，可得：

$$Q_{nm}^{-}=\mathrm{j}\frac{\pi a}{2}\delta_{nm}\left[J_m(k_1\rho')\frac{\partial H_n^{(2)}(k_2\rho')}{\partial n'}-\frac{\varepsilon_2}{\varepsilon_1}H_n^{(2)}(k_2\rho')\frac{\partial J_m(k_1\rho')}{\partial n'}\right]\Bigg|_{\rho'=a}=$$
$$\mathrm{j}\frac{\pi a}{2}\delta_{nm}\left[k_2J_m(k_1a)H_n^{(2)\prime}(x)|_{x=k_2a}-k_1\frac{\varepsilon_2}{\varepsilon_1}H_n^{(2)}(k_2a)J_m'(x)|_{x=k_1a}\right] \tag{11a}$$

$$Q_{nm}^{+}=\mathrm{j}\frac{\pi a}{2}\delta_{nm}\left[J_m(k_1\rho')\frac{\partial J_n(k_2\rho')}{\partial n'}-\frac{\varepsilon_2}{\varepsilon_1}J_n(k_2\rho')\frac{\partial J_m(k_1\rho')}{\partial n'}\right]\Bigg|_{\rho'=a}=$$
$$\mathrm{j}\frac{\pi a}{2}\delta_{nm}\left[k_2J_m(k_1a)J_n{}'(x)|_{x=k_2a}-k_1\frac{\varepsilon_2}{\varepsilon_1}J_n(k_2a)J_m'(x)|_{x=k_1a}\right] \tag{11b}$$

根据T矩阵的定义，可以得到：

$$T_{nm}=-\sum_{k=-N}^{N}Q_{nk}^{+}(Q_{km}^{-})^{-1}=$$
$$\delta_{nm}\frac{\eta_1J_n(k_2a)J_m'(k_1a)-\eta_2J_m(k_1a)J_n'(k_2a)}{\eta_2J_m(k_1a)H_n^{(2)}(k_2a)-\eta_1H_n^{(2)}(k_2a)J_m'(k_1a)} \tag{12}$$

式中，$\eta=\sqrt{\mu/\varepsilon}$ 为波阻抗；$a_m=\exp[-\mathrm{j}m(\phi_i+\pi/2)]$ 为入射场的展开系数。因此，最终得到散射场的展开系数为：

$$b_n=\delta_{nm}\frac{[\eta_1J_n(k_2a)J_m'(k_1a)-\eta_2J_m(k_1a)J_n'(k_2a)]}{[\eta_2J_m(k_1a)H_n^{(2)\prime}(k_2a)-\eta_1H_n^{(2)}(k_2a)J_m'(k_1a)]}\times$$
$$\exp[-\mathrm{j}m(\phi_i+\pi/2)] \tag{13}$$

该结果与经典解析解一致[16]。这样，当散射体的边界趋于理想圆柱边界时，本文完成了由T矩阵方法的数值解到经典解析解的一致性过渡。在该理想情形下，T矩阵是一个对角矩阵。

## 3　结　论

尽管介质散射与导体散射的边界条件和T矩阵构造元不同，但本文研究表明，散射体边界趋于理想圆柱边界时，介质体散射也能够实现由数值解过渡到经典解析解。同时该研究还能够得出如下结论：1) 基于T矩阵方法的构造是合理的；2) T矩阵方法能够充分利用具有良好解析性态的波函数。因此，它的物理本质是解析解与扩展边界技术的混合。

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