Fuzzy Impulsive Control of the Hyperchaotic Lü System Via a Time-Dependent Lyapunov Function Approach

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Abstract　A novel analysis method for the fuzzy impulsive control of general chaotic systems is presented by using the time-dependent Lyapunov function-based technique. Compared with the time-independent Lyapunov function method, the proposed method can make full use of the information about the impulsive intervals and provides less conservative results. Being different from the existing results, the derived condition for global exponential stability of the system under consideration depends both on the upper bound and the lower bound of the impulsive intervals. This condition is expressed in terms of linear matrix inequalities (LMIs). By solving a set of LMIs, a fuzzy impulsive state feedback controller can easily be obtained. Finally, the proposed impulsive control scheme is applied to the stabilization of the hyperchaotic Lü system. The simulation results demonstrate the effectiveness of the developed method.

Key words　hyperchaotic systems;　impulse control;　time-dependent Lyapunov function;　T-S fuzzy models

超混沌Lü系统的模糊脉冲控制：时变Lyapunov函数方法

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【摘要】针对一般混沌系统模糊脉冲控制问题，提出了一种基于时变Lyapunov函数的分析方法。与时不变的Lyapunov函数方法相比，该方法能充分利用脉冲区间的信息，从而推导出具有较少保守性的结果。不同于已有的结果，所得到系统的全局指数稳定性条件同时依赖于脉冲区间的上界和下界。该稳定性条件表示为线性矩阵不等式形式。通过求解一组线性矩阵不等式，得到镇定混沌系统的模糊脉冲状态反馈控制器。提出的脉冲镇定方案应用于超混沌吕氏系统的镇定问题，所得结果证实了该方法的有效性。

关　键　词　超混沌系统;　脉冲控制;　时变Lyapunov函数;　T-S 模糊模型

中图分类号　TP273　　文献标志码　A　　doi:10.3969/j.issn.1001-0548.2013.05.014

Chaos is a typical phenomenon in many nonlinear systems whose phase has a dimension not lower than three. Since the dynamic chaos leads to irregular and unpredictable behavior, controlling chaos has been an interesting field in academic research and practical applications[1].

In recent years, fuzzy logic control based on the Takagi-Sugeno (T-S) model[2-9] has attracted great attention for complex systems. It can provide an effective solution to the model and control of plants that are complex, uncertain, and ill-defined. In the so-called T-S type fuzzy model, a complex system is

Received date：2011－10－17;　Reviced date：2012－05－10

收稿日期：2011－10－17;　修回日期：2012－05－10

Foundation items: Supported by National Natural Science Foundation of China (60864002, 61164016); Guangxi Natural Science Foundation (2011GXNSFA018141)

基金项目：国家自然科学基金(60864002, 61164016);　广西省自然科学基金((2011GXNSFA018141)

Biography：CHEN Wu-hua was born in 1967, and his research interests include time-delay systems, robust control, neural networks, impulsive systems and networked control systems.

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decomposed into several linear models according to fuzzy rules. And the complex system is approximated by the overall fuzzy linear models. Then we can apply linear control theory for analysis and synthesis of complex nonlinear systems by means of T-S fuzzy models at a certain state-space domain.

On the other hand, stability theory of impulsive systems and its application to impulsive control and impulsive synchronization of chaotic systems have gained renewed interests[10-14]. Impulsive control method allows the stabilization of a chaotic system using only small control impulses generated by samples of the state variables at discrete time instants. This drastically reduces the amount of stabilization information transmitted from the plant to the impulsive controller and increases the efficiency of bandwidth usage, which makes this method more efficient and thus useful in practical applications.

Recently, the fuzzy logical control theory combined with the impulsive control approach was applied to design the stabilization control strategy of chaotic systems. Some interesting results based on linear matrix inequalities (LMIs) or matrix norms have been reported in Ref. [15-20]. We note that the existing fuzzy impulsive control methods are based on stability analysis via time-independent Lyapunov functions. The time-independent Lyapunov function method may neglect some useful information concerning impulsive interval and thus the resulting stability criteria may be conservative.

The objective of the present paper is to develop a novel time-dependent Lyapunov-based technique for fuzzy impulsive control of chaotic systems which can be represented by T-S fuzzy models. The introduced time-dependent Lyapunov functions lead to a new stability condition of impulsive T-S systems. Different from the existing stability conditions for fuzzy impulsive systems, our new stability condition depends both on the lower bound and the upper bound of impulsive intervals, which make full use of the information about impulsive intervals. Through an example concerning fuzzy impulsive control of the hyperchaotic Lü system, we show that considering a lower bound of impulsive interval can significantly reduce conservatism. Based on the derived stability criterion, the fuzzy impulsive state feedback controller can be obtained by solving a set of LMIs.

1 Fuzzy impulsive control

Consider a continuous-time nonlinear control system of the form

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})\boldsymbol{u} \tag{1}$$

where $\boldsymbol{x} \in R^n$ is the state vector, $\boldsymbol{u} \in R^m$ is the input vector, $\boldsymbol{f}(\boldsymbol{x})$ and $\boldsymbol{g}(\boldsymbol{x})$ are continuous functions. Constructing a T-S fuzzy control model for system (1) in the following form (see Ref. [2-3]).

Rule i: IF $z_1(t)$ is F_1^i and … and $z_s(t)$ is F_s^i, then:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}_i\boldsymbol{x} + \boldsymbol{B}_i\boldsymbol{u} \qquad i = 1, 2, \cdots, r \tag{2}$$

where $F_j^i\,(j = 1, 2, \cdots, s)$ is the fuzzy set, r is the number of fuzzy rule， $z_j(t), (j = 1, 2, \cdots, s)$ is the premise variables, $\boldsymbol{x} \in R^n$ is the state vector, $\boldsymbol{u} \in R^m$ is the input vector, and $\boldsymbol{A}_i \in R^{n\times n}$ and $\boldsymbol{B}_i \in R^{n\times m}$ are known constant matrices.

By the singleton fuzzifier, product inference and the center average defuzzifier, the final output of the fuzzy system (2) for system (1) can be represented as:

$$\dot{\boldsymbol{x}} = \sum_{i=1}^{r} h_i(\boldsymbol{z}(t))(\boldsymbol{A}_i\boldsymbol{x} + \boldsymbol{B}_i\boldsymbol{u}) \tag{3}$$

where $\boldsymbol{z}(t) = (z_1(t), z_2(t), \cdots, z_s(t)), h_i(\boldsymbol{z}(t)) = \omega_i(\boldsymbol{z}(t))/\varpi(t)$ with $\varpi(t) = \sum_{i=1}^{r} \omega_i(\boldsymbol{z}(t))$ and $\omega_i(\boldsymbol{z}(t)) = \prod_{j=1}^{s} F_j^i(z_j(t))$,

$F_j^i(z_j(t))$ denotes the grade of membership of $z_j(t)$ in F_j^i. Note that:

$$\sum_{i=1}^{r} h_i(\boldsymbol{z}(t)) = 1 \qquad h_i(\boldsymbol{z}(t)) \geqslant 0 \qquad i = 1, 2, \cdots, r \tag{4}$$

for all t, $h_i(\boldsymbol{z}(t))$ can be regarded as the normalized weight of the IF-THEN rules.

Fuzzy impulsive controllers for stabilizing the fuzzy system (3) can be designed via parallel distributed control (PDC)[4]. In PDC, fuzzy impulsive controllers share the same premise parts with system (3). That is, the impulsive controller for Rule i is:

IF $z_1(t)$ is F_i^1 and …, $z_s(t)$ is F_i^r, then:

$$\boldsymbol{u}(t) = \sum_{k=1}^{\infty} \delta(t - t_k)\boldsymbol{K}_i\boldsymbol{x}(t^-) \qquad i = 1, 2, \cdots, r \tag{5}$$

where $\delta(t)$ is the Dirac delta function, $\boldsymbol{K}_i \in R^{m\times n}$ are the impulsive feedback gains, $\{t_k\}$ is an impulsive time sequence satisfying $0 = t_0 < t_1 < \ldots < t_k < \ldots$ with

$\lim_{k\to\infty} t_k = \infty$, $\boldsymbol{x}(t^-)$ denotes the limit from the left at time t. Then, the overall output of this fuzzy impulsive controller is:

$$\boldsymbol{u}(t) = \sum_{i=1}^{r} h_i(\boldsymbol{z}(t)) \sum_{k=1}^{\infty} \delta(t-t_k)\boldsymbol{K}_i \boldsymbol{x}(t^-) \tag{6}$$

where $h_i(\boldsymbol{z}(t))$ is the same as that of the $\boldsymbol{i}$ th rule of the fuzzy system (3). By substituting system (6) into system (3), we get:

$$\dot{\boldsymbol{x}}(t) = \sum_{i=1}^{r} h_i(\boldsymbol{z}(t))(\boldsymbol{A}_i\boldsymbol{x}(t) + \boldsymbol{B}_i\boldsymbol{K}_i \sum_{k=1}^{\infty}\delta(t-t_k)\boldsymbol{x}(t^-)) \tag{7}$$

Integrating system (7) on both sides from $t_k - h$ to $t_k + h$, and letting $h \to 0^+$, we can obtain:

$$\Delta\boldsymbol{x}(t_k) \triangleq \boldsymbol{x}(t_k^+) - \boldsymbol{x}(t_k^-) = \sum_{i=1}^{r} h_i(\boldsymbol{z}(t_k))\boldsymbol{B}_i\boldsymbol{K}_i\boldsymbol{x}(t_k^-)$$

where $\boldsymbol{x}(t^+)$ denotes the limit from the right at time t. Here, we assume that $\boldsymbol{x}(t^+) = \boldsymbol{x}(t)$. Therefore, we can represent the fuzzy system (7) as the following equivalent form:

$$\begin{cases} \dot{\boldsymbol{x}} = \sum_{i=1}^{r} h_i(\boldsymbol{z}(t))\boldsymbol{A}_i\boldsymbol{x}(t) & t \neq t_k \\ \Delta\boldsymbol{x}(t) = \sum_{i=1}^{r} h_i(\boldsymbol{z}(t))\boldsymbol{B}_i\boldsymbol{K}_i\boldsymbol{x}(t^-) & t = t_k, k \in N \end{cases} \tag{8}$$

where N denotes the positive integer set.

2 Fuzzy impulsive controller design

In this section, we will discuss the design of fuzzy impulsive stabilizing controller for fuzzy system (8). There are some works[15-19] on the stability analysis of the fuzzy impulsive system (8), where time-independent Lyapunov functions are used. To exploit more information on the impulsive intervals, we develop a time-dependent Lyapunov function approach to analyze the stability of system (8). For this purpose, let us introduce some notations.

For given positive scalars τ_1, τ_2, τ, where $\tau_1 \leqslant \tau_2$, we use the notations $S(\tau_1, \tau_2)$ and $S(t)$ to denote the class of impulsive time sequences that satisfy $\tau_1 \leqslant t_{k+1} - t_k \leqslant \tau_2$. For given impulsive time sequence $\{t_k\} \in S(\tau_1, \tau_2)$, we introduce the following two piecewise linear functions $\rho, \rho_1 : [t_0, \infty) \to R^+$:

$$\rho_1(t) = 1/(t_{k+1} - t_k) \quad \rho(t) = (t - t_k)\rho_1(t) \tag{9}$$

for $t \in [t_k, t_{k+1})$. It is clear that $\rho(t) \in [0,1]$, and there exists a function $\rho_2(t) \in [0,1]$ such that:

$$\rho_1(t) = (1 - \rho_2(t))/\tau_1 + \rho_2(t)/\tau_2 \tag{10}$$

A sufficient condition for the existence of local state feedback gains $\boldsymbol{K}_i$ is provided in the following theorem.

Theorem 1 Consider the fuzzy impulsive system (8) with $\{t_k\} \in S(\tau_1, \tau_2)$. For given $\mu \in (0,1]$, if there exist $n \times n$ matrices $\boldsymbol{P}_1 > 0$ and $\boldsymbol{P}_2 > 0$ such that the following LMIs are satisfied:

$$\boldsymbol{A}_i^{\mathrm{T}}\boldsymbol{P}_j + \boldsymbol{P}_j\boldsymbol{A}_i + \frac{\ln\mu}{\tau_2}\boldsymbol{P}_1 + \frac{1}{\tau_l}(\boldsymbol{P}_2 - \boldsymbol{P}_1) < 0 \tag{11}$$

$$\begin{bmatrix} -\mu\boldsymbol{P}_2 & (\boldsymbol{I} + \boldsymbol{B}_i\boldsymbol{K}_i)^{\mathrm{T}}\boldsymbol{P}_1 \\ * & -\boldsymbol{P}_1 \end{bmatrix} \leqslant 0 \tag{12}$$

for $i = 1, 2, \cdots, r$, $j, l = 1, 2$, then the fuzzy impulsive system (8) is globally exponentially stable.

Proof: By system (11), for some scalar $\theta > 1$, we have:

$$\boldsymbol{N}_{jl} \triangleq \boldsymbol{A}_i^{\mathrm{T}}\boldsymbol{P}_j + \boldsymbol{P}_j\boldsymbol{A}_i + \frac{\ln\theta\mu}{\tau_2}\boldsymbol{P}_1 + \frac{1}{\tau_l}(\boldsymbol{P}_2 - \boldsymbol{P}_1) < 0 \tag{13}$$

for $i = 1, 2, \cdots, r$, $j, l = 1, 2$, Let $\boldsymbol{P}(t) = \boldsymbol{P}_1 + \rho(t)(\boldsymbol{P}_2 - \boldsymbol{P}_1)$, where $\rho(t)$ is defined in system (9). We claim that the inequalities in system
(13) imply:

$$\boldsymbol{P}(t)\boldsymbol{A}_i + \boldsymbol{A}_i^{\mathrm{T}}\boldsymbol{P}(t) + \rho_1(t)(\boldsymbol{P}_2 - \boldsymbol{P}_1) + \frac{\ln(\theta\mu)}{\tau_2}\boldsymbol{P}(t) < 0 \tag{14}$$

In fact, by system (10) and system (13), we have:

$$\boldsymbol{P}(t)\boldsymbol{A}_i + \boldsymbol{A}_i^{\mathrm{T}}\boldsymbol{P}(t) + \rho_1(t)(\boldsymbol{P}_2 - \boldsymbol{P}_1) + \frac{\ln(\theta\mu)}{\tau_2}\boldsymbol{P}(t) =$$
$$(1 - \rho_2(t))[(1 - \rho(t))\boldsymbol{N}_{11} + \rho(t)\boldsymbol{N}_{21}] +$$
$$\rho_2(t)[(1 - \rho(t))\boldsymbol{N}_{12} + \rho(t)\boldsymbol{N}_{22}] < 0$$

Now we choose a Lyapunov function for system (8) as $V(t, \boldsymbol{x}) = \boldsymbol{x}^{\mathrm{T}}\boldsymbol{P}(t)\boldsymbol{x}$. Set $V(t) = V(t, \boldsymbol{x}(t))$.

For $t \in (t_k, t_{k+1})$, the derivative of $V(t)$ along the trajectory of system (8) is:

$$\dot{V}(t) = \sum_{i=1}^{r} h_i(\boldsymbol{z}(t))\boldsymbol{x}^{\mathrm{T}}(t)(\boldsymbol{A}_i^{\mathrm{T}}\boldsymbol{P}(t) + \boldsymbol{P}(t)\boldsymbol{A}_i +$$
$$\rho_1(t)(\boldsymbol{P}_2 - \boldsymbol{P}_1))\boldsymbol{x}(t)$$

Then, it follows from system (4) and system (14) that:

$$\dot{V}(t) \leqslant -\frac{\ln(\theta\mu)}{\tau_2}\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}(t)\boldsymbol{x}(t) = -\frac{\ln(\theta\mu)}{\tau_2}V(t)$$

for $t \in (t_k, t_{k+1})$. This implies that:

$$V(t) \leqslant V(t_k)\exp(-\frac{\ln(\theta\mu)}{\tau_2}(t - t_k)) \quad t \in [t_k, t_{k+1}) \tag{15}$$

On the other hand, from system (4) and system (12), it is easy to see that:

$$\begin{bmatrix} -\mu \boldsymbol{P}_2 & \sum_{i=1}^{r} h_i(\boldsymbol{z}(t))(\boldsymbol{I}+\boldsymbol{B}_i\boldsymbol{K}_i)^{\mathrm{T}}\boldsymbol{P}_1 \\ * & -\boldsymbol{P}_1 \end{bmatrix} \leqslant 0$$

Then using Schur complement, we have:

$$-\mu \boldsymbol{P}_2 + (\sum_{i=1}^{r} h_i(\boldsymbol{z}(t))(\boldsymbol{I}+\boldsymbol{B}_i\boldsymbol{K}_i)^{\mathrm{T}})\boldsymbol{P}_1 \times (\sum_{i=1}^{r} h_i(\boldsymbol{z}(t))(\boldsymbol{I}+\boldsymbol{B}_i\boldsymbol{K}_i)) \leqslant 0 \tag{16}$$

Noticing that $\boldsymbol{P}(t_k)=\boldsymbol{P}_1$ and $\boldsymbol{P}(t_k^-)=\boldsymbol{P}_2$, it follows from system (16) that:

$$V(t_k)=\boldsymbol{x}^{\mathrm{T}}(t_k)\boldsymbol{P}_1\boldsymbol{x}(t_k)= \boldsymbol{x}^{\mathrm{T}}(t_k^-)(\sum_{i=1}^{r} h_i(\boldsymbol{z}(t))(\boldsymbol{I}+\boldsymbol{B}_i\boldsymbol{K}_i)^{\mathrm{T}})\boldsymbol{P}_1 \times (\sum_{i=1}^{r} h_i(\boldsymbol{z}(t))(\boldsymbol{I}+\boldsymbol{B}_i\boldsymbol{K}_i))\boldsymbol{x}(t_k^-) \leqslant \mu \boldsymbol{x}^{\mathrm{T}}(t_k^-)\boldsymbol{P}_2\boldsymbol{x}(t_k^-)=\mu V(t_k^-) \tag{17}$$

For any $t>0$, there exists $k\in N$ such that $t\in[t_k, t_{k+1})$. Combining system (15) and system (17) together yields:

$$V(t)\leqslant \mu^k V(t_0)\exp(-\frac{\ln(\theta\mu)}{\tau_2}(t-t_0))$$

Considering $k \geqslant (t-t_0)/\tau_2 - 1$, we have:

$$V(t)\leqslant \frac{1}{\mu} V(t_0)\exp(-\frac{\ln\theta}{\tau_2}(t-t_0))$$

which implies that the fuzzy impulsive system (8) is globally exponentially stable.

Remark 1 In the communication systems based on impulsive control, the efficiency of bandwidth utilization can be improved by increasing impulsive intervals. Thus, a larger upper bound τ_2 on the impulsive intervals for which the impulsively controlled system is of globally exponential stability is of interest. It is noted that the LMIs in Theorem 1 are affine in $1/\tau_2$.When the lower bound τ_1 of impulsive interval is known, the allowable maximum value of τ_2 can be derived by solving the LMIs in Theorem 1 with tuning the parameter $\mu\in(0,1]$.

Corollary 1 Consider the fuzzy impulsive system (8) with $\{t_k\}\in S(0,\tau)$. For given $\mu\in(0,1]$, if there exists a $n\times n$ matrix $\boldsymbol{P}>0$ such that the following linear matrix inequalities (LMIs) are satisfied:

$$\boldsymbol{A}_i^T\boldsymbol{P}+\boldsymbol{P}\boldsymbol{A}_i+\frac{\ln\mu}{\tau}\boldsymbol{P}<0 \tag{18}$$

$$\begin{bmatrix} -\mu\boldsymbol{P} & (\boldsymbol{I}+\boldsymbol{B}_i\boldsymbol{K}_i)^{\mathrm{T}}\boldsymbol{P} \\ * & -\boldsymbol{P} \end{bmatrix}\leqslant 0 \tag{19}$$

for $i=1,2,\cdots,r$, then the fuzzy impulsive system (8) is globally exponentially stable.

Proof: Suppose that conditions (18)～(19) are feasible. Choosing $\boldsymbol{P}_2=\boldsymbol{P}_1=\boldsymbol{P}$ and $\tau_2=\tau$, then one can verify that conditions (11)-(12) in Theorem 1 are also feasible for all scalars τ_1 satisfying $0<\tau_1\leqslant\tau_2$.

Remark 2 Corollary 1 is the LMI version of the Theorem 1 in Ref. [16]. Note that the condition of Corollary 1 does not take into consideration specific knowledge on low bound of the impulsive intervals. This results in more conservative stability conditions and consequently lower values for upper bound of the impulsive interval. We note that condition (19) implies that $\rho(\boldsymbol{I}+\boldsymbol{B}_i\boldsymbol{K}_i)\leqslant\sqrt{\mu}<1$ for $i=1,2,\cdots,r$, while this restriction is unnecessary in condition (12).

The next Theorem presents a solution to the fuzzy impulsive control problem for the system (8).

Theorem 2 Consider the fuzzy impulsive system (8) with $\{t_k\}\in S(\tau_1,\tau_2)$. For given positive scalars $\mu\in(0,1]$, ε_1 and ε_2, if there exist $n\times n$ matrices $\boldsymbol{X}_j>0$, $j=1,2$, $m\times n$ matrices $\bar{\boldsymbol{K}}_i$, $i=1,2,\cdots,r$, such that the following linear matrix inequalities (LMIs) are satisfied:

$$\begin{bmatrix} \boldsymbol{X}_1\boldsymbol{A}_i^{\mathrm{T}}+\boldsymbol{A}_i\boldsymbol{X}_1+\frac{\ln\mu}{\tau_2}\boldsymbol{X}_1-\frac{1}{\tau_l}\boldsymbol{X}_1 & \frac{1}{\sqrt{\tau_l}}\boldsymbol{X}_1 \\ * & -\boldsymbol{X}_2 \end{bmatrix}<0 \tag{20}$$

$$\boldsymbol{X}_2\boldsymbol{A}_i^{\mathrm{T}}+\boldsymbol{A}_i\boldsymbol{X}_2+\frac{\ln\mu}{\tau_2}\boldsymbol{X}_2+\frac{1}{\tau_l}\boldsymbol{X}_2+\frac{1}{\tau_l}(-2\varepsilon_1\boldsymbol{X}_2+\varepsilon_1^2\boldsymbol{X}_1)<0 \tag{21}$$

$$\begin{bmatrix} -\mu\boldsymbol{X}_2 & \boldsymbol{X}_2+\bar{\boldsymbol{K}}_i^{\mathrm{T}}\boldsymbol{B}_i^{\mathrm{T}} \\ * & -\boldsymbol{X}_1 \end{bmatrix}\leqslant 0 \tag{22}$$

for $i=1,2,\cdots,r$, $l=1,2$, then system (8) with $\boldsymbol{K}_i=\bar{\boldsymbol{K}}_i^{\mathrm{T}}\boldsymbol{X}_2^{-1}$ is globally exponentially stable.

Proof: Define $\boldsymbol{K}_i=\bar{\boldsymbol{K}}_i\boldsymbol{P}_2$, $\boldsymbol{P}_j=\boldsymbol{X}_j^{-1}$. Pre- and Post-multiplying both sides of the LMIs in system (20) with $\mathrm{diag}\{\boldsymbol{P}_1,\boldsymbol{P}_2\}$, the LMIs in system (21) with $\boldsymbol{P}_2$, the LMIs in system (22) with $\mathrm{diag}\{\boldsymbol{P}_2,\boldsymbol{P}_1\}$, and using Schur complement and the matrix inequality:

$$-\boldsymbol{P}_1\leqslant -2\varepsilon_j\boldsymbol{P}_2+\varepsilon_j^2\boldsymbol{P}_2\boldsymbol{P}_1^{-1}\boldsymbol{P}_2 \qquad j=1,2$$

we obtain system (11)～(12). Since all the conditions of Theorem 1 are satisfied, system (8) with $\boldsymbol{K}_i = \bar{\boldsymbol{K}}_i \boldsymbol{X}_2^{-1}$ is globally exponentially stable.

3 Fuzzy impulsive stabilization of the hyperchaotic Lü system

In this section, we apply the proposed method to the fuzzy impulsive stabilization of the hyperchaotic Lü system [20], which will be used as a simulation platform in this paper. The dynamics of the hyperchaotic Lü system are described by:

$$\begin{cases} \dot{x}(t) = a(y(t) - x(t)) + w(t) \\ \dot{y}(t) = -x(t)z(t) + cy(t) \\ \dot{z}(t) = x(t)y(t) - bz(t) \\ \dot{w}(t) = x(t)z(t) + dw(t) \end{cases} \tag{23}$$

where $x(t), y(t), z(t), w(t)$ are state variables. and $a = 36,\ b = 3,\ c = 20,\ d = 1.3$.

In order to construct a T-S fuzzy model, the range of state variable x is set to be $[-m, m]$ with $m = 40$. These boundaries are determined from the numerical simulation of the chaotic attractor. Using the exact T-S fuzzy modeling method in Ref. [16], system (23) can be represented as follows:

Rule i: IF x is F_i, then $\dot{\xi} = \boldsymbol{A}_i \xi$, $i = 1, 2$, where $\xi = (x, y, z, w)^{\mathrm{T}}$:

$$\boldsymbol{A}_1 = \begin{bmatrix} -a & a & 0 & 1 \\ 0 & c & m & 0 \\ 0 & -m & -b & 0 \\ 0 & 0 & -m & d \end{bmatrix}$$

$$\boldsymbol{A}_2 = \begin{bmatrix} -a & a & 0 & 1 \\ 0 & c & m & 0 \\ 0 & m & -b & 0 \\ 0 & 0 & -m & d \end{bmatrix} \tag{24}$$

and the membership functions are:

$$F_1(x) = \frac{x + m}{2m}, \quad F_2(x) = \frac{-x + m}{2m} \tag{25}$$

The defuzzified output of the T-S fuzzy model of the hyperchaotic Lü system (23) with the membership functions (25) can be can be represented as:

$$\dot{\xi}(t) = \sum_{i=1}^{2} F_i(\boldsymbol{x}(t)) \boldsymbol{A}_i \xi(t) \tag{26}$$

One can verify that the fuzzy system (26) exactly represents the hyperchaotic Lü system (23).

The T-S model of the controlled system (23) is:

Rule i: IF x is F_i, then $\dot{\xi} = \boldsymbol{A}_i \xi + \boldsymbol{B}_i \boldsymbol{u}$, $i = 1, 2$, where $\boldsymbol{A}_i$ and $F_i, i = 1, 2$, are defined in system (24) and system (25), respectively. From system (5), the fuzzy impulsive controller can be represented as follows:

Control Rule i: IF x is F_i, then:

$$\boldsymbol{u}(\boldsymbol{t}) = \sum_{k=1}^{\infty} \delta(t - t_k) \boldsymbol{K}_i \xi(t^-) \quad i = 1, 2 \tag{27}$$

Then the overall fuzzy impulsive control system of system (23) can be rewritten in the form of system (8) with $r = 2$.

First, we assume that $\boldsymbol{B}_1 = \boldsymbol{B}_2 = \boldsymbol{I}$, and $\boldsymbol{K}_1$ = –diag{0.5, 0.6, 0.7, 0.6}, $\boldsymbol{K}_2$ = – diag{0.5, 0.7, 0.7,0.5}. For $\{t_k\} \in S(0, \tau)$, by Theorem 1 in Ref. [16] or our Corollary 1 with $\mu = 0.26$, the allowable maximum value of τ is 0.033 6. Now we assume that $\{t_k\} \in S(\tau_1, \tau_2)$ and the lower bound τ_1 of impulsive interval is known. When τ_1 is chosen to be 0.001, 0.005, 0.01 and 0.02 successively, by applying our Theorem 1 with μ =0.25, 0.22, 0.19, and 0.16 successively, it has been found the corresponding maximum value of τ_2 is 0.043 6, 0.044 6, 0.045 4 and 0.045 8, respectively. This shows that when the specific knowledge on the lower bound of impulsive interval is known, the value for upper bound of impulsive interval obtained from Theorem 1 is significantly larger than that obtained in Ref. [16].

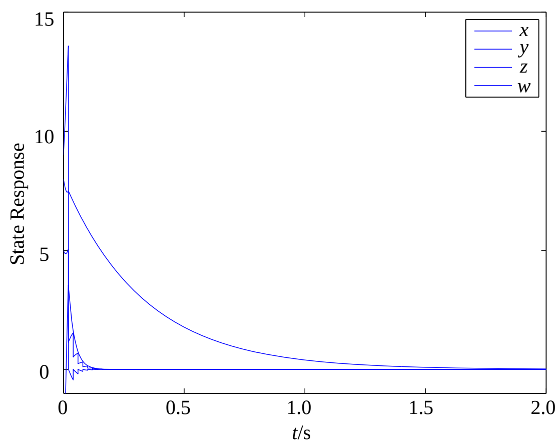

Fig.1　State trajectory of the controlled hyperchaotic system

Next we assume that:

$$\boldsymbol{B}_1 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \qquad \boldsymbol{B}_2 = \begin{bmatrix} 0 & 0 \\ 0.9 & 0 \\ 0 & 0 \\ 0 & 0.8 \end{bmatrix}$$

Without lose of generality, we assume that the impulses are equidistant, i.e., $\{t_k\}\in S(\tau,\tau)$. By applying our Theorem 2, choosing $\mu=0.62$, $\varepsilon_1=\varepsilon_2=1$, it has been found that when $\tau\leqslant 0.022\,8$, system can be impulsively stabilizable. The corresponding impulsive gain matrices are:

$$\boldsymbol{K}_1=\begin{bmatrix}0.003\,5 & -1 & 0 & 0\\ 0.319\,9 & 0.000\,1 & 0 & -1.000\,1\end{bmatrix}$$

$$\boldsymbol{K}_2=\begin{bmatrix}0.003\,9 & -1.111\,1 & 0 & 0\\ 0.399\,9 & 0.000\,1 & 0 & -1.250\,1\end{bmatrix} \quad (28)$$

With the initial value $\boldsymbol{\xi}_0=(-5,9,8,5)^{\mathrm{T}}$, the simulation results are shown in Fig.1. One can see that under the fuzzy impulsive control law system (27) with the gain matrices in system (28), the trajectory asymptotically approaches zero with 2 s.

4 Conclusions

A time-dependent Lyapunov function method has been introduced to the fuzzy impulsive control of chaotic systems represented by T-S models. The main advantage of the new method is that it enables to exploit more information on impulsive interval that is neglected by time-independent Lyapunov function methods. Furthermore, the fuzzy impulsive controller can be derived by solving a set of LMIs. Finally, a numerical example concerning the fuzzy impulsive control of the hyperchaotic Lü system has shown the effectiveness and the superiority of the developed design method.

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编 辑 黄 莘