A Fast Method for the Optimization of Polarimetric Contrast Enhancement in Partially Polarized Condition

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Abstract A general signal to clutter plus noise ratio (SCNR) model containing the partially polarized condition is created. Based on this SCNR model, a fast method for the optimization of the polarimetric contrast enhancement (OPCE) problem with constrained transmitted and received polarization is proposed. The method proves the OPCE problem equivalent to the maximization of a linear cost function. The solving of the maximization of the function is simpler than that of the OPCE problem. Hence, the faster performance searching is achieved. The method is theoretically deduced. The numerical experiments demonstrate the effectiveness of this method. Compared with the conventional global search method (GSM) based on three-step method, the proposed method costs less than 5% of the calculation time.

Key words Kennaugh matrix; optimization of polarimetric contrast enhancement (OPCE); polarization ratio; polarization state

部分极化条件下的极化对比度增强优化的快速方法

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【摘要】介绍了一种包含完全极化情形和部分极化情形在内的通用信号杂波噪声比(SCNR)模型。基于该模型，提出了一种适用于收发极化状态受约束的极化对比度增强优化(OPCE)的快速方法。该方法证明OPCE问题等价于某类线性代价函数的极值问题，且该类线性代价函数的极值问题的求解比OPCE问题的求解容易。从而构建了快速解决OPCE问题的方法。理论分析和数值实验验证了该方法的可靠性和高效性，与基于三步法的全局搜索方法(GSM)相比，该方法仅需要5%的计算时间。

关 键 词 Kennaugh矩阵; 极化对比度增强优化(OPECE); 极化率; 极化状态

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The problem of optimally selecting polarization states of the transmitted waveform is extensively studied as it can enhance the performance in target detection, tracking and identification[1-3]. Scattering properties of targets and clutter are polarization-sensitive; hence, the benign applications of polarization can enhance the polarimetric power contrast. They are known as the optimization polarimetric contrast enhancement (OPCE) problem [4-6].

In partially polarized condition, Sinclair matrix cannot provide the whole polarization information. To cope with the OPCE problem in this condition, Kennaugh matrix which provides the whole polarization information is applied. Usually, there are not analytic solutions to the OPCE problem, and numerical methods are applied. Among those methods, the global search method (GSM), which searches the overall two-dimensional polarization space, is the most common used one[7-8]. This kind of method is time-consuming; especially when fast real-time signal processing is required such as in an accurate tracking of high maneuvering target scenario. Aiming to

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Biography：LEI Shi-wen was born in 1985，and his research interests include radar signal and information processing, polarimetric information processing, adaptive signal processing, etc.

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expedite the process of enhancing the desired targets versus the clutter and noise, various fast methods are proposed. Ref.[9-10] proposed iterative numerical methods for the completely polarized condition 0 and the partially polarized condition 0. Those methods are faster than the GSM. However, the method in 0 can only be used in the condition in which the relationship between the transmitted and received polarization states is not constrained. Ref.[11-12] proposed methods based on polarization ellipse parameters. Those methods require for the information of the angle consisting of the target and the clutter on the Poincare sphere frame as well as the sphere center.

In the paper, assuming the Kennaugh matrices is measured, we emphasize on how to fast solve the OPCE problem in partially polarized condition with constrained transmitted and received polarization. We first introduce a general signal to clutter plus noise ratio (SCNR) model for the partially polarized condition. Then an OPCE problem is deduced and a fast method for the OPCE problem is proposed. The method always converges to the optimal result. It expedites the computation of the OPCE problem by converting the problem into an equivalent maximization linear function.

1 Signal to Clutter Plus Noise Ratio Model

In this section, a model of signal to clutter plus noise ratio (SCNR) defined by the Stokes vector and Kennaugh matrix is created. Stokes vector is defined as

$$\boldsymbol{J}=\begin{bmatrix} g_0 & g_1 & g_2 & g_3 \end{bmatrix}^{\mathrm{T}} \tag{1}$$

where,

$$\begin{cases} g_0=\left\langle \left|E_H\right|^2\right\rangle+\left\langle \left|E_V\right|^2\right\rangle, g_1=\left\langle \left|E_H\right|^2\right\rangle-\left\langle \left|E_V\right|^2\right\rangle \\ g_2=2\left\langle \left|E_H\right|\cdot\left|E_V^*\right|\right\rangle\cos(\phi), g_3=2\left\langle \left|E_H\right|\cdot\left|E_V^*\right|\right\rangle\sin(\phi) \end{cases} \tag{2}$$

$|E_i|$ ($i=H,V$) denotes the amplitude of electric filed in horizontal and vertical directions. $\langle\cdot\rangle$ denotes the ensemble mean. ϕ is the phase difference. In the backward scattering alignment (BSA) convention, polarization received power is defined as 0,

$$P=(kr)^{-2}/2F\boldsymbol{J}_r^{\mathrm{T}}\boldsymbol{K}\boldsymbol{J}_t^{\mathrm{T}} \tag{3}$$

where k is the wave number, r is the distance between the target and the transceiver, $\boldsymbol{K}$ is the Kennaugh matrix, and $\boldsymbol{J}_r$ and $\boldsymbol{J}_t$ are the received and transmitted Stokes vectors, respectively. The superscript "T" denotes the transpose matrix. The factor F is expressed as

$$F(\lambda,\theta,\varphi)=\lambda/(8\pi\eta)G(\theta,\varphi)/|\boldsymbol{E}^r|^2 \tag{4}$$

where λ is the received wavelength, θ and φ are the spherical coordinates of the antenna pointing direction, η is the free space impedance, G and E are the antenna gain and received electric field strength, respectively.

The received power includes the power of the targets, the clutter and the noise, expressed separately as P_T, P_C and P_N. SCNR can be defined as

$$\mathrm{SCNR}=\frac{r_C^2F_C+r_N^2F_N}{r_T^2F_T}\cdot\frac{\boldsymbol{J}_{rT}^{\mathrm{T}}\boldsymbol{K}_T\boldsymbol{J}_{tT}}{\boldsymbol{J}_{rC}^{\mathrm{T}}\boldsymbol{K}_C\boldsymbol{J}_{tC}+\boldsymbol{J}_{rN}^{\mathrm{T}}\boldsymbol{K}_N\boldsymbol{J}_{tN}} \tag{5}$$

where $\boldsymbol{J}_{ri}$ ($i=T,C,N$) and $\boldsymbol{J}_{ti}$ ($i=T,C,N$) denote the received and transmitted Stokes vectors of the targets, clutter, and noise, respectively. A general scattering polarization echo may include the completely polarized component and the unpolarized component; the Stokes vector in (1) can be rewritten as

$$\boldsymbol{J}=\underbrace{\begin{bmatrix}\sqrt{g_1^2+g_2^2+g_3^2}\\ g_1\\ g_2\\ g_3\end{bmatrix}}_{\text{completely polarzed component}}+\underbrace{\begin{bmatrix}g_0-\sqrt{g_1^2+g_2^2+g_3^2}\\ 0\\ 0\\ 0\end{bmatrix}}_{\text{unpolarized component}} \tag{6}$$

The ratio of the completely polarized power to the total power is defined as the polarization ratio

$$p=\sqrt{g_1^2+g_2^2+g_3^2}\Big/g_0 \tag{7}$$

where $0\leq p\leq 1$. Hence, (6) can be normalized as

$$\boldsymbol{J}=g_0\left\{p\underbrace{\begin{bmatrix}1\\ \cos(2\varepsilon)\cos(2\tau)\\ \cos(2\varepsilon)\sin(2\tau)\\ \sin(2\varepsilon)\end{bmatrix}}_{\text{completely polarzed component}}+\underbrace{\begin{bmatrix}1-p\\ 0\\ 0\\ 0\end{bmatrix}}_{\text{unpolarized component}}\right. \tag{8}$$

where ε and τ are the polarization ellipse angle and the polarization inclination angle, respectively.

Let $\boldsymbol{J}_D$ represent a unit polarization state,

$$\boldsymbol{J}_D=\begin{bmatrix}1\\ \vec{g}\end{bmatrix}=\begin{bmatrix}1\\ d_1\\ d_2\\ d_3\end{bmatrix}=\begin{bmatrix}1\\ \cos(2\varepsilon)\cos(2\tau)\\ \cos(2\varepsilon)\sin(2\tau)\\ \sin(2\varepsilon)\end{bmatrix} \tag{9}$$

hence, $d_1^2+d_2^2+d_3^2=1$.

In the detection period, the polarization direction of the antenna to the target, clutter, and noise are the same. Hence, SCNR defined in (5) can be rewritten as

$$\mathrm{SCNR} = \mathrm{SCR} \cdot (\boldsymbol{J}_{rD}^{\mathrm{T}} \cdot \boldsymbol{K}_{Tp} \cdot \boldsymbol{J}_{tD}) / (\boldsymbol{J}_{rD}^{\mathrm{T}} \cdot \boldsymbol{K}_{CNp} \cdot \boldsymbol{J}_{tD}) \quad (10)$$

where SCR denotes signal to clutter ratio, K_{CNP} is the sum of the clutter Kennaugh matrix and the noise Kennaugh matrix. $\boldsymbol{J}_{rD}$ and $\boldsymbol{J}_{tD}$ denote the received and the transmitted polarization states, respectively. The OPCE problem is to select the optimal polarization states to maximize the SCNR in (10).

2 Polarization Optimization Method

In this section, the OPCE problem based on the SCNR is created. Then a polarization states optimization method is proposed.

2.1 Problem Formulation

Considering the radar system receives the echoes of the targets embedded in clutter and noise background, the maximization of the SCNR is choosing the optimization criterion to design antenna. For simplicity, we assume a co-polar condition, i.e., the transmitted Stokes vector is the same to the received Stokes vector (other transmitted and received polarization relationship can be realized by matrix rotation). Hence,

$$\boldsymbol{J}_{rD} = \boldsymbol{J}_{tD} = \boldsymbol{J}_D \quad (11)$$

The fundamental principle of optimal reception is to adaptively adjust the polarization states to maximize the SCNR. The OPCE problem is then converted to be the optimization problem,

$$\begin{aligned} &\max \quad \mathrm{SCNR}(\boldsymbol{J}_D) \\ &\text{s.t.} \quad \left\| d_1^2 + d_2^2 + d_3^2 \right\| = 1 \end{aligned} \quad (12)$$

Constituting (9) and (10) into (12), the optimization problem is transformed to be,

$$\begin{aligned} &\max \quad \mathrm{SCR} \cdot (\boldsymbol{J}_D^{\mathrm{T}} \cdot \boldsymbol{K}_{Tp} \cdot \boldsymbol{J}_D) / (\boldsymbol{J}_D^{\mathrm{T}} \cdot \boldsymbol{K}_{CNp} \cdot \boldsymbol{J}_D) \\ &\text{s.t.} \quad d_1^2 + d_2^2 + d_3^2 = 1 \end{aligned} \quad (13)$$

2.2 A Fast Polarization Optimization Method

In this subsection, we recast (13) in a linear function with two variables which are polarization state $\boldsymbol{J}_D$ and supplementary parameter λ. Then we solve the problem numerically. Let us first define the following function,

$$f(\boldsymbol{J}_D, \lambda) = \mathrm{SCR} \cdot \boldsymbol{J}_D^{\mathrm{T}} \cdot \boldsymbol{K}_{Tp} \cdot \boldsymbol{J}_D - \lambda \cdot \boldsymbol{J}_D^{\mathrm{T}} \cdot \boldsymbol{K}_{CNp} \cdot \boldsymbol{J}_D \quad (14)$$

Define the maximum value of function $f(\boldsymbol{J}_D, \lambda)$ with respect to $\boldsymbol{J}_D$ as

$$f_{\max}(\lambda) = \max_{\boldsymbol{J}_D} f(\boldsymbol{J}_D, \lambda) \quad (15)$$

Lemma 1 $f_{\max}(\lambda)$ in (15) is a monotone decreasing function with respect to λ.

Proof Given two variables λ_1, λ_2 and $\lambda_1 > \lambda_2$, their corresponding maximum values are assumed to be $f_{\max 1}$ and $f_{\max 2}$. Their corresponding polarization states are $\boldsymbol{J}_{D1}$ and $\boldsymbol{J}_{D2}$, respectively. We obtain

$$\begin{cases} f_{\max 1} = \mathrm{SCR} \cdot \boldsymbol{J}_{D1}{}^{\mathrm{T}} \cdot \boldsymbol{K}_{Tp} \cdot \boldsymbol{J}_{D1} - \lambda_1 \cdot \boldsymbol{J}_{D1}{}^{\mathrm{T}} \cdot \boldsymbol{K}_{CNp} \cdot \boldsymbol{J}_{D1} \\ f_{\max 2} = \mathrm{SCR} \cdot \boldsymbol{J}_{D2}{}^{\mathrm{T}} \cdot \boldsymbol{K}_{Tp} \cdot \boldsymbol{J}_{D2} - \lambda_2 \cdot \boldsymbol{J}_{D2}{}^{\mathrm{T}} \cdot \boldsymbol{K}_{CNp} \cdot \boldsymbol{J}_{D2} \end{cases} \quad (16)$$

Since, P_C and P_N are the received power of the clutter and the noise, there are $P_C \geq 0$ and $P_N > 0$, which makes sure $\boldsymbol{J}_D{}^{\mathrm{T}} \cdot \boldsymbol{K}_{CNp} \cdot \boldsymbol{J}_D > 0$. Hence,

$$\begin{aligned} f_{\max 1} - f_{\max 2} &= f(\boldsymbol{J}_{D1}, \lambda_1) - f(\boldsymbol{J}_{D2}, \lambda_2) \leq \\ &f(\boldsymbol{J}_{D1}, \lambda_1) - f(\boldsymbol{J}_{D1}, \lambda_2) = \\ &(\lambda_2 - \lambda_1) \cdot \boldsymbol{J}_{D1}{}^{\mathrm{T}} \cdot \boldsymbol{K}_{CNp} \cdot \boldsymbol{J}_{D1} < 0 \end{aligned} \quad (17)$$

That is $f_{\max 1} < f_{\max 2}$, hence, the Lemma 1. **Q.E.D**

Assuming the polarization state corresponding to $f_{\max} = 0$ is $\boldsymbol{J}_{D\_opt}$; then, the maximum SCNR is

$$\max \mathrm{SCNR} = \mathrm{SCR} \cdot \frac{\boldsymbol{J}_{D\_opt}^{\mathrm{T}} \cdot \boldsymbol{K}_{Tp} \cdot \boldsymbol{J}_{D\_opt}}{\boldsymbol{J}_{D\_opt}^{\mathrm{T}} \cdot \boldsymbol{K}_{CNp} \cdot \boldsymbol{J}_{D\_opt}} \quad (18)$$

According to the forward analyses, the following Theorem 1 can be summarized.

Theorem 1 The maximum SCNR equals to the λ when $f_{\max} = 0$ and the corresponding polarization state is $\boldsymbol{J}_{D\_\mathrm{opt}}$.

When λ is ascertained, the problem of (14) is transformed to be the one in 0. The problem has the form,

$$f(\boldsymbol{J}_D) = \boldsymbol{J}_D^{\mathrm{T}} \cdot \boldsymbol{M} \cdot \boldsymbol{J}_D = \begin{bmatrix} 1 \\ \boldsymbol{g} \end{bmatrix}^{\mathrm{T}} \cdot \begin{bmatrix} k_{0M} & \boldsymbol{a}_M^{\mathrm{T}} \\ \boldsymbol{b}_M & \boldsymbol{Q}_M \end{bmatrix} \cdot \begin{bmatrix} 1 \\ \boldsymbol{g} \end{bmatrix} \quad (19)$$

where $\boldsymbol{M} = \mathrm{SCR} \cdot \boldsymbol{K}_{Tp} - \lambda \cdot \boldsymbol{JK}_{CNp}$. A faster numerical solver will be proposed for maximizing (19). Assuming a Lagrange multiplier is v, the cost function is,

$$L = f(\boldsymbol{J}_D) - v(\boldsymbol{g}^{\mathrm{T}} \boldsymbol{g} - 1) \quad (20)$$

Let the derivation of the cost function with respect to $\boldsymbol{g}$ equal to zero, we obtain

$$\begin{aligned} \boldsymbol{g} &= -(\boldsymbol{Q}_M + \boldsymbol{Q}_M^{\mathrm{T}} - 2\boldsymbol{I}_3 \cdot v)^{-1} (\boldsymbol{a}_M + \boldsymbol{b}_M) \square \\ &-(\boldsymbol{M} - \boldsymbol{I}_3 \cdot v)^{-1} \boldsymbol{V} \end{aligned} \quad (21)$$

According to matrix decomposition theory, there is a unitary matrix $\boldsymbol{U}$ and a diagonal matrix $\boldsymbol{\Lambda}$ such that $\boldsymbol{M}=\boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{U}^{\mathrm{T}}$. Let

$$[\beta_1,\beta_2,\beta_3]^{\mathrm{T}}=\boldsymbol{U}\boldsymbol{V},\ \boldsymbol{\Lambda}=\mathrm{diag}\{\lambda_1,\lambda_2,\lambda_3\} \tag{22}$$

According to $\|\boldsymbol{g}\|=1$, we obtain

$$\sum_{i=1}^{3}[\beta_i/(\lambda_i-v)]^2=1 \tag{23}$$

The domain of Lagrange multiplier $v_{\max}$ that maximizes (19) is 0,

$$v_L\le v_{\max}\le v_U \tag{24}$$

where,

$$\begin{cases} v_L=\max\left\{\left\{\min\left\{\lambda_i+\sqrt{3}|\beta_i|\right\},\max\left\{\lambda_i+|\beta_i|\right\}\right\}\right\} \\ v_U=\max\left\{\lambda_i+\sqrt{3}|\beta_i|\right\},\quad i=1,2,3 \end{cases} \tag{25}$$

and,

$$\begin{cases} [\beta_1\quad \beta_2\quad \beta_3]^{\mathrm{T}}=\dfrac{1}{4}(\boldsymbol{Q}_M+\boldsymbol{Q}_M^{\mathrm{T}})\cdot(\boldsymbol{a}_M+\boldsymbol{b}_M) \\ \mathrm{diag}\{\lambda_1\quad \lambda_2\quad \lambda_3\}=\mathrm{eigs}\left(\dfrac{1}{2}(\boldsymbol{Q}_M+\boldsymbol{Q}_M^{\mathrm{T}})\right) \end{cases} \tag{26}$$

"eigs" denotes the eigenvalues of matrix.

Substituting (21) into (19), there is:

$$\max f(v)=\frac{1}{2}\left[k_{0M}-\sum_{i=1}^{3}\frac{\beta_i^2}{\lambda_i-v}+\sum_{i=1}^{3}\frac{\beta_i^2 v}{(\lambda_i-v)^2}\right] \tag{27}$$

Given two variables v_1 and v_2, $v_1>v_2$, according to Cauchy-Schwartz inequality theory, there is：

$$\max f(v_1)-\max f(v_2)= \\ \frac{1}{2}\left\{(v_1-v_2)+\sum_{i=1}^{3}\left[\frac{\beta_i^2(v_1-v_2)}{(\lambda_i-v_2)(\lambda_i-v_1)}\right]\right\}> \\ \frac{1}{2}(v_1-v_2)\cdot\left\{1-\sum_{i=1}^{3}\left[\frac{\beta_i^2}{2(\lambda_i-v_2)^2}+\frac{\beta_i^2}{2(\lambda_i-v_1)^2}\right]\right\}=0 \tag{28}$$

Hence, the function (19) is monotone increasing with respect to the variable v. The conclusion can be established by the following theorem 2.

Theorem 2 The maximization of (19) can be calculated by the Lagrange method in (20). The function is a monotone increasing one with respect to the Lagrange multiplier v in (20).

Since the two variables in $f(\boldsymbol{J}_D,\lambda)$, i.e., $\boldsymbol{J}_D$ and λ, are mutual independent, the problem of searching the optimal polarization state that maximizing the SCNR can be divided into the following two steps: 1) selecting λ to construct the function (14); 2) selecting $\boldsymbol{J}_D$ to obtain $f_{\max}$ of (15). According to Lemma 1, (15) is monotone decreasing with respect to λ. According to theorem 2, the (19) is monotone increasing with respect to Lagrange multiplier v. Hence, bisection method can be applied to both the two steps.

Given the maximum value of the numerator and the minimum value of the denominator of (10) are $P_{T\max}$ and $P_{CN\min}$, respectively, the upper bound of λ is:

$$\lambda_U\le P_{T\max}/P_{CN\min} \tag{29}$$

Given the minimum value of the numerator and the maximum value of the denominator of (10) are $P_{T\min}$ and $P_{CN\max}$, respectively, the low bound of λ is:

$$\lambda_L\ge P_{T\min}/P_{CN\max} \tag{30}$$

Hence the search intervals of λ is:

$$\lambda_L\le\lambda\le\lambda_U \tag{31}$$

The procedures for the proposed method are:

1) set $\lambda=(\lambda_L+\lambda_U)/2$ (see (29) and (30));

2) solve $f_{\max}$ in (15) by bisection search in $[v_L,v_U]$ (cf. (25)).

3) if $f_{\max}>0$, $\lambda=(\lambda_U+\lambda)/2$;

else if $f_{\max}<0$, $\lambda=(\lambda_L+\lambda)/2$;

else if $f_{\max}=0$, the maximum SCNR is λ, end.

4) else go to 2).

3 Numerical Experiments

Experiments are accomplished by Matlab 2010 code running on a 32-bit computer with CPU AMD Athlon 3.0GHz, RAM 4G. Monte Carlo simulation time is 100. Let us consider the following Kennaugh matrix: the target Kennaugh matrix $\boldsymbol{K}_T$ in 0 and the clutter Kennaugh matrix $\boldsymbol{K}_C$ in 0.

$$\boldsymbol{K}_T=\begin{bmatrix} 13.0625 & -5.8125 & 3\sqrt{2} & \sqrt{2} \\ -5.8125 & 9.3125 & -\sqrt{2} & -3\sqrt{2} \\ 3\sqrt{2} & -\sqrt{2} & 10.25 & 0 \\ \sqrt{2} & -3\sqrt{2} & 0 & -6.25 \end{bmatrix} \tag{32}$$

$$\boldsymbol{K}_C=\begin{bmatrix} 2.1000 & 0.2545 & 0.3798 & 0.1528 \\ 0.0524 & 1.4364 & 0.8664 & -0.023 \\ 0.1798 & 0.6664 & -0.5604 & 0.2192 \\ -0.0472 & -0.223 & 0.0192 & 0.8238 \end{bmatrix} \tag{33}$$

Some assumptions are: 1) the distances of the target and the clutter to the antenna are the same, i.e., $r_T = r_C$. 2) the antenna gains are the same, i.e., $G_T = G_C$. Hence, $\text{SCR} = |E_T|^2 / |E_C|^2$ and $\text{CNR} = |E_C|^2 / |E_N|^2$. Equation (10) shows the SCR does not affect the selection of the optimal polarization state, we choose SCR = 10 dB. Three different clutter to noise ratios(CNRs), i.e., CNR = 10 dB, 0 dB and −10 dB are tested to validate the proposed method.

Considering the target is completely polarized, i.e., $p_T = 1$. To test the performance of proposed method in the partially polarized condition, three conditions are operated: 1) Low polarization ratio $p_C = 0.01$; 2) Middle polarization ratio $p_C = 0.5$; 3) High polarization ratio $p_C = 0.99$. The results obtained by the GSM in a small search-step, i.e., $\pi/720$, is considered to be the real ones.

Experiment I: Low Polarization Ratio $p_C = 0.01$

The maximum SCNRs corresponding to CNR=10 dB, 0 dB and −10 dB are 22.333 9 dB, 19.824 3 dB and 12.509 3 respectively. Their corresponding polarization states are (−0.754 4, 0.633 0, 0.173 6), (−0.771 5, 0.608 2, 0.186 5) and (−0.771 5, 0.608 2, 0.186 5), respectively.

The average time consumed by the proposed method is about 5% of that consumed by the GSM with the similar calculation accuracy. The optimal polarization sates to different CNRs are similar to each other.

TABLE I　the Proposed Method V.S. the GSM with p_c=0.01

Method	CNR/dB	Polarization State	SCNR/dB	Time/s
GSM	10	(−0.754 4, 0.633 0, 0.173 6)	22.332 4	0.040 6
	0	(−0.754 4, 0.633 0, 0.173 6)	19.823 0	0.040 2
	−10	(−0.754 4, 0.633 0, 0.173 6)	12.508 1	0.040 3
Proposed Method	10	(−0.773 6, 0.605 7, 0.185 7)	22.333 6	0.002 4
	0	(−0.772 3, 0.607 6, 0.185 6)	19.824 4	0.002 3
	−10	(−0.770 8, 0.609 3, 0.185 5)	12.509 3	0.001 9

Experiment II: Middle Polarization Ratio $p_C = 0.5$

The maximum SCNRs corresponding to CNR = 10 dB, 0 dB and –10 dB are 22.2646 dB, 19.7747 dB and 12.497 3, respectively. Their corresponding polarization states are (–0.674 8, 0.730 0, 0.108 9), (–0.709 3, 0.691 0, 0.139 2), and (–0.756 6, 0.629 2, 0.177 9) respectively.

TABLE II　the Proposed Method V.S. the GSM with p_c=0.5

Method	CNR/dB	Polarization State	SCNR/dB	Time /s
GSM	10	(−0.665 5, 0.739 1, 0.104 5)	22.264 3	0.040 3
	0	(−0.715 4, 0.690 9, 0.104 5)	19.771 0	0.040 1
	−10	(−0.754 4, 0.663 0, 0.173 6)	12.497 3	0.040 3
Proposed Method	10	(−0.674 6, 0.730 3, 0.107 0)	22.264 3	0.002 3
	0	(−0.708 9, 0.691 6, 0.138 3)	19.774 7	0.002 2
	−10	(−0.757 7, 0.628 4, 0.176 3)	12.497 3	0.001 9

The average time consumed by the proposed method is about 5% of that consumed by the GSM with the similar calculation accuracy. The optimal polarization states to different CNRs are different.

Experiment III: High Polarization Ratio $p_C = 0.99$

The maximum SCNRs corresponding to CNR = 10 dB, 0 dB and −10 dB are 22.910 8 dB, 20.045 6 dB, and 12.504 3, respectively. Their corresponding polarization states are (−0.488 6, 0.872 4, −0.013 1), (−0.536 9, 0.842 7, 0.039 3), and (−0.693 2, 0.705 4, 0.147 8) respectively.

TABLE III the Proposed Method V.S. the GSM with p_c=0.99

Method	CNR/dB	Polarization State	SCNR/dB	Time/s
GSM	10	(−0.5, 0.866, 0)	22.908 5	0.069 3
	0	(−0.543 9, 0.843 2, 0.040 2)	20.504 0	0.070 3
	−10	(−0.698 4, 0.698 4, 0.156 4)	12.462 0	0.069 3
Proposed Method	10	(−0.486 7, 0.873 6, −0.014)	22.911 9	0.002 3
	0	(−0.536 0, 0.843 4, 0.040 2)	20.045 6	0.002 3
	−10	(−0.694 2 , 0.704 5, 0.147 6)	12.5043	0.001 9

The average time consumed by the proposed method is about 3% of that consumed by the GSM with the same calculation accuracy. The optimal polarization states to different CNRs are greatly different.

The proposed method has been proved to be able to obtain the optimal polarization states for all the partially polarized conditions. Compared with the GSM, the proposed method is less time-consuming and more accurately.

4　Conclusions

In the paper, the OPCE problem with constrained transmitted and received polarization state relationship in partially polarized condition is discussed. A general SCNR model is first created to contain the partially polarized condition. A fast method for the OPCE problem is proposed based on the SCNR model. The method has converted the OPCE problem into the maximization problem of a linear function. Hence, the computational burden is greatly reduced. The numerical experiments have demonstrated the proposed method is better and has higher efficiency than the GSM. This method is easily extended to other polarization states conditions, such as the cross-polarize condition, by matrix rotation.

In the following work, we will research on the fast polarization optimization methods for the OPCE problem with unconstrained relationship between the transmitted polarization state and the received polarization state.

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