Shearlet

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(PSNR)
(SSIM)

; ; ; Shearlet
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Shearlet Sparsity Regularized Image Reconstruction Based on Nonlocal Self-Similarity

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Abstract In this paper, a Shearlet sparsity and nonlocal self-similarity based image reconstruction model is proposed. In the new model, the energy error between the observed image and the image to be reconstructed is used as fidelity term. The Shearlet sparsity and non-local similarity are used as hybrid a regularization term, which takes into account transformation and structural characteristics of images. Furthermore, an efficient variable splitting augmented Lagrangian algorithm is developed to solve the above combined sparsity and non-local regularization constrained problem. Image deblurring and image inpaint are used as examples to test the performance of the proposed method. Experimental results show that the proposed method can preferably reconstruct the images and achieve improvement over the state-of-the-art methods in Peak-Signal-to-Noise-Ratio (PSNR) and structural similarity (SSIM) index.

Key words augmented Lagrangian; image recovery; nonlocal self-similarity; Shearlet transform

Tikonov [1]
[2] [3]
[4] [5]
[6-7]

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(1974 –) .

[2,12] [3-5,13]

[8-10]

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1 Shearlet

Shearlet [11]

$$\mathcal{A}_{AB}(\boldsymbol{\psi}) =$$

$$\left\{\boldsymbol{\psi}_{j,l,k}(x) = \left|\det \boldsymbol{A}\right|^{j/2} \psi(\boldsymbol{B}^{l}\boldsymbol{A}^{j}\boldsymbol{x} - k) : j, l \in Z, k \in Z^{2}\right\} \quad (1)$$

$\boldsymbol{\psi} \in L^2(R^2)$ $\boldsymbol{A}$ $\boldsymbol{B}$ 2×2 , $\left|\det \boldsymbol{B}\right| = 1$ $\boldsymbol{A}^j$ $\boldsymbol{B}^l$

( )

$$\boldsymbol{A}_a = \begin{bmatrix} a & 0 \\ 0 & \sqrt{a} \end{bmatrix} \quad \boldsymbol{B}_s = \begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix}$$

$$\left\{\boldsymbol{\psi}_{ast}(x) = a^{-\frac{3}{4}}\boldsymbol{\psi}(\boldsymbol{A}_a^{-1}\boldsymbol{B}_s^{-1}\boldsymbol{x} - t), a \in R^{+}, s \in R, t \in R^{2}\right\} \quad (2)$$

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$\boldsymbol{f} \in L^2(R^2)$ Shearlet

$$\mathrm{SH}_{\psi}\boldsymbol{f}(a,s,t) = \left\langle \boldsymbol{f}, \psi_{ast} \right\rangle,\ a \in R^{+},\ s \in R,\ t \in R^{2} \quad (3)$$

2

2.1

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n} \quad (4)$$

$\boldsymbol{x}$ $\boldsymbol{H}$ $\boldsymbol{y}$

$\boldsymbol{n}$

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{x}} \left\{\frac{1}{2}\left\|\boldsymbol{y} - \boldsymbol{H}\boldsymbol{x}\right\|_2^2 + \lambda\varphi(\boldsymbol{x})\right\} \quad (5)$$

$\left\|\boldsymbol{y} - \boldsymbol{H}\boldsymbol{x}\right\|_2^2$ $\varphi(x)$ λ

Tikonov[1]

[2,12] [3-5,13]

[12-13]

[4]

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2.2

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$$\hat{\boldsymbol{x}}=\arg\min_{\boldsymbol{x}}\left\{\frac{1}{2}\|\boldsymbol{y}-\boldsymbol{H}\boldsymbol{x}\|_2^2+\lambda\|\langle\boldsymbol{x},\psi_{\text{ast}}\rangle\|_1+\gamma\|\boldsymbol{x}-\boldsymbol{w}\boldsymbol{x}\|_2^2\right\} \tag{6}$$

$\|\langle\boldsymbol{x},\psi_{\text{ast}}\rangle\|_1$ Shearlet $\|\boldsymbol{x}-\boldsymbol{w}\boldsymbol{x}\|_2^2$

γ $\boldsymbol{w}$

(6)

$$\hat{\boldsymbol{x}}=\arg\min_{\boldsymbol{x}}\left\{\frac{1}{2}\|\boldsymbol{y}-\boldsymbol{H}\boldsymbol{x}\|_2^2+\lambda\|\boldsymbol{S}\boldsymbol{x}\|_1+\gamma\|\boldsymbol{x}-\boldsymbol{w}\boldsymbol{x}\|_2^2\right\} \tag{7}$$

$\boldsymbol{S}$ Shearlet

[8-10]

$\boldsymbol{w}$

$$\boldsymbol{w}(i,j)=\begin{cases} w_{ij} & 1 \le j \le J \\ 0 & 其他 \end{cases} \tag{8}$$

$w_{i,j}$ $\boldsymbol{x}_i$ j

$\boldsymbol{x}_i^j$ $w_{i,j}$

$$w_{ij}=\frac{1}{c_i}\exp(-\|\boldsymbol{x}_i-\boldsymbol{x}_i^j\|_2^2/h^2) \tag{9}$$

h c_i

2.3

(6) (7)

[12]

$\boldsymbol{u}$ (7)

$$\hat{\boldsymbol{x}}=\arg\min_{\boldsymbol{x}}\left\{\frac{1}{2}\|\boldsymbol{y}-\boldsymbol{H}\boldsymbol{u}\|_2^2+\lambda\|\boldsymbol{S}\boldsymbol{x}\|_1+\gamma\|\boldsymbol{u}-\boldsymbol{w}\boldsymbol{u}\|_2^2\right\} \quad \text{s.t.} \quad \boldsymbol{u}=\boldsymbol{x} \tag{10}$$

$$\mathcal{L}(\boldsymbol{u},\boldsymbol{x},\boldsymbol{d})=\frac{1}{2}\|\boldsymbol{y}-\boldsymbol{H}\boldsymbol{u}\|_2^2+\lambda\|\boldsymbol{S}\boldsymbol{x}\|_1+\gamma\|\boldsymbol{u}-\boldsymbol{w}\boldsymbol{u}\|_2^2+\frac{\mu}{2}\|\boldsymbol{x}-\boldsymbol{u}-\boldsymbol{d}\|_2^2 \tag{11}$$

$\mu>0$ $\boldsymbol{d}/\mu$

$\mathcal{L}(\boldsymbol{u},\boldsymbol{x},\boldsymbol{d})$

(10) (10)

$$(\boldsymbol{u}_{k+1},\boldsymbol{x}_{k+1})=\arg\min_{\boldsymbol{u},\boldsymbol{x}}\mathcal{L}(\boldsymbol{u},\boldsymbol{x},\boldsymbol{d}) \tag{12}$$

$$\boldsymbol{d}_{k+1}=\boldsymbol{d}_k-(\boldsymbol{x}_{k+1}-\boldsymbol{u}_{k+1}) \tag{13}$$

(12)

$\boldsymbol{x}$ (12)

$$\boldsymbol{u}_{k+1}=\arg\min_{\boldsymbol{x}}\frac{1}{2}\|\boldsymbol{y}-\boldsymbol{H}\boldsymbol{u}\|_2^2+\gamma\|\boldsymbol{u}-\boldsymbol{w}\boldsymbol{u}\|_2^2+\frac{\mu}{2}\|\boldsymbol{x}-\boldsymbol{u}-\boldsymbol{d}\|_2^2 \tag{14}$$

(14)

$$\boldsymbol{u}_{k+1}=[\boldsymbol{H}^{\mathrm{T}}\boldsymbol{y}-\mu(\boldsymbol{x}_k-\boldsymbol{d}_k)]/[\boldsymbol{H}^{\mathrm{T}}\boldsymbol{H}-2\gamma(\boldsymbol{I}-\boldsymbol{w})^{\mathrm{T}}(\boldsymbol{I}-\boldsymbol{w})-\mu] \tag{15}$$

$\boldsymbol{u}$ (12)

$$\boldsymbol{x}_{k+1}=\arg\min_{\boldsymbol{u}}\lambda\|\boldsymbol{S}\boldsymbol{x}\|_1+\frac{\mu}{2}\|\boldsymbol{x}-\boldsymbol{u}-\boldsymbol{d}\|_2^2 \tag{16}$$

(16) ℓ_1

$$\boldsymbol{x}_{k+1}=\boldsymbol{S}^{-1}(\text{soft}(\boldsymbol{S}^{\mathrm{T}}\boldsymbol{u}_{k+1}-\boldsymbol{S}^{\mathrm{T}}\boldsymbol{d}_k,\lambda/\mu)) \tag{17}$$

2.4

$\boldsymbol{y}$

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1) $k=0$ γ μ $\boldsymbol{d}_0$ $\boldsymbol{u}_0$ $\boldsymbol{x}_0$ K

2) (15) (17) (13) $\boldsymbol{u}_{k+1}$ $\boldsymbol{x}_{k+1}$ $\boldsymbol{d}_{k+1}$

3) $k = k+1$

4) $k \le K$ 2)

$\boldsymbol{x}_{k+1}$

3

512×512

Lena Barbara Boats Bridge

$K=100$

$\gamma=\mu=1$ (PSNR)

(SSIM)[14]

3.1 PSNR SSIM TwIST

H 1 7×7 [13] C-SALSA [12]

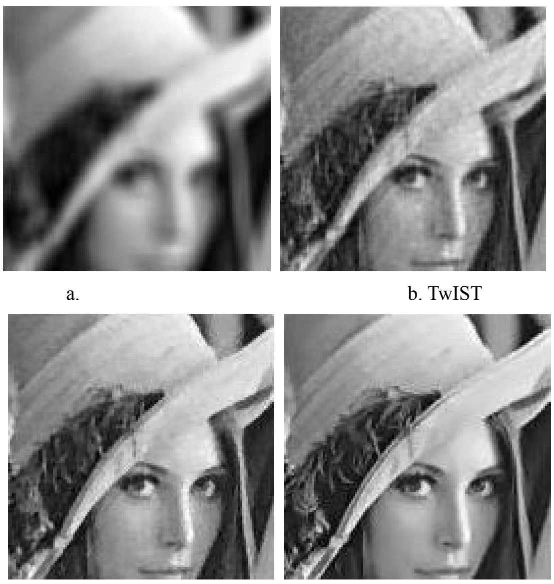
1

40 dB 1 PSNR SSIM

1 **PSNR SSIM**

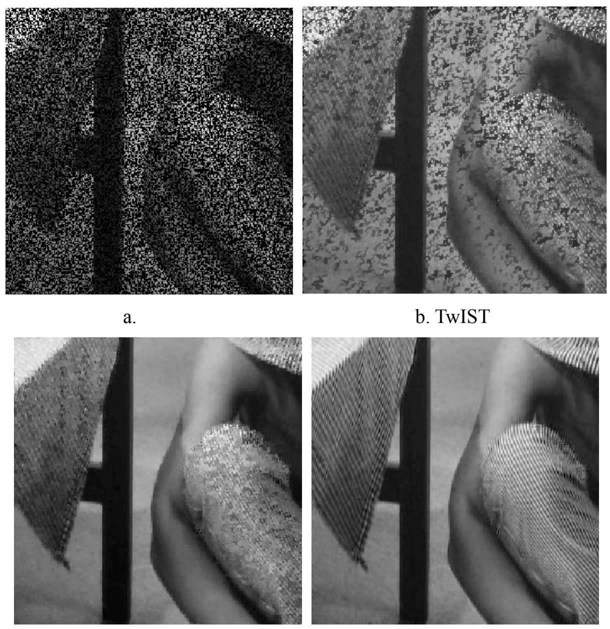
	PSNR/dB				SSIM			
	Lena	Barbara	Boats	Bridge	Lena	Barbara	Boats	Bridge
TwIST	25.35	23.05	23.09	24.08	0.645	0.703	0.728	0.739
C-SALSA	25.92	24.76	24.56	25.10	0.732	0.781	0.805	0.783
	27.32	26.41	26.14	26.89	0.812	0.847	0.861	0.837

1 3 Lena PSNR SSIM 2 1 TwIST PSNR SSIM TwIST C-SALSA C-SALSA 2 Barbara 60% 3 2b 2c TwIST C-SALSA



a. b. TwIST c. C-SALSA d.

1 3 Lena



a. b. TwIST c. C-SALSA d.

3.2

40% 60% 2 40% 2 60% 3

2 40% PSNR SSIM

	PSNR/dB				SSIM			
	Lena	Barbara	Boats	Bridge	Lena	Barbara	Boats	Bridge
TwIST	27.64	26.71	29.28	28.12	0.748	0.776	0.788	0.783
C-SALSA	33.02	28.76	32.51	30.34	0.972	0.918	0.964	0.959
	33.92	32.13	33.15	32.04	0.985	0.978	0.981	0.975

3.3

TwIST

C-SALSA Shearlet Shearlet

$O(N\log N)$ N

3

$O(N)$

Shearlet

TwIST

Shearlet

$O(N\log N)$ C-SALSA

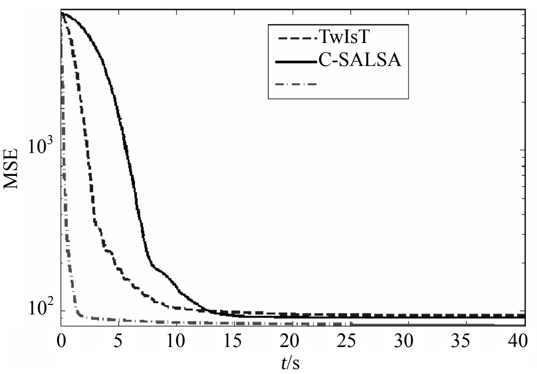
Shearlet

$O(N\log N)$

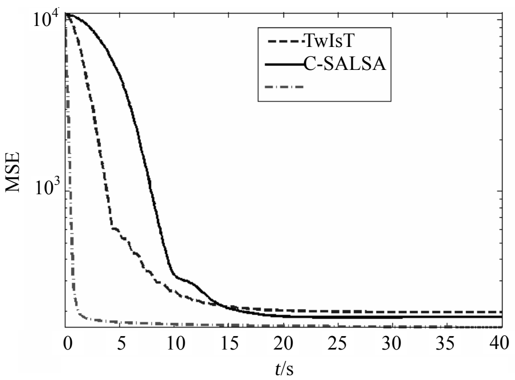
Shearlet

$O(N\log N)+$

$O(N)$



a. 40% MSE



b. 60% MSE

3 60% 40% 3 MSE

3

40% 60% Barbara

(MSE) 3

MSE

MSE MSE

4

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PSNR SSIM

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