Design and Evaluation of GPS Code Pseudorage Smoothing Algorithm Based on Carrier Observables*

Deng Qiang Huang Shunji
(Dept. of Electronic Engineering, UEST of China Chengdu 610054)

Abstract The goal in designing a GPS code pseudorange smoothing algorithm is to smooth the random noise of the code pseudorange by means of the carrier observables and to improve the positioning accuracy when the requirement for real-time kinematic point determination is satisfied. The performance of smoothing algorithm is dominated by three factors of the receiver performance, the choice of weight factor, and the recursive time. Theoretic discussion of evaluation of the smoothing algorithm is made in this paper. And a smoothing algorithm with optimal weight factor is given here, which has the fastest noise reduction performance.

Key words smoothing algorithm; code pseudorange; carrier phase observable; error estimation; performance evaluation

The basic positioning models for GPS users are code pseudorange and carrier phase pseudorange positioning, the detailly mathemaic models of which are given by B. Hofmann-Wellenhof et. al[1]. The advantage of using GPS carrier phase measurements is that they are precise to a few millimeters. The code pseudorange smoothing technique by GPS carrier phase observables overcomes the shorts of the bad accuracy of code pseudorange positioning and the time consumption of carrier phase positioning. The first extensive investigation of this subject was provided by Hatch[2].

1 The Smoothing Algorithm Description[1,2]

For an epoch t_k, the smoothed code pseudorange is given by

$$l_s(t_k) = wl(t_k) + (1 - w)l_e(t_k) \tag{1}$$

where w is a weight factor; $l(t_k)$ is the ionosphere-free combination of the dual-frequency P-code pseudoranges $P_{L1}(t_k)$ and $P_{L2}(t_k)$, writing as

$$l(t_k) = \frac{f_1^2 P_{L1}(t_k) - f_2^2 P_{L2}(t_k)}{f_1^2 - f_2^2} \tag{2}$$

The extrapolated value $l_e(t_k)$ as the forward estimation for code pseudorange is given by

$$l_e(t_k) = l_s(t_{k-1}) + W(t_k) \tag{3}$$

where $l_s(t_{k-1})$ is the last code pseudorange smoothing value; and $W(t_k)$ is the correction term consisting of carrier phase observables. By introducing the wide signal

$$H(t) = H_{L1}(t) - H_{L2}(t) \quad \text{(cycles)} \tag{4}$$

Received May 20, 1996 and revised April 24, 1996
* The subject surpported by Electronic Industrial Ministry Advanced Research Foundation

we have the common-error-free correction term

$$W(t_k) = \lambda H(t_k) - \lambda H(t_{k-1}) \qquad \text{(meters)} \tag{5}$$

where λ is the wavelength of the wide lane signal.

A recursive algorithm of smoothing technique is obtained from Eq. (1)~ (5) under the initial condition $l(t_0) = l_e(t_0) = l_s(t_0)$.

2 The Error Estimation of Observables[3,4]

The P-code pseudorange measuring error is composed of systematic group delay bias and random noise. The system biases on L_1 and L_2 channels are given by

$$\Delta l_{g1} = W_{i1} + W_{t} + W_{c} + W_{e1} + W_{ep} \tag{6}$$

$$\Delta l_{g2} = W_{i2} + W_{t} + W_{c} + W_{e2} + W_{ep} \tag{7}$$

where W_{i1} and W_{i2} are the ionospheric delay ranges on carrier frequencies L_1 and L_2, respectively; W_t is the tropospheric delay range which holds the same value for L_1 and L_2 channels; W_c is the corresponding range of the difference of the satellite clock and receiver clock; W_{e1} and W_{e2} are the equipment group delay ranges which have a little bias between L_1 and L_2 frequencies and can be removed by introducing a numeric model; W_{ep} is the epheride error.

According to Eq. (2), the ionosphere-free combined code pseudorange error $\Delta l(t_k)$ has a mean value for arbitrary epoch

$$\__1 = W_t + W_c + W_e + W_{ep} \tag{8}$$

with assumption that $W_e = W_{e1} = W_{e2}$. The approximate value for $\__1$ is 5. 6 m. The noise of $l(t_k)$ is increased by the factor of 3, thus the variance of $\Delta l(t_k)$ is obtained from

$$e_1 = 3e_p \tag{9}$$

where e_p is the variance of random noise for P-code pseudorange which has an approximate value of 1 m. So e_1 holds the value of 3 m.

Because the correction term of Eq. (5) derived from the wide lane signal is the differential range of the changed distances measured on L_1 and L_2 channels, the $\Delta W(t_k)$ behavouring as a normal distributional variable with the mean and variance are

$$\__2 = 0 \tag{10}$$

$$e_2 = 1.7e_c \tag{11}$$

where e_c is the variance of the carrier phase measurement, and holds about 2 millimeters. So the correction term has an approximately noise level of 5 millimeters.

3 Performance Evaluation

Denoting the mean value and variance of the code pseudorange smoothing value error $\Delta l_s(t_k)$ as $\_(k)$ and $e(k)$ respectively, and assuming the independence is between the observables, the resursive of $\_(k)$ and $e(k)$ are give by

$$\_(k) = w\__1 + (1-w)\_(k-1) \tag{12}$$

$$e(k) = \sqrt{w^2 e_1^2 + (1-w)^2 e^2(k-1) + (1-w)^2 e_2^2} \tag{13}$$

With the initial condition $\_(0)= \_{1}$ and $e(0)= e_1$, we have the closed equation for $\_(k)$ and $e(k)$ as follows

$$\_(k) = \_{1} \tag{14}$$

$$e(k) = \sqrt{\sum_{j=0}^{k-1} w^2(1-w)^{2j} e_1^2 + (1-w)^{2k} e_1^2 + \sum_{j=1}^{k} (1-w)^{2j} e_2^2} \tag{15}$$

with the assumption that the weight factor w holds as a constant. According to Eq. (15), and because $0 < w < 1$, we have

$$e(\infty) = \sqrt{\frac{w}{2-w} e_1^2 + \frac{(1-w)^2}{1-(1-w)^2} e_2^2} \tag{16}$$

From above, a conclusion is derived that the smoothing technique can reduce the random code pseudorange measuring noise, but has no ability to eliminate the systematic bias given in Eq. (8).

The first part of the right-hand side of the Eq. (16) decreases with the reduced w, while the second part increases with the reduced. Because $e_2 \ll e_1$, it is reasonable to reduce w greatly. Fig. 1 gives the illustrations of the noise levels with incorporating different weight factor into the algorithm given in the first section of this paper.

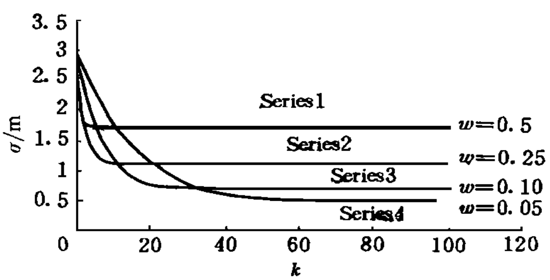

Figure1 The illustrations of noise reduction

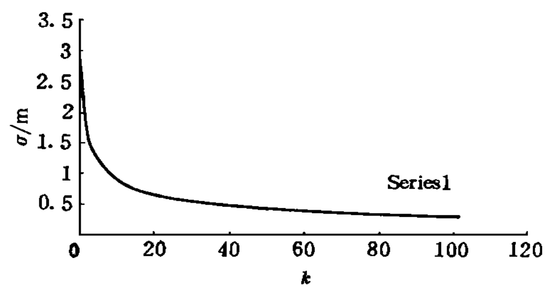

Figure2 The illustration of noise reduction with the optimal weight factor

4 Choice of the Weight Factor

The criterion of the optimal weight factor choice is that the algorithm has the fastest noise reduction performance. According to Eq. (13), we have

$$e^2(k) = w^2(k) e_1^2 + (1-w(k))^2 e^2(k-1) + (1-w(k))^2 e_2^2 \tag{17}$$

Let $\frac{\partial e^2(k)}{\partial w(k)} = 0$, the optimal weight factor w_{opt} is derived as follows

$$w_{opt}(k) = \frac{e^2(k-1) + e_2^2}{e_1^2 + e^2(k-1) + e_2^2} \tag{18}$$

Fig. 2 gives the illustration of the noise level with the optimal weight factor. Table 1 gives the relationship between the smoothing and the resursive time when optimal weight factor is available.

Table 1 Performance of the smoothing algorithm with the optimal weight factor

k	0	5	10	20	30	50	100	1 000
e/m	3.0	1.22	0.90	0.65	0.54	0.42	0.30	0.13

5 Conclusions

The smoothing technique is an effective method to smooth the measuring noise embeded in code pseudorange. The theoretical discussions and tests illustrate that the smoothing accuracy depends on the choice of weight factor, the recerver noise, and the recuresive time. As a result, when recursive time is larger than 100, a smoothing accuracy of 30 cm is derived with the optimal weight factor incorporated into the smoothing algorithm and with the conditions given in the second section of this paper.

参 考 文 献

1 Hofmann-Wellenhof B, Lichtenegger H, Collins J. Global positioning system theroy and practice. New York: Springer-Verlag Wien, 1992

2 Hatch R. The synergism of GPS code and carrier measurements. Proc of the 3rd International Symposium on Satellite Doppler Positioning, 1992, 2: 1 213~ 1 231

3 Brown. A expended differential GPS. Navigation, 1989, 36(3): 265~ 286

4 Deng Qiang, Huang Shunji. Pseudorange differetial GPS error analysis (in Chinese). Journal of University of Electronic Science and Technology of China, 1995, 24(5): 457~ 460

基于载波相位的 GPS码伪距平滑算法设计*

邓　强**　　黄顺吉

(电子科技大学电子工程系　成都　610054)

【摘要】 文中给出了全球定位系统接收机可获取观测量的误差估计,基于此讨论了平滑算法的性能。平滑算法的精度与接收机性能,权重因子和平滑递归的次数有关。给出了载波相位平滑伪码伪距的具有最快的噪声对消的递归算法结构,并且对其性能进行了评估。当递归次数大于 100,码伪距和载波相位的测量精度分别为 5.6 m和 5 mm时,具有最优加权系数的平滑算法可提供优于 30 cm的伪距测量精度。

关　键　词　伪距;　载波相位;　平滑;　递归结构;　最优权重;　性能评估

中图分类号　TN967.1

编辑　叶　红

1996年 3月 20日收稿

* 电子部预研基金资助项目

** 男　26岁　博士生