An Inverse QR Decomposition Algorithm for Full Adaptive Array*

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Abstract QR decomposition LS algorithm (QRD) is a widely used algorithm for adaptive arrays recently, which has good numerical stability and can be easily mapped to Systolic structure for parallel processing. However, for full adaptive array where optimal weights are needed, this method typically requries three Systolic modulars to produce optimal weight. This paper presents a new method to solve the nulling problem of a full adaptive array using inverse QR decomposition LS algorithm (IQRD) and its Systolic implementation. It can easily obtain optimal weight vector using only one Systolic array. Since it still directly deals with the input data matrix and is based on unitary transformation, the IQRD algorithm has the same numerical stability as that of the QRD algorithm.

Key workds adaptive algorithm; adaptive array; signal processing; systolic array

Beacuse QR Decomposition LS Algorithm (QRD) has good numerical stability[1]. which is widely used in adaptive array processing, it typically needs three Systolic modulars for a full adaptive array. The inverse QR Decomposition (IQRD) can obtain the optimal weight vector by one Systolic modular. The idea of IQRD was first presented by C. T. Pan et. al[2]. Alexander et. al.[3] applied it to a time-domain adaptive filter. This paper applies the idea to space domain processing, presents a new IQRD algorithm for a full adaptive array, and derives the recursive equation for the algorithm using multiple complex variable.

1 Recursive Equation for the Optimal Weight

The QRD method[1] is an effective algorithm to obtain the optimal weight vector in the sense of maximizing SNJR for an array. It transforms the input matrix X into an upper triangular matrix R by an unitary matrix Q

$$Q(n)B(n)X(n) = \begin{bmatrix} R(n) \\ 0 \end{bmatrix} \tag{1}$$

The optimal array weight vector w_{opt} is obtained as follows

$$w_{opt} = \frac{R^{-1}(n)p(n)}{\| p(n)\|^2} \tag{2}$$

where

$$p(n) = R^{-H}(n)s^* \tag{3}$$

and s is the aiming vector. Schreiber et. al.[4] presented a Systolic structure based on this algorithm, using three Systolic modulars to obtain w_{opt}.

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Carefully observeing Eq. (2), one finds that it is easy to obtain w_{opt} if directly deal with $R^{-1}(n)$. This is the idea of IQRD. Its recursive equations are given below.

1) $R^{-1}(n)$

The recursive equation for QRD algorithm is

$$G(n)\begin{bmatrix} \mathrm{U}R(n-1) \\ x^T(n) \end{bmatrix} = \begin{bmatrix} R(n) \\ 0^T \end{bmatrix} \tag{4}$$

where $G(n)$ is a series of Givens rotations. Let

$$G(n) = \begin{bmatrix} A & b \\ c^H & \mathrm{W} \end{bmatrix} \tag{5}$$

and since $G(n)$ is an unitary matrix, we have

$$b = -\frac{1}{\mathrm{W}}Ac \tag{6}$$

Substituting Eq. (5) into Eq. (4) and taking inverse of the upper sub-square matrix of both side, we get

$$R^{-1}(n) = \left[\mathrm{U}R(n-1) - \frac{1}{\mathrm{W}}cx^T(n)\right]^{-1}A^{-1} \tag{7}$$

2) $p(n)$ and $w(n)$

Form Eq. (3), we have, at the moment $(n-1)$

$$s^* = R^H(n-1)p(n-1) \tag{8}$$

Then

$$s^* = \mathrm{U}^{2}[\mathrm{U}R^H(n-1) \quad 0 \quad x^*(n)]G^H(n)G(n)\begin{bmatrix} \mathrm{U}p(n-1) \\ \mathrm{U}v(n-1) \\ 0 \end{bmatrix} \tag{9}$$

Let

$$G(n)\begin{bmatrix} \mathrm{U}p(n-1) \\ \mathrm{U}v(n-1) \\ 0 \end{bmatrix} = \begin{bmatrix} d(n) \\ \mathrm{U}v(n-1) \\ z(n) \end{bmatrix} \tag{10}$$

Substituting Eq. (10) into Eq. (9), there results

$$s^* = \mathrm{U}^{-2}R^H(n)d(n) \tag{11}$$

From Eq. (11) and Eq. (3). we have

$$d(n) = \mathrm{U}^{-2}p(n) \tag{12}$$

Substituting Eq. (12) into Eq. (10), there results

$$G(n)\begin{bmatrix} \mathrm{U}p(n-1) \\ 0 \end{bmatrix} = \begin{bmatrix} \mathrm{U}^{2}p(n) \\ z(n) \end{bmatrix} \tag{13}$$

From Eq. (13) and Eq. (5), one obtains

$$p(n) = \frac{1}{\mathrm{U}}Ap(n-1) \tag{14}$$

Substituting Eq. (7) and Eq. (14) into Eq. (2), there results

$$w(n) = \frac{\| p(n-1)\|^2}{\| p(n)\|^2}\left\{\frac{1}{\mathrm{U}^{2}}w(n-1) - \frac{u(n)}{b(n)}\frac{1}{\mathrm{U}}\hat{e}(n|n-1)\right\} \tag{15}$$

where

$$b^2(n) = 1 + a^H(n)a(n) \quad (16)$$

$$u(n) = \frac{R^{-1}(n-1)a(n)}{U b(n)} \quad (17)$$

$$\hat{e}(n|n-1) = x^T(n)w(n-1) \quad (18)$$

$$a(n) = \frac{R^{-H}(n)x^*(n)}{U} \quad (19)$$

Eq. (18) has the clear physical meanings: prediction of the error at the moment n by that at the monment $n-1$.

3) $\| p(n)\|^2$

From Eq. (13) and Eq. (5), we have

$$\| p(n)\|^2 = \frac{1}{U^2}\| p(n-1)\|^2 - \frac{1}{U^4}| z(n)|^2 \quad (20)$$

and

$$z(n) = - WU a^H(n)p(n-1) \quad (21)$$

On substituting Eq. (16)~ (19) and Eq. (21) into Eq. (20), there results

$$\| p(n)\|^2 = \frac{1}{U^2}\| p(n-1)\|^2 - \frac{1}{U^4 b^2(n)}\| p(n-1)\|^4 |\hat{e}(n|n-1)|^2 \quad (22)$$

Thus the recursive equation of $w(n)$ is obtained

$$w(n) = k_0(n)\{w(n-1) - k_1(n)\hat{e}(n|n-1)\} \quad (23)$$

where

$$k_0(n) = \frac{\| p(n-1)\|^2}{\| p(n)\|^2}, k_1(n) = \frac{u(n)}{U b(n)} \quad (24)$$

and $\| p(n)\|^2$ is obtained by Eq. (22).

2 IQRD and Its Systolic Implementation

From Eq. (23) and Eq. (24), we can see that all the unknown parameters depend on $R^{-1}(n-1)$. So the key of the algorithm is to obtain the recursive equation of $R^{-1}(n)$. Through the following analysis, it shows that the recursive equation of $R^{-1}(n)$ is similar to that of $R(n)$ in QRD algorithm.

Taking the inverse of the upper sub-square matrix of both side of Eq. (4) and using Eq. (16) ~ (19), we have

$$R^{-1}(n)R^{-H}(n) + u(n)u^H(n) = \frac{R^{-1}(n-1)R^{-H}(n-1)}{U^2} \quad (25)$$

or

$$G(n)\begin{bmatrix} U^{-1}R^{-H}(n-1) \\ 0 \end{bmatrix} = \begin{bmatrix} R^{-H}(n) \\ u^H(n) \end{bmatrix} \quad (26)$$

From Eq. (16)~ (19), we have

$$\left\| \begin{pmatrix} a(n) \\ 1 \end{pmatrix} \right\| = \left\| \begin{pmatrix} 0 \\ b(n) \end{pmatrix} \right\| \quad (27)$$

Based on the matrix theory, it is known that there must exist an unitary matrix $G'(n)$ statisfing

$$G'(n)\begin{bmatrix} a(n) \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ b(n) \end{bmatrix} \quad (28)$$

where $G'(n)$ is also a series of Givens Rotations. It can be proved that

$$\dot{G}'(n) = G(n) \quad (29)$$

thus

$$G(n)\begin{bmatrix} a(n) & \mho^{-1}R^{-H}(n-1) \\ 1 & 0^T \end{bmatrix} = \begin{bmatrix} 0 & R^{-H}(n) \\ b(n) & u^H(n) \end{bmatrix} \quad (30)$$

Eq. (30) is the key recuresive equation for IQRD algorithm which is very similar to the one for QRD algorithm, Eq. (4). Therefore the Systolic structure of IQRD is the same as that of QRD, except the following:

(1) The recursive object of QRD is the upper triangular $R(n)$, whereas IQRD' s is the lower triangular $R^{-1}(n)$.

(2) The input for QRD systolic array is data matrix $X(n)$, whereas the one for IQRD' s is $a(n)$, which is derived form $X(n)$.

From the Systolic array of the IQRD, we can obtain $b(n)$, $u(n)$ and $\| p(n)\|^2$. Substituting them into the liner recursive equation of $w(n)$, Eq. (23), the w_{opt} will be derived.

For the IQRD algorithm, the initialization of the Systolic is very important. The methods we used is called Diagonal Loading Intialization:

$$R^{-H}(0) = \lambda E \quad (31)$$

$$\lambda \ll 1 \quad (32)$$

The value of $\frac{1}{\lambda}$ is related to the SNJR of the input data.

Correspondingly

$$w(0) = s^* \quad (33)$$

$$\| p(0)\|^2 = \lambda^2 \| s\|^2 \quad (34)$$

3 Summary

This paper presents an Inverse QR Decomposition LS Algorithm for a Full Adaptive Array. It can easily obtain w_{opt} by using only one Systolic array. At the same time because it still directly deals with the input data matrix and is based on unitary transformation, IQRD remains the similar numerical stability of the QRD algorithm.

Reference

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全自适应阵列中的逆 QR分解算法*

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【摘要】 提出了一种在全自适应阵列中的逆 QR分解算法,给出了它的 Sysrolic实现方法。该算法可以很容易地映射到 Systolic结构上,仅需一个Systolic阵便能得到最佳权矢量。并且同 QR分解算法一样,它是直接对数据矩阵进行处理,且变换采用酉阵,保持了很好的数值稳定性。

关 键 词 自适应算法; 自适应阵列; 信号处理; Systolic阵列
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· 科研成果介绍 ·

片式高频晶界层电容层

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该课题采用独创的低温烧结晶界偏析法,利用常规多层陶瓷电容器制作工艺,选用钯银合金作内电极材料,成功地制作出了电容量大于 1μF的 $SrTiO_3$为基础的片式晶界层电容器。其特点是: (1)实现了 $SrTiO_3$的低温半导化烧结及晶界偏析绝缘。(2)低烧瓷料可与钯银内电极浆料匹配烧结。(3)与常规的多层陶瓷电容器生产工艺有良好的工艺适应性。

片式晶界层电容器性能与 1993年国际上报道的用纯钯作内电极的相似产品性能比较,具有烧结温度低,介电常数大,温度变化率小的优点。

并行处理容错技术研究

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并行处理容错技术研究的核心包括三部分: 分布式 (x.1)概念冗余容错技术 ($X=2,3$);故障检测与诊断;断点恢复实现技术。

分布式 (x,1)概念冗余容错系统: 由 X个机器按 X冗余机制构成一个结点,然后再由这种结点按分布式机制构成系统,同时在系统范围内另设 N个备份机。若某一结点有一机器不能正常工作,则选一备用机取代以保持结点继续正常运行。我们称这种系统为分布式 (x.1)概念冗余系统。

故障检测与诊断 该系统对故障诊断采用了两种方法: 周期性诊断和非周期性诊断。周期性诊断由一个按一定周期运行的诊断任务完成;非周期性诊断是在特定条件下启动执行的一个过程。

动态拓扑变换及断点恢复技术: 一旦系统检测出硬件故障或软件故障后,系统奖采取两个方面的措施: 1) 屏蔽故障处理单元; 2) 迁移故障机对应的备份任务环境到一可用备份机。该系统通过引入可编程控制开关,并对其相应控制、动态拓朴变换,用以完成对故障单元的屏蔽,并实现逻辑结点的重构。把故障机上对应的备份任务迁移到一可用备份机,并使其从“断点”处继执行。

分布式 (x.1)概念冗余容错算法及其实现技术在国内属于首创,已接近国际 90年代水平。

· 科 卞 ·

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