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Electrostatic Force Between Two Parallel Strip Transmission Lines in A Plane*

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Abstract The electrostatic field region on the cross section of two charged parallel strip transmission lines in a plane is transformed into the interior of a rectangle with the conformal mapping. The capacitance and the electrostatic energy per unit length of the transmission lines are obtained. On the basis of the principle of virtual work, the electrostatic force between the two lines is calculated. Variation of the normalized force verses according to the geometric parameters is illustrated. The typical special cases are analyzed, including the force between charged strings, narrow lines and broad lines.

Key words transmission lines; conformal mapping; principle of virtual work; electrostatic force

It has both theoretic and engineering significance to study electrostatic force on transmission lines[1]. In this paper, the electrostatic force between two charged parallel strip transmission lines in a plane is studied. With the conformal mapping method, the field region, infinite on the cross section, is transformed into the interior of a rectangle. The capacitance is obtained and the expression of the electrostatic field energy is established. The principle of virtual work is employed to achieve the electrostatic forces between two lines. Finally, some typical cases are treated as special examples.

1 Transformation of Field Region

The cross section of two parallel strip transmission lines in a plane is sketched in Fig.1 . The voltage between them is V. Their widths are $AB = l_1$ and $CD = l_2$. The distance between them is $BC = l_{12}$. Assume the length of the line is much longer than l_1, l_2 and l_{12}, the electric field may be regarded as two-dimensional on the cross section due to its uniformity along the lines.

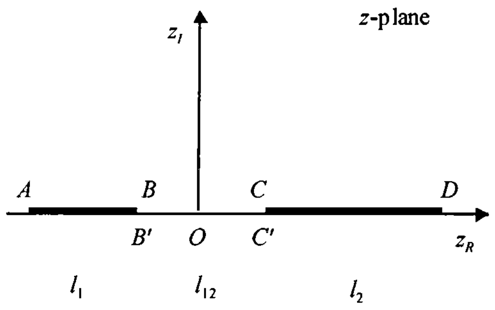

Fig. 1 The cross section of two parallel strip lines

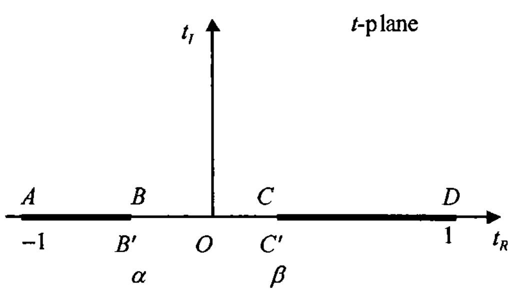

Fig. 2 The t plane

On the z plane, we put the original O at the midpoint of AD as shown in Fig.1. Making the transformation

$$z = \frac{1}{2}(l_1 + l_{12} + l_2)t \tag{1}$$

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Maps the z plane onto the t plane in Fig.2, where the coordinates of A, B, C and D are $-1, \alpha, \beta$ and 1, respectively. Two parameters α and β are

$$\alpha = \frac{l_1 - l_{12} - l_2}{l_1 + l_{12} + l_2} \tag{2}$$

$$\beta = \frac{l_1 + l_{12} - l_2}{l_1 + l_{12} + l_2} \tag{3}$$

Referring to Fig 3, the electrostatic field region on the t plane is transformed into the interior of rectangle BCC'B on the w plane by

$$t = \alpha + \frac{(1-\alpha)(1+\alpha)}{2sn^2(w,k) + \alpha - 1} \tag{4}$$

in which $sn(w,k)$ is Jacobi elliptic function[2] and the modulus k is

$$k = \sqrt{\frac{2(\beta-\alpha)}{(1-\alpha)(1+\beta)}} \tag{5}$$

2 The Force Between Two Lines

From Fig.3, we obtain the capacitance per unit length of the lines

$$C = 2\varepsilon_0 \frac{K'(k)}{K(k)} \tag{6}$$

where $K(k)$ is the first complete elliptic integral and

$$K'(k) = K(k') \tag{7}$$

k' is the complementary modulus of k, e.g.

$$k' = \sqrt{\frac{(1+\alpha)(1-\beta)}{(1-\alpha)(1+\beta)}} \tag{8}$$

Hence the electrostatic energy per unit length of the lines is

$$W = \frac{1}{2} C V^2 \tag{9}$$

Making use of the principle of the virtual work[3], we get the force between two lines per unit length

$$F = \left.\frac{\partial W}{\partial l_{12}}\right|_{V=c} = \frac{1}{2} V^2 \frac{\partial C}{\partial l_{12}} = \frac{1}{2} V^2 \frac{\partial C}{\partial k}\left(\frac{\partial k}{\partial \alpha}\frac{\partial \alpha}{\partial l_{12}} + \frac{\partial k}{\partial \beta}\frac{\partial \beta}{\partial l_{12}}\right) \tag{10}$$

Eq.(6) gives

$$\frac{\partial C}{\partial k} = -\frac{\pi\varepsilon_0}{kk'^2 K^2(k)} \tag{11}$$

Substituting Eqs.(2), (3), (5) and Eq.(11) into Eq.(10), we achieve

$$F = -\frac{\pi\varepsilon_0 V^2}{4K^2(k)}\left(\frac{1}{l_{12}} + \frac{1}{l_1 + l_{12} + l_2}\right) \tag{12}$$

The minus on the right side of Eq.(12) indicates that the force between two lines is attractive.

Introduce the normalized force as

$$f = \frac{F}{-\dfrac{\pi\varepsilon_0 V^2}{4l_1}} = \frac{l_1}{K^2(k)}\left(\frac{1}{l_{12}} + \frac{1}{l_1 + l_{12} + l_2}\right) \tag{13}$$

The manifold of f verses l_{12}/l_1 and l_2/l_1 is plotted in Fig.4, where we can see the effect of l_{12} on f is much more than that of l_2.

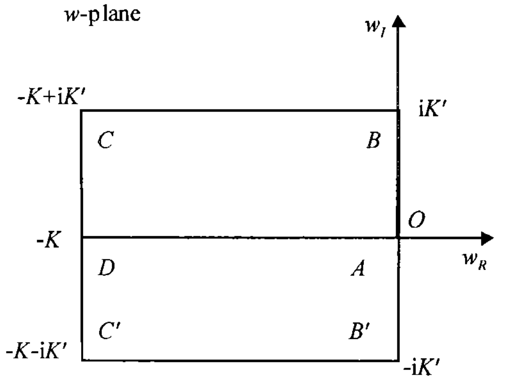

Fig.3　The w plane

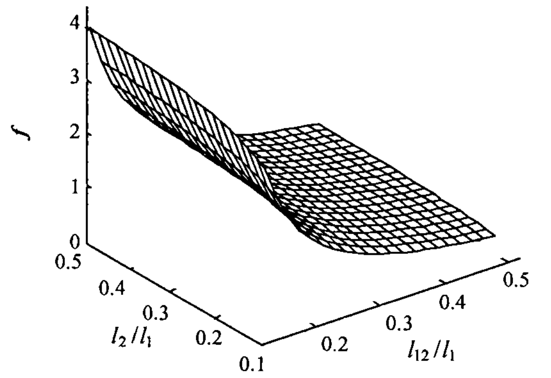

Fig. 4　The manifold of f verses l_{12}/l_1 and l_2/l_1

3　Discussion for Typical Cases

The general result in Eq.(12) provides us the basis to discuss following typical cases.

3.1　Equal Widths and Distance

Suppose the widths of two strips and the distance between them are equal. Substituting the condition $l_1 = l_2 = l_{12} = l$ into Eqs.(2), (3), and (5) we get

$$\alpha = -\frac{1}{3} \qquad \beta = \frac{1}{3} \qquad k = \frac{\sqrt{3}}{2} \tag{14}$$

Thus Eq.(12) reduces to

$$F = -\frac{\pi \varepsilon_0 V^2}{3 l K^2(\sqrt{3}/2)} \tag{15}$$

Under this condition the force F is inversely proportional to l.

3.2　Narrow Strip Line

From Eq.(6), the charge magnitude per unit length on each line is

$$q = CV = 2\varepsilon_0 \frac{K'(k)}{K(k)} V \tag{16}$$

Then Eq.(12) can be rewritten as

$$F = -\frac{\pi q^2}{16\varepsilon_0 K'^2(k)} \left(\frac{1}{l_{12}} + \frac{1}{l_1 + l_{12} + l_2} \right) \tag{17}$$

Suppose one strip line is narrow , for instance $l_1 << l_{12}$, we have $k' \to 0,\ K'(k) \to \frac{\pi}{2}$. Substituting above into Eq.(17), we get

$$F = -\frac{q^2}{4\pi\varepsilon_0} \left(\frac{1}{l_{12}} + \frac{1}{l_{12} + l_2} \right) \tag{18}$$

Putting $l_2 << l_{12}$ into Eq.(18) gives the force between the two narrow strip lines per unit length

$$F = -\frac{q^2}{2\pi\varepsilon_0 l_{12}} \tag{19}$$

that is as same as the force between two parallel charged strings which is acquainted to us.

From the above two equations we can draw following conclusion: the force between a charged string and a strip line is equal to the average of forces between the string and another pair of strings located on the two edges of the strip with the condensed charge of that strip line.

3.3 Broad Strip Lines

When the widths of the strip lines are so broad that $l_1 \sim l_2 >> l_{12}$, we have $k=0$ and $K(k)=\dfrac{\pi}{2}$ approximately. Hence from Eq.(12) the approximate expression of the force is

$$F=-\frac{\varepsilon_0 V^2}{\pi l_{12}} \tag{20}$$

3.4 One Strip Line Is Narrow, the Other Is Broad

Applying the condition $l_1 << l_{12} << l_2$ on Eq.(18) produces

$$F=-\frac{q^2}{4\pi\varepsilon_0 l_{12}} \tag{21}$$

which is a half of the one in Eq.(19).

4 Conclusion

For the two charged parallel strip transmission lines in a plane, the universal electrostatic field on the cross section can be transformed into the interior of a rectangle on the complex plane with conformal mapping method. The principle of virtual work is a handy way to achieve the force between two lines per unit length. The effect of geometric parameters of two lines on the force are illustrated and discussed. The general result includes some essential special cases, such as the force between strings, narrow lines, broad lines, etc..

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共面平行微带传输线间的静电力*

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【摘要】 将两共面平行荷电微带传输线在横截面内的场区用保形映射变换为矩形内域， 从而获得单位纵长传输线的电容和静电能量。以虚功原理计算荷电微带线间的静电力，图示出规范力随系统几何参数的变化，并分析一些典型特例，其中包括荷电弦、窄带和宽带间的静电力。

关 键 词 传输线; 保形映射; 虚功原理; 静电力

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