Magnetic Force Between Two Coaxial Square Coils Carrying Currents*

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Abstract In order to achieve the magnetic force between two coaxial square coils carrying currents,the coefficient of mutual induction is computed.Then principle of virtual work is employed as a handy way to calculate the force.The extreme value property of the force is analyzed and illustrated.The preeminent mechanical state where no magnetic tension force exerts on either of the two concentric square coils carrying currents is described.

Key words coaxial square coils; magnetic force; Neumann's formula; principle of virtual work

It has engineering significance to calculate the magnetic force between two coils carrying currents.The magnetic force between two circular currents has been studied in Ref.[1].The aim of this paper focuses on the magnetic force between two square coils carrying currents.

Based upon Neumann's formula,the coefficient of the mutual induction is computed first.Then the expression of the magnetic force between two coils is achieved by employing the principle of virtual work.The extreme value position of the force is analyzed and illustrated.The condition in which one of the concentric square coils in a plane carrying current has a preeminent mechanical state is discussed,where no magnetic tension force exerts.

1 Coefficient of the Mutual Induction

Fig.1 shows two parallel coaxial square coils perpendicular to the axis OO'.The lower coil has the side length of $2a$ and the current of I.The upper coil has $2a'$ and I'.$OO'=h$ is the distance between them.Suppose the directions of I and I' are the same and $a \geqslant a'$.

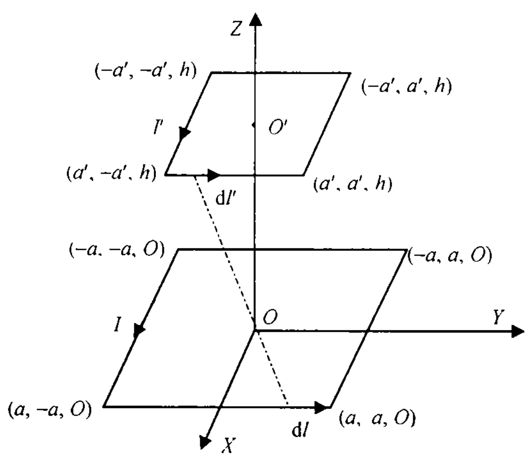

Fig.1 Two parallel coaxial square coils carry currents

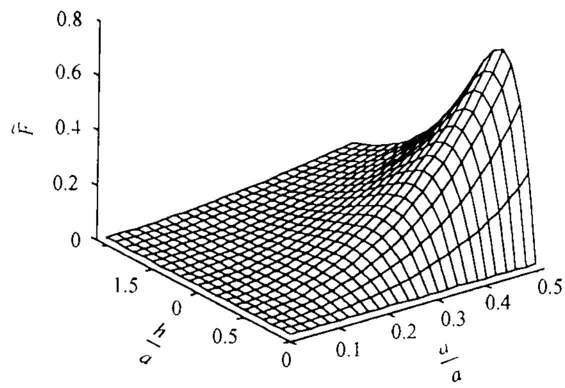

Fig.2 $\tilde{F}$ verses $\frac{a'}{a}$ and $\frac{h}{a}$

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Denote the line element on the lower and upper coils as dl and dl', respectively. The distance between the two elements is R.According to Neumann's formula[2] the coefficient of mutual induction between two coils is expressed by

$$M = \frac{\mu_0}{4\pi} \oint_{l'} \oint_{l} \frac{\mathrm{d}\, l \cdot \mathrm{d} l'}{R} \tag{1}$$

Obviously,we have $\mathrm{d}l \cdot \mathrm{d}l'=0$ on the perpendicular sides of the lower and upper square coils.Referring Fig.1 and considering the symmetry,we can rewrite Eq.(1) as

$$M = \frac{\mu}{\pi} \int_{-a}^{a} \int_{-a'}^{a'} \left[\frac{1}{\sqrt{(a-a')^2 + (y-y')^2 + h^2}} - \frac{1}{\sqrt{(a+a')^2 + (y-y')^2 + h^2}} \right] \mathrm{d}y' \mathrm{d}y \tag{2}$$

Calculating the integration in above equation gives

$$M = \frac{\mu_0}{\pi} \left[(a-a') \ln \frac{(a-a'+\sqrt{2(a^2+a'^2)+h^2})(-a+a'+\sqrt{2(a-a')^2+h^2})}{(-a+a'+\sqrt{2(a^2+a'^2)+h^2})(a-a'+\sqrt{2(a-a')^2+h^2})} \right] +$$
$$(a+a') \ln \frac{(a-a'+\sqrt{2(a^2+a'^2)+h^2})(-a-a'+\sqrt{2(a+a')^2+h^2})}{(-a-a'+\sqrt{2(a^2+a'^2)+h^2})(a+a'+\sqrt{2(a+a')^2+h^2})} +$$
$$2\sqrt{2(a-a')^2+h^2} - 4\sqrt{2(a^2+a'^2)+h^2} + 2\sqrt{2(a+a')^2+h^2} \tag{3}$$

2 Magnetic Force Between Coils

The symmetry indicates that the magnetic force between the two coils carrying currents has z-component only.The handy method to calculate this force is to use the principle of the virtual work[3]

$$F = F_z = II' \frac{\partial M}{\partial h} \tag{4}$$

Substituting Eq.(3) into above equation we get,after simplification

$$F = \frac{2\mu_0 II' h}{\pi} \left[\frac{\sqrt{2(a^2-a'^2)+h^2}}{(a-a')^2+h^2} - \frac{2(a^2-a'^2+h^2)\sqrt{2(a^2+a'^2)+h^2}}{[(a-a')^2+h^2][(a+a')^2+h^2]} + \frac{\sqrt{2(a^2+a'^2)+h^2}}{(a+a')^2+h^2} \right] \tag{5}$$

It is not difficult to prove $F<0$ which demonstrates the force between two coils is attractive when the currents have the same direction[4].

Define the normalized force as

$$\widetilde{F} = \frac{|F|}{\frac{2\mu_0 I_1 I_2}{\pi}} = h \left[\frac{2(a^2+a'^2+h^2)\sqrt{2(a^2+a'^2)+h^2}}{[(a-a')^2+h^2][(a+a')^2+h^2]} - \frac{\sqrt{2(a^2-a'^2)+h^2}}{(a-a')^2+h^2} - \frac{\sqrt{2(a^2+a'^2)+h^2}}{(a+a')^2+h^2} \right] \tag{6}$$

The function surface of $\widetilde{F}$ verses $\frac{a'}{a}$ and $\frac{h}{a}$ is ploted in Fig.2.

Eq.(5) indicates that both cases of $h \to 0$ and $h \to \infty$ lead to $F \to 0$.It means $|F|$,as a function of h,has some extreme value.To determine the extreme value position of $|F|$ as a and a',let

$$\frac{\partial |F|}{\partial h} = 0 \tag{7}$$

Substituting Eq.(5) into above equation produces

$$\frac{(a-a')^4}{[(a-a')^2+h^2]^2\sqrt{2(a-a')^2+h^2}}+\frac{(a+a')^4}{[(a+a')^2+h^2]^2\sqrt{2(a+a')^2+h^2}}-$$
$$2\frac{(a^4+a'^4)^2+2(a^6-5a^4a'^2-5a^2a'^4+a^6)h^2+(a^4-6a^2a'^2+a'^4)h^4}{[(a-a')^2+h^2][(a+a')^2+h^2]\sqrt{2(a^2+a'^2)+h^2}}=0 \quad (8)$$

This is the condition that h satisfies when $|F|$ reach its maximum.We can solve Eq.(8) with numerical method.For example,as $a'=0.5a$,the numerical solution of Eq.(8) is h=0.402 913a.

If the two currents on the coils are in opposite direction,the results are the same as above mentioned except direction of the magnetic force between two coils is repulsive.

3 Two Coils in A Plane

A typical special case is that the two coils are in the same plane.Substituting the condition h=0 into Eq.(5) arrives at F=0.It just means that the resultant of the magnetic force between two coils is zero.However,each side current of the square coils is still exerted on by some magnetic forces which rise from the currents,whatever the current is on the same square coil or else.Considering the symmetry and using the principle of the virtual work again,we can calculate the magnetic force on a side of the inner coil from the current on the coil itself as

$$f_L{}'=\frac{1}{8}I'^2\frac{\partial L'}{\partial a'} \quad (9)$$

Where L' is the self-inductance of the inner square coil.Denote the radius of the wire as r, L' can be given by Eq.(3) in condition of h=0 and $a=a'-r$.Then we have

$$L'=\frac{2\mu_0}{\pi}\left[r\ln\frac{2a'-r}{\sqrt{2(a'-r)^2+r^2}-r}+(2a'-r)\ln\frac{r}{\sqrt{(2a'-r)^2+r^2}-2a'+r}-\right.$$
$$\left.2a'\ln(\sqrt{2}+1)-2\sqrt{(2a'-r)^2+r^2}+2\sqrt{2}a'\right] \quad (10)$$

Substituting Eq.(10) to Eq.(9) yields

$$f_{L'}=\frac{\mu_0 I'^2}{2\pi}\left[\ln\frac{r}{(\sqrt{2}+1)(\sqrt{(2a'-r)^2+r^2}-2a'+r)}+\sqrt{2}\left(1-\frac{\sqrt{(a'-r)^2+a'^2}}{2a'-r}\right)\right] \quad (11)$$

Similarly,the magnetic force on a side of the inner coil from the current on the outer coil is

$$f_M{}'=\frac{1}{4}I'I\frac{\partial M_0}{\partial a'} \quad (12)$$

Where M_0 is the mutual inductance when h=0. From Eq.(3),it is written as

$$M_0=\frac{2\mu_0}{\pi}\left[(a-a')\ln\frac{a+a'}{\sqrt{2(a^2-a'^2)}-(a-a')}+(a+a')\ln\frac{a-a'}{\sqrt{2(a^2+a'^2)}-(a+a')}-\right.$$
$$\left.2a\ln(\sqrt{2}+1)+2\sqrt{2}a-2\sqrt{2(a^2+a'^2)}\right] \quad (13)$$

Two above equations lead to

$$f_M{}'=\frac{\mu_0 II'}{2\pi}\left[\ln\frac{a^2-a'^2}{(\sqrt{2(a^2+a'^2)}+a-a')(\sqrt{2(a^2+a'^2)}-a-a')}+2\sqrt{2}a'\frac{\sqrt{a'^2+a'^2}}{a^2-a'^2}\right] \quad (14)$$

Where $f_M{}'<0$ reflects that $f_M{}'$ is an attractive force if I and I' have same direction.Combining Eqs.(12) and (14), we obtain the resultant of magnetic force on a side of the inner coil

$$f'=f_L'-f_M'=\frac{\mu_0 I'}{2\pi}\Bigg[I'\ln\frac{r}{(\sqrt{2}+1)(\sqrt{(2a'-r)^2+r^2}-2a'+r)}-$$
$$I\ln\frac{a^2-a'^2}{(\sqrt{2(a^2+a'^2)}+a-a')(\sqrt{2(a^2+a'^2)}-a-a')}+$$
$$\sqrt{2}\left(I'-\frac{\sqrt{(a'-r)^2+a'^2}}{2a'-r}I'-\frac{2a'\sqrt{(a'^2+a'^2)^2}}{a^2-a'^2}I\right)\Bigg] \quad (15)$$

Similarly,the resultant of magnetic force on a side of the outer coil is

$$f=f_L+f_M=\frac{\mu_0 I}{2\pi}\Bigg[I\ln\frac{r}{(\sqrt{2}+1)(\sqrt{(2a-r)^2+r^2}-2a+r)}+$$
$$I'\ln\frac{a^2-a'^2}{(\sqrt{2(a^2+a'^2)}-a+a')(\sqrt{2(a^2+a'^2)}-a-a')}+$$
$$\sqrt{2}\left(I-\frac{\sqrt{(a-r)^2+a^2}}{2a-r}I-\sqrt{2}I'\ln(\sqrt{2}+1)+2I'-\frac{2a\sqrt{a^2+a'^2}}{a^2-a'^2}I'\right)\Bigg] \quad (16)$$

Above equations imply that there may be a case where no magnetic tension force exerts on either side of outer square coil when two coils carry currents in the same direction.Making $f=0$ in Eq.(16),we can determine the condition

$$\frac{I}{I'}=-\frac{\ln\dfrac{r}{(\sqrt{2}+1)(\sqrt{(2a-r)^2+r^2}-2a+r)}+\sqrt{2}\left(1-\dfrac{\sqrt{(a-r)^2+a^2}}{2a-r}\right)}{\ln\dfrac{a^2-a'^2}{(\sqrt{2(a^2+a'^2)}-a+a')(\sqrt{2(a^2+a'^2)}-a-a')}+2\sqrt{2}\left(1-\dfrac{\ln(\sqrt{2}+1)}{\sqrt{2}}-\dfrac{a\sqrt{a^2+a'^2}}{a^2-a'^2}\right)} \quad (17)$$

This is a preeminent mechanical state for the outer coil.The surface of $\frac{I'}{I}$ verses $\frac{a'}{a}$ and $\frac{r}{a}$ in Eq.(17) is sketched in Fig.3.

It is worthy of pointing out that there is a similar state where no magnetic tension force exerts on either side of inner coil when two coils carry currents in opposite direction. In much the same way,that condition can be found out easily.

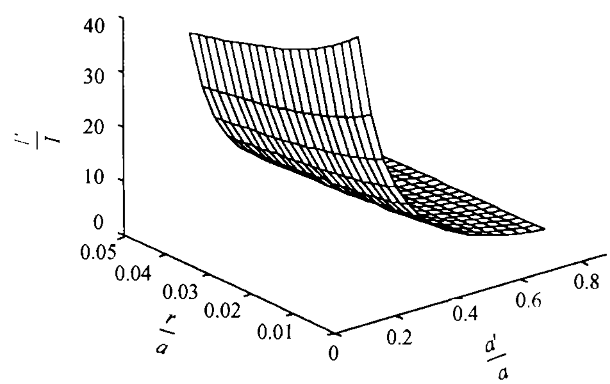

Fig.3 Surface of $\frac{I'}{I}$ verses $\frac{a'}{a}$ and $\frac{r}{a}$ in Eq.(17)

4 Summary

Evidently,it is a handy way to calculate the magnetic force between two coaxial square coils carrying currents employing the principle of virtual work.This force reaches its maximum value when the two coils are located in proper relative position.When the two coils are in a plane,it is possible to prove that no magnetic tension force exerts on the outer or inner coil.

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同轴方形载流线圈间的磁力*

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【摘要】 为获得二同轴方形载流线圈间相互作用磁力，计算出了线圈间的互感系数。将虚功原理作为一种简捷方法加以运用，对该力极值位置进行了分析和图示。讨论平面内二同心方形线圈载流时一线圈不受磁张力的优良力学状态。

关 键 词 同轴方形线；磁力；诺依曼公式；虚功原理

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驻厂军代室网络管理信息系统包括网络系统和管理信息系统两大部分。网络系统是面向地域分散、站点较多等客观情况而建立的一个以光集线器为中心的光纤星型结构，具有网络共享打印、网络文件共享及网络远程透明访问等功能。管理信息系统主要包括8个分系统：办公室管理、计划管理、成本管理、技术管理、热加工组管理、冷加工组管理、装试组管理及玻璃组管理。

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