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Formulation for Large Aspect Ratios Scattering Problems with Elliptic Cylindrical Wave Functions

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Abstract In this paper, a developed *T*-matrix method (Extended *T*-Matrix Method) based on elliptic cylindrical wave function is proposed for two-dimensional scattering problems. Computation results show that the method is well applied in large aspect ratio scattering problems.

Key words *T*-matrix; extended boundary condition; aspect ratios

The original *T*-matrix method is not available to determine the scattering field under the condition of large aspect ratio scattering body of resonant frequency[1]. In this paper, a developed *T*-matrix method (Extended *T*-Matrix Method)based on the elliptic cylindrical wave function is proposed for the two-dimensional scattering problems. The numerical results show that the extended *T*-matrix method can not only overcome the difficulties existing in original *T*-matrix method but also offer a reliable way for solving electromagnetic scattering problems of large aspect ratio scattering body with resonate mode.

1 *T*-Matrix Equation in Elliptic Cylindrical Wave Function

In this section, according to the method of extending boundary condition, we give *T*-matrix formulation on elliptic cylindrical wave function. For simplicity, we only discuss the transform in the case of TM wave incident scattering. As for the TE wave incident, the process is the same[2].

For conductor scattering problems, the process to determine the scattering field with *T*-matrix equation can be divided into two steps. Firstly, according to the fact that the electromagnetic fields are zero in the inner of scattering conductors, we obtain

$$E_Z^i(\boldsymbol{\rho}) = \frac{k_0\eta}{4}\int_\Gamma J_Z(\boldsymbol{\rho}')H_0^{(2)}(k|\boldsymbol{\rho}-\boldsymbol{\rho}'|)\mathrm{d}l \qquad \boldsymbol{\rho}\in\Omega \tag{1}$$

where J_Z is the surface current of conductor, E_Z^i is the incident field.

Therefore the scattering field in outer area $(\rho\notin\Omega)$ is

$$E_Z^s(\boldsymbol{\rho}) = -\frac{k_0\eta}{4}\int_\Gamma J_Z(\boldsymbol{\rho}')H_0^{(2)}(k|\boldsymbol{\rho}-\boldsymbol{\rho}'|)\mathrm{d}l \qquad \boldsymbol{\rho}\notin\Omega \tag{2}$$

Taking elliptic cylindrical function as the base function[2], we get

$$E_Z^i(\boldsymbol{\rho}) = 2j^{-m}\sum_{m=1}^{\infty}[p_{m-1}^{(e)^{-1}}Re_{m-1}^{(1)}(q,\mu)Se_{m-1}(q,v)Se_{m-1}(q,\phi_0)+ p_m^{(o)^{-1}}Ro_m^{(1)}(q,\mu)So_m(q,v)So_m(q,\phi_0)] \tag{3a}$$

where the coefficient $p_{m-1}^{(e)^{-1}}$ 、 $p_m^{(o)^{-1}}$ are identity factors. The factors can be expressed as

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$$p_m^{(e)} = \begin{cases} \dfrac{(-1)^m Se_{2m}(0) Se_{2m}\left(\dfrac{\pi}{2}\right)}{A_0^{2m}} \\ \dfrac{(-1)^{m+1} Se_{2m+1}(0) Se_{2m+1}\left(\dfrac{\pi}{2}\right)}{A_1^{2m+1}} \end{cases} \qquad p_m^{(o)} = \begin{cases} \dfrac{(-1)^m Se'_{2m}(0) Se'_{2m}\left(\dfrac{\pi}{2}\right)}{q^{\frac{1}{2}} B_2^{2m}} \\ \dfrac{(-1)^m Se'_{2m+2}(0) Se'_{2m+2}\left(\dfrac{\pi}{2}\right)}{q^{\frac{1}{2}} B_1^{2m+1}} \end{cases}$$

$E_Z^i(\boldsymbol{\rho})$ can also be expressed as

$$E_Z^i(\boldsymbol{\rho}) = (\boldsymbol{Re}^{(1)}, \boldsymbol{Ro}^{(1)}) \begin{bmatrix} \boldsymbol{A}^{(e)} \\ \boldsymbol{A}^{(o)} \end{bmatrix} \tag{3b}$$

where, $\boldsymbol{Re}^{(1)}$、$\boldsymbol{Ro}^{(1)}$ and $\boldsymbol{A}^{(e)}$、$\boldsymbol{A}^{(o)}$ are single rank and column matrixes respectively, and

$$\boldsymbol{Re}^{(1)} = (Re_0^{(1)}(q,\mu)Se_0(q,v), Re_1^{(1)}(q,\mu)Se_1(q,v), \cdots, Re_{m-1}^{(1)}(q,\mu)Se_{m-1}(q,v), \cdots)$$

$$\boldsymbol{Ro}^{(1)} = (Ro_1^{(1)}(q,\mu)Se_1(q,v), Re_2^{(1)}(q,\mu)Se_2(q,v), \cdots, Re_m^{(1)}(q,\mu)Se_m(q,v), \cdots)$$

$$\boldsymbol{A}^{(e)} = 2\begin{bmatrix} p_0^{(e)^{-1}} Se_0(q,\phi_0) \\ \vdots \\ j^{-m-1} p_{m-1}^{(e)^{-1}} Se_{m-1}(q,\phi_0) \\ \vdots \end{bmatrix}$$

$$\boldsymbol{A}^{(o)} = 2\begin{bmatrix} j^{-1} p_1^{(o)^{-1}} So_1(q,\phi_0) \\ \vdots \\ j^{-m} p_m^{(o)^{-1}} So_m(q,\phi_0) \\ \vdots \end{bmatrix}$$

Since the scattering field is in out, it can be extended as

$$E_Z^s(\boldsymbol{\rho}) = 2j^{-m} \sum_{m=1}^{\infty} [b_m^{(e)} Re_{m-1}^{(4)}(q,\mu)Se_{m-1}(q,v) + b_m^{(o)} Ro_m^{(4)}(q,\mu)So_m(q,v)] \tag{4a}$$

where $b_m^{(e)}$ and $b_m^{(o)}$ are expanding coefficients to be determined. For the same reason, $E_Z^s(\boldsymbol{\rho})$ can be expressed as

$$E_Z^s(\boldsymbol{\rho}) = (\boldsymbol{Re}^{(4)}, \boldsymbol{Ro}^{(4)}) \begin{bmatrix} \boldsymbol{B}^{(e)} \\ \boldsymbol{B}^{(o)} \end{bmatrix} \tag{4b}$$

where $\boldsymbol{Re}^{(4)}$、$\boldsymbol{Ro}^{(4)}$ and $\boldsymbol{B}^{(e)}$、$\boldsymbol{B}^{(o)}$ are single rank and column matrixes respectively.

$$\boldsymbol{Re}^{(4)} = (Re_0^{(4)}(q,\mu)Se_0(q,v), Re_1^{(4)}(q,\mu)Se_1(q,v), \cdots, Re_{m-1}^{(4)}(q,\mu)Se_{m-1}(q,v), \cdots)$$

$$\boldsymbol{Ro}^{(4)} = Ro_1^{(4)}(q,\mu)Se_1(q,v), Re_2^{(4)}(q,\mu)Se_2(q,v), \cdots, Re_m^{(4)}(q,\mu)Se_m(q,v), \cdots)$$

$$\boldsymbol{B}^{(e)} = 2\begin{bmatrix} b_0^{(e)} \\ \vdots \\ b_{m-1}^{(e)} \\ \vdots \end{bmatrix} \qquad \boldsymbol{B}^{(o)} = 2\begin{bmatrix} b_1^{(o)} \\ \vdots \\ b_m^{(o)} \\ \vdots \end{bmatrix}$$

Suppose the surface current can be extended as

$$J_z(\mu', v') = \sum_{m=1}^{\infty} [\alpha_{m-1}^{(e)} \frac{\partial}{\partial n'}(Re_{m-1}^{(1)}(q,\mu')Se_{m-1}(v')) + \alpha_m^{(o)} \frac{\partial}{\partial n'}(Ro_m^{(1)}(q,\mu')So_m(q,v'))] \tag{5a}$$

where $\dfrac{\partial}{\partial n'}$ stands for the differentiation to upright direction of the boundary surface. For the same reason, J_z can be expressed as matrix form

$$J_z=(\boldsymbol{Re}^{(1)'},\boldsymbol{Ro}^{(1)'})\begin{bmatrix}\alpha^{(e)}\\ \alpha^{(o)}\end{bmatrix} \tag{5b}$$

$\boldsymbol{Re}^{(1)'}$ 、 $\boldsymbol{Ro}^{(1)'}$ and $\boldsymbol{\alpha}^{(e)}$ 、 $\boldsymbol{\alpha}^{(o)}$ represent single rank and column matrixes respectively. Here

$$\boldsymbol{Re}^{(1)'}=(\frac{\partial}{\partial n'}(Re_0^{(1)}(q,\mu')Se_0(v')),\cdots,\frac{\partial}{\partial n'}(Re_{m-1}^{(1)}(q,\mu')Se_{m-1}(v')),\cdots)$$

$$\boldsymbol{Ro}^{(1)'}=(\frac{\partial}{\partial n'}(Ro_0^{(1)}(q,\mu')So_0(v')),\cdots,\frac{\partial}{\partial n'}(Ro_m^{(1)}(q,\mu')So_m(v')),\cdots)$$

$$\boldsymbol{\alpha}^{(e)}=\begin{bmatrix}\alpha_0^{(e)}\\ \vdots\\ \alpha_{m-1}^{(e)}\\ \vdots\end{bmatrix}\qquad \boldsymbol{\alpha}^{(o)}=\begin{bmatrix}\alpha_1^{(o)}\\ \vdots\\ \alpha_m^{(o)}\\ \vdots\end{bmatrix}$$

The relation between the coefficient of incident current and the coefficient of current extending can be obtained

$$\begin{cases}\boldsymbol{A}\equiv\begin{bmatrix}A^{(e)}\\ A^{(o)}\end{bmatrix}=[\boldsymbol{V}]\begin{bmatrix}\alpha^{(e)}\\ \alpha^{(o)}\end{bmatrix}\\ \boldsymbol{B}\equiv\begin{bmatrix}B^{(e)}\\ B^{(o)}\end{bmatrix}=-[\boldsymbol{U}]\begin{bmatrix}\alpha^{(e)}\\ \alpha^{(o)}\end{bmatrix}\end{cases} \tag{6}$$

Here $\boldsymbol{V}$ and $\boldsymbol{U}$ are the conjoin matrixes of expanding coefficient of incident wave, scattering wave to surface current expanding coefficient. It can be expressed as

$$\begin{cases}[\boldsymbol{V}]=\begin{bmatrix}V^1 & V^2\\ V^3 & V^4\end{bmatrix}\\ [\boldsymbol{U}]=\begin{bmatrix}U^1 & U^2\\ U^3 & U^4\end{bmatrix}\end{cases} \tag{7}$$

Elements $\boldsymbol{V}$ and $\boldsymbol{U}$ can be derived as

$$V_{mn}^1=\frac{k_0\eta}{2}\int_\Gamma Re_{m-1}^{(4)}(q,\mu')Se_{m-1}(q,v')\frac{\partial}{\partial n'}(Re_{n-1}^{(1)}(q,\mu')Se_{n-1}(v'))\mathrm{d}l'$$

$$V_{mn}^2=\frac{k_0\eta}{2}\int_\Gamma Re_{m-1}^{(4)}(q,\mu')Se_{m-1}(q,v')\frac{\partial}{\partial n'}(Ro_n^{(1)}(q,\mu')So_n(v'))\mathrm{d}l'$$

$$V_{mn}^3=\frac{k_0\eta}{2}\int_\Gamma Ro_m^{(4)}(q,\mu')So_m(q,v')\frac{\partial}{\partial n'}(Re_{n-1}^{(1)}(q,\mu')Se_{n-1}(v'))\mathrm{d}l'$$

$$V_{mn}^4=\frac{k_0\eta}{2}\int_\Gamma Ro_m^{(4)}(q,\mu')So_m(q,v')\frac{\partial}{\partial n'}(Ro_n^{(1)}(q,\mu')So_n(v'))\mathrm{d}l'$$

$$U_{mn}^1=\frac{k_0\eta}{2}\int_\Gamma Re_{m-1}^{(1)}(q,\mu')Se_{m-1}(q,v')\frac{\partial}{\partial n'}(Re_{n-1}^{(1)}(q,\mu')Se_{n-1}(v'))\mathrm{d}l'$$

$$U_{mn}^2=\frac{k_0\eta}{2}\int_\Gamma Re_{m-1}^{(1)}(q,\mu')Se_{m-1}(q,v')\frac{\partial}{\partial n'}(Ro_n^{(1)}(q,\mu')So_n(v'))\mathrm{d}l'$$

$$U_{mn}^3=\frac{k_0\eta}{2}\int_\Gamma Ro_m^{(1)}(q,\mu')So_m(q,v')\frac{\partial}{\partial n'}(Re_{n-1}^{(1)}(q,\mu')Se_{n-1}(v'))\mathrm{d}l'$$

$$U_{mn}^4=\frac{k_0\eta}{2}\int_\Gamma Ro_m^{(1)}(q,\mu')So_m(q,v')\frac{\partial}{\partial n'}(Ro_n^{(1)}(q,\mu')So_n(v'))\mathrm{d}l'$$

From Eq. (6), we can obtain

$$B = -UV^{-1}A \tag{8a}$$

From the definition of T-matrix

$$T = UV^{-1} \tag{8b}$$

We obtained the T-matrix (Extended T-matrix) equation which is based on the elliptic cylindrical wave function with which associated the expanding coefficients of incident field and scattering field.

$$B = -TA \tag{9}$$

2 Conclusion

In order to determine the scattering field under the condition of large aspect ratio scattering body in resonant mode, the Extended T-matrix method based on the elliptic cylindrical wave function is proposed in this paper. With the asymptotic characteristic values under the condition of q being much bigger than 1 in resonant frequency, the computer calculation shows that the developed method can satisfy the engineering requirement[3].

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大纵横比物体电磁散射问题的推广 *T* 矩阵程式

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【摘要】由于原始的 T 矩阵不适用于求解具有大纵横比物体的电磁散射问题，针对二维问题，通过对其格林函数的椭圆柱波函数展开，提出一种求解二维电磁散射问题椭圆柱波函数的 T 矩阵程式(推广 T 矩阵)。结果表明：推广 T 矩阵能够很好地用于处理谐振频域的大纵横比物体的电磁散射问题。

关 键 词 T 矩阵； 扩展边界条件； 纵横比

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