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Transition from *T*-Matrix Method to Canonical Solution

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Abstract　In this paper, the limitation of T-Matrix structure is analyzed. It is found that a solution by *T*-Matrix method can be transformed to a canonical solution when the boundary of the scatterer tends to the cylindrical form and the scatterer is illuminated by E-plane wave**.** The transition from *T*-Matrix method to canonical solution is established. It is concluded that *T*-Matrix is diagonal with the scatter boundary in this limit situation. This is also the limit of numerical solution.

Key words　*T*-matrix method；　canonical solution;　scattering

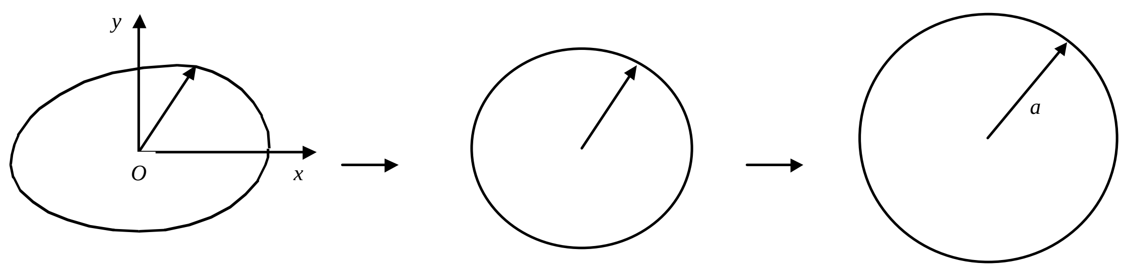
EBCM (Extended Boundary Condition Method) was first proposed by P. C. Waterman to estimate the scattered field by conductors[1]. In the method, the coefficients of the scattered field are linked to that of incident field by a transition matrix. So the method is usually called *T*-Matrix method. Since it has overcome the resonant problem and can save a lot of storage of computer[2], the method has been widely used in acoustic, electromagnetic scattering as well as plastic wave scattering.

In spite of its popularity, the analysis of the mechanism of *T*-Matrix structure is not so complete[3]. Especially in the transition from calculation result by *T*-Matrix method to the analytical solution when the boundary of the scatterer tends to a regular form which is in accordance with the coordinate system. This kind of work helps understanding the calculation results by the numerical method.

In this paper, we give a full analysis to the limitation of the *T*-Matrix method: when the boundary of the scatterer is in cylindrical form and the scatterer is illuminated by E-plane wave form. The transition from *T*-Matrix method to the canonical solution is established.

1　Transition From *T*-matrix Method To Canonical Solution

For simplicity, we only give the transition in case of E-plane wave incident scattering. As for the H-plane wave incident and dielectric scattering, the process is the same.



(a)　Irregular Boundary　　(b)　Sub-Regular Boundary　　(c)　Regular Boundary

Fig 1　The transition from irregular boundary to regular boundary

The coordinate system is shown as in Fig.1a. When the *T*-Matrix method is applied to the scatterer whose boundary tends to cylindrical form, on the boundary satisfies: $\mathrm{d}s'=a\mathrm{d}f'$. Since *T*-Matrix formulation consists of two matrix: one is $\boldsymbol{Q}^{+}$ matrix and the other is the inversion of $\boldsymbol{Q}^{-}$ matrix [4], the

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coefficient of surface current and factor k_0h as well as other factors will be canceled each other, respectively. For a regular boundary, $\boldsymbol{Q}^-$ and $\boldsymbol{Q}^+$ become

$$
\begin{cases}
\boldsymbol{Q}_{nm}^- = \int_S y_n f_m' \mathrm{d}s' = \\
\quad \int_0^{2ð} H_n^{(2)}(k_0a)\mathrm{e}^{-\mathrm{j}nj} \left.\dfrac{\partial J_m(k_0r')}{\partial r'}\right|_{r'=a} \mathrm{e}^{\mathrm{j}nj} a\mathrm{d}j' = \\
\quad aH_n^{(2)}(k_0a) \left.\dfrac{\partial J_m(k_0r')}{\partial r'}\right|_{r'=a} \int_0^{2\pi} \mathrm{e}^{-\mathrm{j}(n-m)j'} \mathrm{d}j' = \\
\quad 2\pi d_{n,m} aH_n^{(2)}(k_0a) \left.\dfrac{\partial J_m(k_0r')}{\partial r'}\right|_{r'=a} \\
\boldsymbol{Q}_{nm}^+ = \int_S y_{nr} f_m' \mathrm{d}s' = \\
\quad \int_0^{2\pi} J_n(k_0a)\mathrm{e}^{-\mathrm{j}nj} \left.\dfrac{\partial J_m(k_0r')}{\partial r'}\right|_{r'=a} \mathrm{e}^{\mathrm{j}nj} a\mathrm{d}j' = \\
\quad aJ_n(ka_0) \left.\dfrac{\partial J_m(k_0r')}{\partial r'}\right|_{r'=a} \int_0^{2\pi} \mathrm{e}^{-\mathrm{j}(n-m)j'} \mathrm{d}j' = \\
\quad 2\pi d_{n,m} aJ_n^{(2)}(k_0a) \left.\dfrac{\partial J_m(k_0r')}{\partial r'}\right|_{r'=a}
\end{cases}
\tag{1}
$$

where $d_{n,m}$ is the Kroneker sign

$$
d_{n,m} = \begin{cases} 1 & n = m \\ 0 & n \neq m \end{cases}
\tag{2}
$$

From Eqs.(1), we can see that $\boldsymbol{Q}^-$ and $\boldsymbol{Q}^+$ are both diagonal. The inversion of $\boldsymbol{Q}^-$ can be found easily

$$
\left[Q^-\right]_{nm}^{-1} = d_{n,m}\left[2\pi aH_n^{(2)}(k_0a) \left.\frac{¶J_m(k_0r')}{¶r'}\right|_{r'=a}\right]^{-1}
\tag{3}
$$

The elements of *T*-Matrix is

$$
\boldsymbol{T}_{n,m} = -\sum_{k=-N}^{N} (Q^+)_{n,k}(Q^-)_{k,m}^{-1} = -d_{n,m}\frac{J_n(k_0a)}{H_n^{(2)}(k_0a)}
\tag{4}
$$

Here, N represents the cutting off to the infinite series. From Eq. (4), we can see that the *T*-Matrix is diagonal.

To get the coefficient of scattering field, we find the coefficient of incident wave. Assuming the incident wave direction $j_i = 0$, and the plane wave is expressed into Bessel series, we get the incident wave coefficient [4]

$$
a_m = \mathrm{e}^{-\frac{\pi}{2}m\mathrm{j}}
\tag{5}
$$

By use of Eq.(4), the coefficient of scattered field

$$
\begin{aligned}
b_n &= -\sum_{m=-N}^{N} T_{n,m}a_m = -\sum_{m=-N}^{N} d_{n,m}\frac{J_n(k_0a)}{H_n^{(2)}(k_0a)}\mathrm{e}^{-\frac{\pi}{2}m\mathrm{j}} = \\
&-\frac{J_n(k_0a)}{H_n^{(2)}(k_0a)}\mathrm{e}^{-\frac{\pi}{2}m\mathrm{j}} = -(-\mathrm{j})^n\frac{J_n(k_0a)}{H_n^{(2)}(k_0a)}
\end{aligned}
\tag{6}
$$

The above result is the same as canonical solution. In this way we have completed the transition from *T*-Matrix method to the classical solution when the boundary of the scatterer tends to cylindrical boundary.

In the limit boundary, from Eq.(6), the *T*-Matrix is diagonal, all of the elements not situated in diagonal position will be zero. This is also the limit of numerical solution.

2 Conclusion

Although *T*-Matrix method is used to treat irregular boundary problem, when the boundary tends to the cylindrical form, the numerical solution obtained by *T*-Matrix method coincides with the canonical solution. Therefore we can see that, first, the structure kernels of *T*-Matrix method are rational. Second, the *T*-Matrix method differs from other numerical method, which is a combination of classical and numeric method. The combination has absorbed the merit of wave function, which satisfies wave equation and has good analytical property.

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*T*矩阵方法的解析解实现

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【摘要】*T*-MATRIX的结构分析及数值解的过渡特征与解析解的一致性研究有助于判别数值结果的可靠性。该文系统地分析了*T*-MATRIX的极限问题；讨论了二维散射体在理想边界情况下，E波入射时从*T*-MATRIX法到经典解析解的极限过渡。结果表明，在理想边界下，数值结果与经典理论完全吻合。

关 键 词 *T*矩阵法； 解析解； 散射

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