# Optimizing the Construction Relations of Chiral Slab via Genetic Algorithm<sup>\*</sup>

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**Abstract** A novel method of optimizing the continuous variable constitutive relation of multilayered chiral slab based on genetic algorithm is discussed in this paper. The method derive automatically the continuous variable constitutive relations. The reflection coefficient is calculated by wave-splitting method. Further more, the genetic algorithm is used to search the maximum absorbability of the chiral slab with certain thickness. The one point crossover GA and two points crossover GA have both been tested in the research, the latter one is really more stable than the first one and it's very important to set the proper crossover ration and mutation ration. The advantage of GA would be presented more obviously in designing artificial chiral material model with more variables.

Key words genetic algorithm; chiral slab; wave-splitting method; constitutive relation

Chiral materials was discovered in last century, the artificial microwave chiral media was also created by embedding randomly oriented conducting helixes in lossy dielectric host slab which is illustrated in Fig1<sup>[1]</sup>. When the linearly polarized plane wave passed through the material the plane of polarization would be rotated and part of the energy be absorbed by the dielectric host slab.

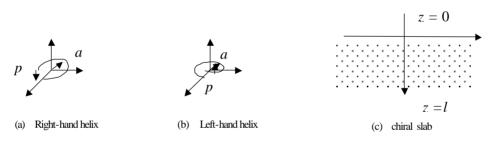


Fig.1 Helixes and chiral slab

Many papers reported that chiral media exhibits the enhanced absorption characteristics<sup>[2,3]</sup>. This paper proposes a method based on genetic algorithm to optimize the continuous variable constitutive relation of the stratified chiral absorber.

### **1** Reflection coefficient

There are several equivalent constitutive relations of chiral media. One of the common constitutive relations is

$$\begin{cases} \boldsymbol{D}(z) = \boldsymbol{e}(z)\boldsymbol{E}(z) \cdot i\boldsymbol{k}(z)\boldsymbol{H}(z) \\ \boldsymbol{B}(z) = \boldsymbol{m}(z)\boldsymbol{H} + i\boldsymbol{k}(z)\boldsymbol{E}(z) \\ \boldsymbol{E} = [E_x \quad , E_y]' \\ \boldsymbol{H} = [H_x \quad , H_y]' \end{cases}$$
(1)

Substitute (1) into the Maxwell equations<sup>[1]</sup>

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$$\partial_{z} \begin{bmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{bmatrix} = \begin{bmatrix} 0 & \boldsymbol{k} \, \boldsymbol{w} \sqrt{\boldsymbol{e}_{0} \boldsymbol{m}_{0}} & 0 & -i \boldsymbol{w} \boldsymbol{m} \\ -\boldsymbol{k} \, \boldsymbol{w} \sqrt{\boldsymbol{e}_{0} \boldsymbol{m}_{0}} & 0 & i \boldsymbol{w} \boldsymbol{m} & 0 \\ 0 & i \boldsymbol{w} \boldsymbol{e} & 0 & \boldsymbol{k} \, \boldsymbol{w} \sqrt{\boldsymbol{e}_{0} \boldsymbol{m}_{0}} \\ -i \boldsymbol{w} \boldsymbol{e} & 0 & -\boldsymbol{k} \, \boldsymbol{w} \sqrt{\boldsymbol{e}_{0} \boldsymbol{m}_{0}} & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{bmatrix} = \boldsymbol{W} \begin{bmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{bmatrix}$$
(2)

According to the wave-splitting method, the electromagnetic field is decomposed into downward and up-going modes satisfying the following equation<sup>[4,5]</sup>

$$\partial_{z} \begin{bmatrix} \mathbf{F}^{+} \\ \mathbf{F}^{-} \end{bmatrix} = \mathbf{T}_{I} \mathbf{W} \mathbf{T}_{1}^{-1} \begin{bmatrix} \mathbf{F}^{+} \\ \mathbf{F}^{-} \end{bmatrix} - \mathbf{T}_{I} (\partial_{z} \mathbf{T}_{1}^{-1}) \begin{bmatrix} \mathbf{F}^{+} \\ \mathbf{F}^{-} \end{bmatrix} = \begin{bmatrix} \mathbf{a} & \mathbf{b} \\ \mathbf{g} & \mathbf{d} \end{bmatrix} \begin{bmatrix} \mathbf{F}^{+} \\ \mathbf{F}^{-} \end{bmatrix}$$
(3)  
$$\mathbf{a} = -\mathbf{i} \mathbf{w} \mathbf{I} + \frac{1}{4} (\frac{\mathbf{m}_{z}}{\mathbf{m}} - \frac{\mathbf{e}_{z}}{\mathbf{e}}) \mathbf{I} \qquad \mathbf{b} = -\mathbf{i} \mathbf{w} \mathbf{c} \sqrt{\mathbf{e}_{0}} \mathbf{m}_{0} \mathbf{I}_{2} - \frac{\mathbf{i}}{4} (\frac{\mathbf{m}_{z}}{\mathbf{m}} - \frac{\mathbf{e}_{z}}{\mathbf{e}}) \mathbf{I}_{I}$$
$$\mathbf{g} = -\mathbf{i} \mathbf{w} \mathbf{c} \sqrt{\mathbf{e}_{0}} \mathbf{m}_{0} \mathbf{I}_{2} + \frac{\mathbf{i}}{4} (\frac{\mathbf{m}_{z}}{\mathbf{m}} - \frac{\mathbf{e}_{z}}{\mathbf{e}}) \mathbf{I}_{I} \qquad \mathbf{d} = \mathbf{i} \mathbf{w} \mathbf{I} + \frac{1}{4} (\frac{\mathbf{m}_{z}}{\mathbf{m}} - \frac{\mathbf{e}_{z}}{\mathbf{e}}) \mathbf{I}$$
$$\mathbf{I} = \begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{bmatrix} \qquad \mathbf{I}_{1} = \begin{bmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} \end{bmatrix} \qquad \mathbf{I} = \begin{bmatrix} \mathbf{I}_{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{2} \end{bmatrix} \qquad \mathbf{I}_{2} = \sqrt{\mathbf{e} \mathbf{m}} - \mathbf{k} \sqrt{\mathbf{e}_{0}} \mathbf{m}_{0}$$

In order to solve Eq. (3), two matrixes  $g^+(z)$  and  $g^-(z)$  of Green's functions was introduced

$$\begin{bmatrix} \boldsymbol{F}^{+} \\ \boldsymbol{F}^{-} \end{bmatrix} (z) = \begin{bmatrix} \boldsymbol{g}^{+}_{(z)} \boldsymbol{F}^{+}_{(0-)} \\ \boldsymbol{g}^{-}_{(z)} \boldsymbol{F}^{+}_{(0-)} \end{bmatrix} \qquad 0 < z < l$$

$$(4)$$

For the arbitrary incident modes, the Green's matrixes obey the following ODEs

$$\begin{cases} \partial_z g^- = gg^+ + dg^- \\ \partial_z g^+ = ag^+ + bg^- \end{cases}$$
(5)

$$\begin{cases} \boldsymbol{g}_{(0)}^{+} = \frac{1}{2} \left[ 1 + \sqrt{\frac{\boldsymbol{e}(0^{-})/\boldsymbol{m}(0^{-})}{\boldsymbol{e}(0^{+})/\boldsymbol{m}(0^{+})}} \right] \boldsymbol{I} + \frac{i}{2} \left[ 1 - \sqrt{\frac{\boldsymbol{e}(0^{-})/\boldsymbol{m}(0^{-})}{\boldsymbol{e}(0^{+})/\boldsymbol{m}(0^{+})}} \right] \boldsymbol{I}_{I} \boldsymbol{r}(0^{-}) \\ \boldsymbol{g}_{(0)}^{-} = -\frac{i}{2} \left[ 1 - \sqrt{\frac{\boldsymbol{e}(0^{-})/\boldsymbol{m}(0^{-})}{\boldsymbol{e}(0^{+})/\boldsymbol{m}(0^{+})}} \right] \boldsymbol{I}_{I} + \frac{1}{2} \left[ 1 + \sqrt{\frac{\boldsymbol{e}(0^{-})/\boldsymbol{m}(0^{-})}{\boldsymbol{e}(0^{+})/\boldsymbol{m}(0^{+})}} \right] \boldsymbol{r}(0^{-}) \end{cases}$$

$$z=0$$

Where the matrix r(z) is defined according to

$$\boldsymbol{F}_{(z)}^{-} = \boldsymbol{r}(z)\boldsymbol{F}_{(z)}^{+} = \begin{bmatrix} r_{11}(z) & r_{12}(z) \\ r_{21}(z) & r_{22}(z) \end{bmatrix} \boldsymbol{F}_{(z)}^{+}$$
(6)

Substituting (6) into (4), because the incident modes are arbitrary, one can obtain

$$\partial_{z} \boldsymbol{r} = \boldsymbol{g} + (\boldsymbol{d}\boldsymbol{r} - \boldsymbol{r}\boldsymbol{a}) - \boldsymbol{r}\boldsymbol{b}\boldsymbol{r}$$

$$\boldsymbol{r}(l) = i \frac{\sqrt{\boldsymbol{e}(l^{+})/\boldsymbol{m}(l^{+})} - \sqrt{\boldsymbol{e}(l^{-})/\boldsymbol{m}(l^{-})}}{\sqrt{\boldsymbol{e}(l^{+})/\boldsymbol{m}(l^{+})} + \sqrt{\boldsymbol{e}(l^{-})/\boldsymbol{m}(l^{-})}} \boldsymbol{I}_{I}$$

$$(7)$$

In this paper the Runge-Kutta method is used to solve the ODE (7) and got  $r(0^+)$ . The physical reflection coefficient  $r(0^-)$  is calculated by<sup>[1]</sup>

$$\mathbf{r}(0^{-}) = \left[ -i \left( 1 - \sqrt{\frac{\mathbf{e}(0^{+})/\mathbf{m}(0^{+})}{\mathbf{e}(0^{-})/\mathbf{m}(0^{-})}} \right) \mathbf{I}_{I} + \left( 1 + \sqrt{\frac{\mathbf{e}(0^{+})/\mathbf{m}(0^{+})}{\mathbf{e}(0^{-})/\mathbf{m}(0^{-})}} \right) \mathbf{r}(0^{+}) \right] \times$$

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第30卷

$$\left[ \left( 1 + \sqrt{\frac{e(0^+)/m(0^+)}{e(0^-)/m(0^-)}} \right) \mathbf{I} + i \left( 1 - \sqrt{\frac{e(0^+)/m(0^+)}{e(0^-)/m(0^-)}} \right) \mathbf{I}_I \mathbf{r}(0^+) \right]^{-1}$$
(8)

### 2 Genetic Algorithm

The classic genetic algorithm is applied in this paper, the permittivity permeability and chirality parameter of the first layer is fixed in advance according to the material and frequency. The variable process of the electromagnetic parameters of multi-layered slab is coded by binary string. Here (i)  $\mu$  (i) is the permittivity and permeability of layer *i*'(Table 1), the and are the small amount changes of permittivity and permeability from layer *ii* to *ii*+1',and to set the region for (i)  $\mu$  (i) which is practicable and abort the values out of this region. By this way, the characters of the multi-layered chiral slab can be assembled.

Number of layer	Code of	Value of each	Code of µ	Value of each µ
1	1	(1)+	0	μ (1)-
2	1	(1)+	1	μ <sub>(2)</sub> +
3	1	(1)+	0	μ (1)-
<i>n</i> -1	0	(4)-	1	$\mu_{(2)}+$
n	1	(1)+	0	μ (1)-

Table 1 Construct the electromagnetic parameters of multi-layered slab

The design requires the reflection R to be a minimum, so it sets the value of 1-R as the fitness of the accordingly chromosome. In a generation a series of fitness is obtained, and to sum up the series fitness to the total fitness of the population

$$F_{Sum} = \sum_{1}^{P_{Size}} R_i \tag{9}$$

In classic GA, the ratio of each chromosome to the total fitness was calculated by

$$Ra_{i} = \frac{R_{i}}{F_{sum}}$$
(10)

The probability of selection is

$$P_{ri} = \sum_{n=1}^{i} Ra_n \tag{11}$$

After the roulette wheel (the probability) is created, it can generate  $P_s'$  of random values  $T_{ii}$  among the range [0,1]. Then compared the  $T_i$  with the probability  $P_{ri}$ , the chromosomes whose probability is the nearest larger than  $T_i$  are selected to create the next generation. By this way, the chromosomes with larger fitness have more opportunities to propagate their good characters. Obviously, some chromosomes with small fitness would be extinct. The new chromosomes in new generation are created by crossover and mutation. In this paper to set the probability of crossover  $P_x' = 0.8$ . which means that 80% of chromosomes would undergo crossover. For every selected chromosome generating a random value x' from the range [0,1] and taking those chromosomes whose x<0.8 for crossover, can mate them into two pairs(if the number is odd, remove one from it). Generate two random integers from range [1,n-1]. The two integers determined the crossing points, exchange the four parts of mated chromosomes according the crossing point, then two new chromosomes can be constructed. Mutation is also a necessarily process of

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GA to get the optimized value among overall situation. By set the probability of mutation  $P_m = 0.15$ , a random value 'x' in range [0,1] is produced for each chromosome in the new generation, when the value'x'<0.15, it can construct a new chromosome randomly to substitute this one accordingly.

# **3** Examples

In this section, two numerical examples were presented. At first, one point crossover GA was applied. Then two points crossover GA is demonstrated.

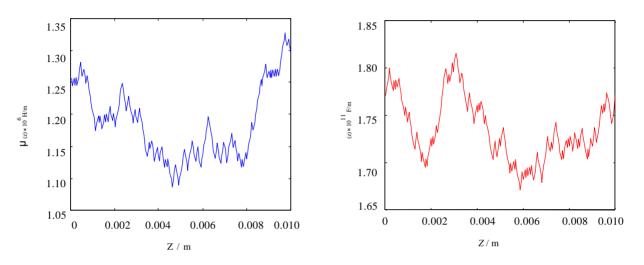


Fig.2 Constructive relations given by one point crossover GA

As we all know that the chiral slab divided into more layers the more cases could be simulated, but the more computational time is needed. According to the numeric experiments we divided the slab into 200 layers. The amount of chromosomes for each generation is 50, propagating for 500 generations. And  $P_x = 0.8$ ,  $P_m = 0.15$ . On frequency 20 GHz the first layer's permeability  $\mu_{(1)} = \mu_0$  permittivity (1)=2  $_0$  and chirality parameter =0.2; and are the small amount changes of permittivity and

permeability from layer *i*' to *i*+1'. =0.01  $_{\circ}$ ; =0.008  $\mu_{\circ}$ . The result is shown in Fig2 which get the reflect coefficient R= 20lg  $r_{(0^{-})}$  = -17.538 781dB.

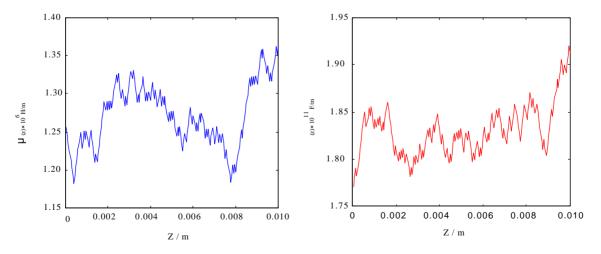


Fig.3 Constructive relations given by two point crossover GA.

As We also explored two point crossover GA, in this experiment the same parameters as one point crossover was used except  $P_m$ =0.1. We got the reflect coefficient  $R=20lg(r_{(0^-)})=-22.524830$  dB from the constructive relations given by two point crossover GA (shown in Fig.3). When the center

frequency is 20 GHz, this constructive relation's frequency band is 2.7 GHz with which the reflect coefficient is less than -20dB. Of course the genetic method can be used to reconstruct a new constructive relations in a broad frequency band instead of a single frequency point if the computer can work steadily.

#### 4 Discussion

Benefited from GA, the method demonstrated here could synthesize continuous abnormal stratified chiral absorber though it cost a long time to get the result. During the experiments, we find that two points crossover is really more stable than one point and it's very important to set the proper crossover ration and mutation ration to get the better result. The advantage of GA would be displayed more obviously if a mathematical model with more variables for artificial chiral material could be quoted to the design.

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# 遗传算法对多层旋波媒质本构关系的优化\*

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【摘要】运用遗传算法对连续变化的多层旋波媒质的本构关系进行了优化,设计了新的编码方案,用波分 法对与编码相应的多层旋波媒质的电磁散射特性进行计算,通过进化筛选,得到了由多层旋波媒质材料构成 的具有较好吸波性能的本构关系曲线。结果表明遗传算法在优化设计中的编码灵活,具有全局搜索等优越性. 关 键 词 遗传算法; 旋波媒质; 波分方法; 本构关系

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