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模糊粗糙集的分解定理*

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【摘要】 根据模糊粗糙集(FR 集)中上下近似都是模糊集合的构造性质，利用模糊集不同截集定义和分解定理，给出了模糊粗糙集(FR 集)截集定义和相应的分解定理，并证明了模糊粗糙集(FR 集)集的分解定理，分解定理揭示了模糊粗糙集和普通集之间的关系，在理论和应用中发挥着重要的作用。

关 键 词 模糊集； 粗糙集； 模糊粗糙集； 分解定理

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粗糙集理论和模糊集理论是研究信息系统中知识的不完善[1~4]、不准确问题。但粗糙集理论的解决问题的出发点是信息系统中知识的不可分辨性，而模糊集理论着眼于集合的模糊性，其解决问题的出发点是信息系统中知识的模糊性。将二者结合研究就形成了模糊粗糙集。文献[5]给出了关于模糊粗糙集的概念，但关于模糊粗糙集的截集定义及相应的分解定理并没有给出。本文主要在文献[6]的基础上引入了模糊粗糙集的截集定义，并给出这些截集相应的分解定理，进一步的完善了模糊粗糙集的理论。

1 模糊粗糙集的截集定义

定义 1[5] 若 R 为粗糙集之集合，$X=\{X_{Low}, X_{Up}\}\in R$, L 是格，则 X 中一个模糊粗糙集 $A=(A_{Low}, A_{Up})$ 由一对映射 $m_{A_{Low}}$ 与 $m_{A_{Up}}$ 来刻划

$$m_{A_{Low}}: X_{Low}\to L \quad m_{A_{Up}}: X_{Up}\to L \qquad \text{且 } m_{A_{Low}}\le m_{A_{Up}} \quad \forall x\in X_{Up}$$

定义 2 若 $A_{Low}: X_{Low}\to L$（记为 $A_{Low}\in L^{X_{Low}}$） $A_{Up}: X_{Up}\to L$ （记为 $A_{Up}\in L^{X_{Up}}$），(X_{Low}, X_{Up})为粗糙集，L为格，定义 $\forall l\in L$

$$A_{Low}^{l}=\{x\in X_{Low}\mid A_{Low}(x)\le l\}\subseteq X_{Low} \qquad A_{Up}^{l}=\{x\in X_{Up}\mid A_{Up}(x)\le l\}\subseteq X_{Up}$$

$$A_{Low}^{i}=\{x\in X_{Low}\mid A_{Low}(x)< l\}\subset X_{Low} \qquad A_{Up}^{i}=\{x\in X_{Up}\mid A_{Up}(x)< l\}\subset X_{Up}$$

则 $(A_{Low}^{l}, A_{Up}^{l})$ 称 FR 集 (A_{Low}, A_{Up}) 的 l _下截集 $(A_{Low}^{i}, A_{Up}^{i})$ 称 FR 集 (A_{Low}, A_{Up}) 的 l _强下截集。

定义 3 若 $A_{Low}: X_{Low}\to L$（记为 $A_{Low}\in L^{X_{Low}}$）$A_{Up}: X_{Up}\to L$（记为 $A_{Up}\in L^{X_{Up}}$），(X_{Low}, X_{Up}) 为粗糙集，L 为格，定义 $\forall l\in L$

$$A_{Low[l]}=\{x\in X_{Low}\mid A_{Low}(x)\ge 1-l\}\subseteq X_{Low} \qquad A_{Up[l]}=\{x\in X_{Up}\mid A_{Up}(x)\ge 1-l\}\subseteq X_{Up}$$

$$A_{Low[i]}=\{x\in X_{Low}\mid A_{Low}(x)> 1-l\}\subset X_{Low} \qquad A_{Up[i]}=\{x\in X_{Up}\mid A_{Up}(x)> 1-l\}\subset X_{Up}$$

则 $(A_{Low[l]}, A_{Up[l]})$ 称 FR 集 (A_{Low}, A_{Up}) 的 l _下重截集 $(A_{Low[i]}, A_{Up[i]})$ 称 FR 集 (A_{Low}, A_{Up}) 的 l _强下

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下重截集。

定义 4　若 $A_{Low}: X_{Low} \to L$（记为 $A_{Low} \in L^{X_{Low}}$）$A_{Up}: X_{Up} \to L$（记为 $A_{Up} \in L^{X_{Up}}$），$(X_{Low},\ X_{Up})$为粗糙集，*L* 为格，$\forall l \in L$ 定义

$$A_{Low}^{[l]} = \{x \in X_{Low} \mid A_{Low}(x) \le 1 - l\} \subseteq X_{Low} \qquad A_{Up}^{[l]} = \{x \in X_{Up} \mid A_{Up}(x) \le 1 - l\} \subseteq X_{Up}$$

$$A_{Low}^{[i]} = \{x \in X_{Low} \mid A_{Low}(x) < 1 - l\} \subset X_{Low} \qquad A_{Up}^{[i]} = \{x \in X_{Up} \mid A_{Up}(x) < 1 - l\} \subset X_{Up}$$

则$\left(A_{Low}^{[l]}, A_{Up}^{[l]}\right)$称 FR 集$(A_{Low}, A_{Up})$的 l _上重截集，$\left(A_{Low}^{[i]}, A_{Up}^{[i]}\right)$ 称 FR 集(A_{Low}, A_{Up})的 l _强上重截集。

在本文中，给出了对应不同截集的分解定理。为了简洁起见，本文对所有结论不做具体证明，读者不难从有关文献中得出证明。

2　模糊粗糙集的分解定理

如果 *A*=(A_{Low}, A_{Up})是 *X* 上的 FR 集，*L*=[0，1]，对于 $l \in L$ 及 *X* 上的子集 *B*=(B_{Low}, B_{Up})，分别定义 *X* 的模糊子集 lB 及 $l\Theta B$ 如下：

$$(lB)(x) = \begin{cases} l, & x \in B \\ 0, & x \notin B \end{cases} \qquad (l\Theta B)(x) = \begin{cases} l & x \in B \\ 1 & x \notin B \end{cases}$$

于是有下列的分解定理。

定理 1　如果 *L* 是完备格，$A_{Low} \in L^{X_{Low}}$，$A_{Up} \in L^{X_{Up}}$，则

$$\begin{cases} A_{Low} = \bigwedge_{l\in L} l\Theta A_{Low}^{l} & A_{Low} = \bigwedge_{l\in L} l\Theta A_{Low}^{l} \\ A_{Low} = \bigwedge_{l\in L} l\Theta A_{Low}^{i} & A_{Low} = \bigwedge_{l\in L} l\Theta A_{Up}^{i} \end{cases} \tag{1}$$

$$\begin{cases} A_{\mathrm{Low}}^{\mathrm{c}} = \bigvee_{l\in L} l^{\mathrm{c}} A_{\mathrm{Low}}^{l} & A_{\mathrm{Up}}^{\mathrm{c}} = \bigvee_{l\in L} l^{\mathrm{c}} A_{\mathrm{Up}}^{l} \\ A^{\mathrm{c}}{}_{\mathrm{Low}} = \bigvee_{l\in L} l^{\mathrm{c}} A_{\mathrm{Low}}^{i} & A^{\mathrm{c}}{}_{\mathrm{Up}} = \bigvee_{l\in L} l^{\mathrm{c}} A_{\mathrm{Up}}^{i} \end{cases} \tag{2}$$

定理 2　若L是稠密的完备格 $A_{Low} \in L^{X_{Low}}$, $A_{Up} \in L^{X_{Up}}$, $R_{Low} = \{X_{Low} \mid (X_{Low},\ X_{Up}) \in R\}$。$R_{Up} = \{X_{Up} \mid (X_{Low},\ X_{Up}) \in R\}$影射 $H_{\mathrm{Low}}: L \to P(R_{\mathrm{Low}})$，$H_{Up}: FL \to P(R_{Up})$（$P$ 表示幂集）且适合 $\forall l \in L$，$A_{\mathrm{Low}}^{i} \subseteq H_{\mathrm{Low}}(l) \subseteq A_{\mathrm{Low}}^{l}$，$A_{Up}^{i} \subseteq H_{Up}(l) \subseteq A_{Up}^{l}$，则有

$$\begin{cases} A_{\mathrm{Low}}^{\mathrm{c}} = \bigvee_{l\in L} l^{\mathrm{c}} H_{\mathrm{Low}}(l) & A_{Up}^{c} = \bigvee_{l\in L} l^{\mathrm{c}} H_{Up}(l) \\ A_{\mathrm{Low}} = \bigwedge_{l\in L} l\Theta H_{\mathrm{Low}}(l) & A_{Up} = \bigwedge_{l\in L} l\Theta H_{Up}(l) \end{cases} \tag{3}$$

$$l_1 < l_2 \Rightarrow H_{Low}(l_1) \subseteq H_{Low}(l_2) \tag{4}$$

$$\begin{cases} A_{Low}^{l} = \bigwedge_{a<l} H_{Low}(a) & A_{Up}^{l} = \bigwedge_{a<l} H_{Up}(a) \\ A_{Low}^{i} = \bigvee_{a<l} H_{Low}(a) & A_{Up}^{i} = \bigvee_{a<l} H_{Up}(a) \end{cases} \tag{5}$$

定理 3 若 L 是完备格，$A_{Low}\in L^{X_{Low}}$，$A_{Up}\in L^{X_{Up}}$，则

$$\begin{cases} A_{Low}=\bigvee_{l\in L} l^c A_{Low[l]} & A_{Up}=\bigvee_{l\in L} l^c A_{Up[l]} \\ A_{Low}=\bigvee_{l\in L} l^c A_{Low[i]} & A_{Up}=\bigvee_{l\in L} l^c A_{Up[i]} \end{cases} \tag{6}$$

$$\begin{cases} A_{Low}^c=\bigwedge_{l\in L} l\Theta A_{Low[l]} & A_{Up}^c=\bigwedge_{l\in L} l\Theta A_{Up[l]} \\ A_{Low}^c=\bigwedge_{l\in L} l\Theta A_{Low[i]} & A_{Up}^c=\bigwedge_{l\in L} l\Theta A_{Up[i]} \end{cases} \tag{7}$$

定理 4 若 L 是稠密的完备格 $A_{Low}\in L^{X_{Low}}$，$A_{Up}\in L^{X_{Up}}$，$R_{Low}=\{X_{Low}\mid(X_{Low},X_{Up})\in R\}$，$R_{Up}=\{X_{Up}\mid(X_{Low},X_{Up})\in R\}$ 影射 $H_{Low}: L\to P(R_{Low})$，$H_{Up}: L\to P(R_{Up})$（P 表示幂集）且适合 $\forall l\in L$，$A_{Low[i]}\subseteq H_{Low}(l)\subseteq A_{Low[l]}$，$A_{Up[i]}\subseteq H_{Up}(l)\subseteq A_{Up[l]}$，则有

$$\begin{cases} A_{Low}^c=\bigwedge_{l\in L} l\Theta H_{Low}(l) & A_{Up}^c=\bigwedge_{l\in L} l\Theta H_{Up}(l) \\ A_{Low}=\bigwedge_{l\in L} l^c H_{Low}(l) & A_{Up}=\bigwedge_{l\in L} l^c H_{Up}(l) \end{cases} \tag{8}$$

$$l_1<l_2\Rightarrow H_{Low}(l_1)\subseteq H_{Low}(l_2)\quad H_{Up}(l_1)\subseteq H_{Up}(l_2) \tag{9}$$

定理 5 若 L 是完备格，$A_{Low}\in L^{X_{Low}}$，$A_{Up}\in L^{X_{Up}}$，则

$$\begin{cases} A_{Low[l]}=\bigwedge_{a>l} H_{Low}(a) & A_{Up[l]}=\bigwedge_{a>l} H_{Up}(a) \\ A_{Low[i]}=\bigvee_{a<l} H_{Low}(a) & A_{Up[i]}=\bigvee_{a<l} H_{Up}(a) \end{cases} \tag{10}$$

$$\begin{cases} A_{Low}=\bigwedge_{l\in L} l^c\Theta A_{Low}^{[l]} & A_{Up}=\bigwedge_{l\in L} l^c\Theta A_{Up}^{[l]} \\ A_{Low}=\bigwedge_{l\in L} l^c\Theta A_{Low}^{[i]} & A_{Up}=\bigwedge_{l\in L} l^c\Theta A_{Up}^{[i]} \end{cases} \tag{11}$$

$$\begin{cases} A_{Low}^c=\bigvee_{l\in L} lA_{Low}^{[l]} & A_{Up}^c=\bigvee_{l\in L} lA_{Up}^{[l]} \\ A_{Low}=\bigvee_{l\in L} lA_{Low}^{[i]} & A_{Up}=\bigvee_{l\in L} lA_{Up}^{[i]} \end{cases} \tag{12}$$

定理 6 若 L 是稠密的完备格 $A_{Low}\in L^{X_{Low}}$，$A_{Up}\in L^{X_{Up}}$，$R_{Low}=\{X_{Low}\mid(X_{Low},X_{Up})\in R\}$，$R_{Up}=\{X_{Up}\mid(X_{Low},X_{Up})\in R\}$ 影射 $H_{Low}: L\to P(R_{Low})$，$H_{Up}: L\to P(R_{Up})$，（P 表示幂集）且适合 $A_{Low}^{[i]}\subseteq H_{Low}(l)\subseteq A_{Low}^{[l]}$，$A_{Up}^{[i]}\subseteq H_{Up}(l)\subseteq A_{Up}^{[l]}$，则

$$A_{Low}^c=\bigvee_{l\in L} lH_{Low}(l)\qquad A_{Up}^c=\bigvee_{l\in L} lH_{Up}(l) \tag{13}$$

$$A_{Low}=\bigwedge_{l\in L} l^c\Theta H_{Low}(l)\quad A_{Up}=\bigwedge_{l\in L} l^c\Theta H_{Up}(l) \tag{14}$$

$$l_1<l_2\Rightarrow H_{Low}(l_1)\supseteq H_{Low}(l_2)\quad H_{Up}(l_1)\supseteq H_{Up}(l_2) \tag{15}$$

$$\begin{cases} A_{Low}^{[l]} = \underset{a>l}{\wedge} H_{Low}(a) & A_{Up}^{[l]} = \underset{a>l}{\wedge} H_{Up}(a) \\ A_{Low}^{[i]} = \underset{a>l}{\vee} H_{Low}(a) & A_{Up}^{[i]} = \underset{a>l}{\vee} H_{Up}(a) \end{cases} \tag{16}$$

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Decomposition Theorems of Fuzzy Rough Sets

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Abstract Based on the structural property of Fuzzy Rough Sets which a pair of lower and upper approximations are Fuzzy sets, and the different concepts of cut sets and decomposition theorems of Fuzzy sets are used to define the concepts of cut sets of Fuzzy Rough Sets and relevant decomposition theorems of Fuzzy Rough sets. Through the different decomposition theorems of Fuzzy sets, the different decomposition theorems of Fuzzy Rough Sets are proved. The relationship between Fuzzy Rough sets and normal sets is well explained by the decomposition theorem of Fuzzy Rough Sets. The decomposition theorem presented in this paper is very import and well applied both in theory and practice .

Key words fuzzy sets ; rough sets; fuzzy rough sets; decomposition theorem