Reflection Characteristics of Impregnated Absorbent Honeycomb under Normal Incidence of Plane Wave*

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Abstract A numerical method for calculating the reflection coefficient of the impregnated absorbent honeycomb with plane wave normal incident to its broad face is presented. By introducing a concept of impedance for the honeycomb wall and by the assistance of the periodic Green's function, the Electric Field Integral Equation for the infinite honeycomb surface is established and the unknown surface current is solved by the method of moment. Numerical results are obtained for various heights, different absorbent parameters and various impregnated thickness within the frequency band 8 ~12 GHz. It shows the proposed method is of high efficiency and can be used in the honeycomb fabrication and application of structural absorbers.

Key words honeycomb; absorbent; reflection coefficient; method of moment; periodic Green's function

平面波垂直入射浸渍吸收剂蜂窝的反射特性*

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【摘要】提出一个数值方法，计算当平面波垂直于蜂窝板入射时浸渍吸收剂的蜂窝的反射系数。在引入蜂窝壁表面阻抗后，借助于周期格林函数对无限大蜂窝表面建立了电场积分方程，用矩量法求解未知的表面电流。在8~12 GHz频段内，得到的不同蜂窝高度、不同吸收剂和不同浸渍厚度的数值结果表明，所提出的方法是高效的，可用于蜂窝制备和吸波结构的工作中。

关 键 词 蜂窝; 吸波; 反射系数; 矩量法; 周期格林函数

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Impregnated absorbent honeycomb has been widely used for fabricating parts of modern war plane, such as the front wedge of the wing, vertical stabilizer of the tail wing and the wings of missiles by taking advantages of its low reflection for reducing the radar cross section of the parts, low density and high mechanical strength. Although it had been widely used since 50 decades last century, the problem of searching a method to estimate the reflection coefficient from its geometric and material parameters was reminded until 1991. Jorgenson proposed a method by using Method of Moment(MOM) for calculating its reflection coefficient[1,2]. But the literatures only gave a brief description of the method. We used the method of moment to solve this problem, involving the method of calculation of surface impedance, the acceleration of convergence of periodic Green's function, the simplification of the calculation of the integrals involving rooftop basis function and razor test function. Also the method for

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utilizing the symmetry of the structure is to reduce the CPU time and store required. The numerical results show that the solver is reliable. A full analysis of how the various height of the honeycomb and various hole wide, different absorbent parameters and various impregnated thicknesses control its reflection coefficients was presented in details.

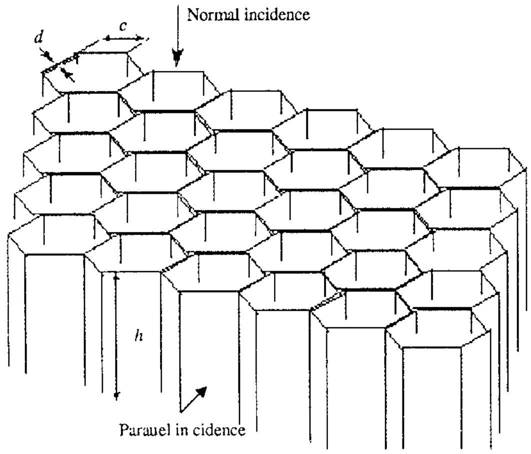


Fig.1 The outside view of honeycomb structure

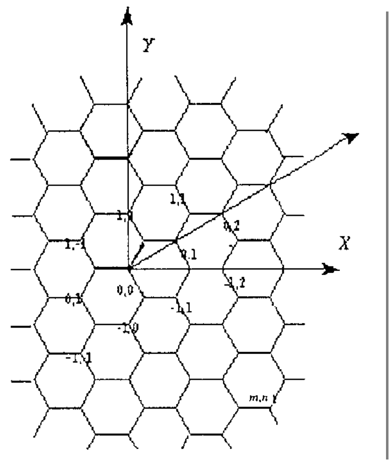


Fig.2 Board view of honeycomb

Fig.1 shows an outside view of a honeycomb board, the plane wave vector with arbitrary incidence angle may be divided into a normal component and a parallel component and be analyzed separately. In Fig.1, *c* is side wall width, *h* is height of board.

Fig.2 shows an infinite honeycomb, it may be viewed as formed by infinite number of *Y*-sections, all the *Y*-sections have the same current distribution under the normal incidence of plane wave. We choose one of these *Y*-sections, called the fundamental unit for calculation and use the periodic Green's function to consider the interactions of all the other sections. The infinite series of periodic Green's function are truncated at suitable length to give the results for engineering purpose. In Fig.2, $m = -\infty \cdots +\infty, n = -\infty \cdots +\infty$, which counts along *x* and *y* to locate in the center position of each *Y* section.

1 Method for Solving Surface Current

The refection coefficient of the honeycomb can be determined easily when the surface current distribution on the honeycomb is known. The following are the main procedures for calculating the surface current and some approaches we adopted in calculation.

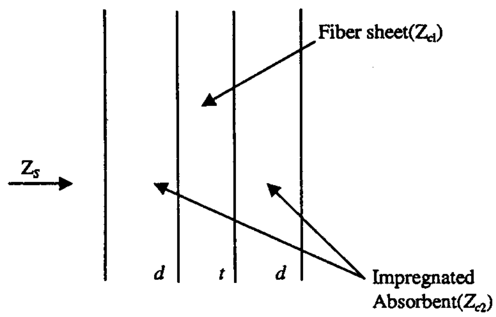


Fig.3 The honeycomb wall

1.1 Surface Impedance and EFIE of the Honeycomb

The electric field on the wall surface satisfies the following relation

$$ZJ = E^{inc} + E^{s} \tag{1}$$

where E^{inc} and E^{s} are the incidence field and scattered field respectively. *Z* is the surface impedance, which is introduced for describing the field which draw the current flowing on the surface. *Z* can be calculated by $Z_s = Z_{c2}[Z_2 + Z_{c2}\mathrm{th}(a_2 + \mathrm{j}b_2)d]/[Z_{c2} + Z_2\mathrm{th}(a_2 + \mathrm{j}b_2)d]$, $Z_2 = Z_{c1}[Z_1 + Z_{d}\,\mathrm{th}(a_1 + \mathrm{j}b_1)t]/[Z_{c1} + Z_1\mathrm{th}(a_1 + \mathrm{j}b_1)t]$, $Z_1 = Z_{c2}[Z_0 + Z_{c2}\mathrm{th}(a_2 + \mathrm{j}b_2)d]/[Z_{c2} + Z_0\mathrm{th}(a_2 + \mathrm{j}b_2)d]$, where

$$Z_{ci} = \sqrt{(m'_{ri} - \mathrm{j}m''_{ri})/(e'_{ri} - \mathrm{j}e''_{ri})} Z_0$$

which is wave impedance of the *i*th material. E^{s} has two components: the fields produced by current $\boldsymbol{J}(\boldsymbol{r}')$, and produced by charge $r_s(\boldsymbol{r}') = -\nabla' \cdot \mathrm{J}(\boldsymbol{r}')/\mathrm{j}we_0$ as

$$E^s(r) = -\mathrm{j}wm_0\int G_p(r/r')\boldsymbol{J}(r')\mathrm{d}S' + \frac{1}{\mathrm{j}we_0}\nabla\int \boldsymbol{G}_p(r/r')\nabla'\cdot\boldsymbol{J}(r')\mathrm{d}S' \quad (2)$$

where G_p is the periodic Green' s function defined by[3]

$$G\left(\frac{r}{r'}\right) = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\left\{\mathrm{e}^{-\mathrm{j}k_0\left[(r_0-r'-r_{mn})^2+(z_0-z')^2\right]^{1/2}}\right\}\Big/\left\{4\pi\left[(r_0-r'-r_{mn})^2+(z_0-z')^2\right]^{1/2}\right\} \quad (3)$$

(r_0, z_0) is the coordinates of the field point, $(\tilde{\boldsymbol{n}}' + \tilde{\boldsymbol{n}}_{mn}, z')$ is the coordinates of the source point, substituting them into Eq. (1),we get the Electric Field Integral Equation(EFIE)

$$\boldsymbol{E}^{inc}(r) = Z\boldsymbol{J}(r) + \mathrm{j}wm_0\left[\boldsymbol{J}(r') * G_p(r/r')\right] - \frac{1}{\mathrm{j}we_0}\nabla\left[\nabla'\cdot\boldsymbol{J}(r') * G_p(r/r')\right] \quad (4)$$

1.2 Moment Method

We use the method of moment to solve Eq(4).Divide the three dielectric plates of the fundamental unit into subdomains, the length of each subdomain does not exceed $l/10$, l is the wave length in free space, as shown in Fig.4.

Choose rooftop function as basic function to expand the unknown surface current on the dielectric plates. The rooftop function is shown in Fig.5a, each rooftop occupies two subdomains as[4]

$$J(r') = \sum[sj_{si}b_{si} + zj_{zi}b_{zi}] \qquad i = 1,2\cdots,N$$

where b_{si} and b_{zi} are the expanding coefficients.

Then choose razor function as test function, a function of field point coordinates, razor function is shown in Fig.5b.Take the inner product of the test function with Eq.(4), where the unknown current has been expanded by basic function, we have

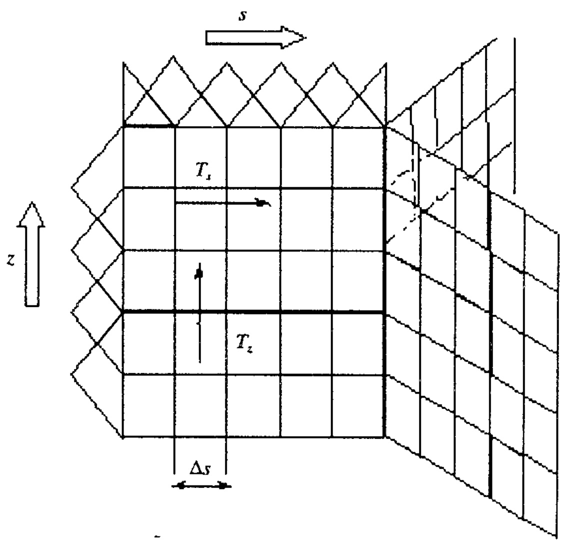
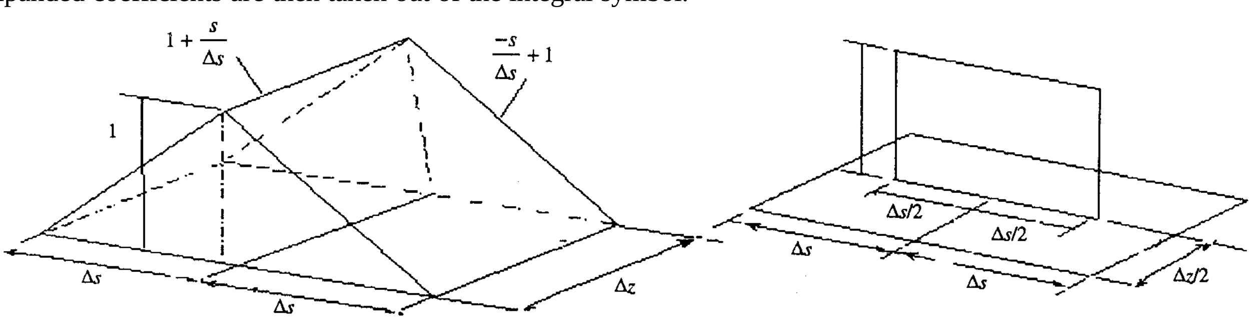


Fig.4 Basic functions and Test functions

$$\int_{si}\boldsymbol{T}_{si}\cdot\boldsymbol{E}_i^{inc}\mathrm{d}S_i = \int_{si}\boldsymbol{T}_{si}Z_s\boldsymbol{J}(r)_{si}\mathrm{d}S_i + \int_{si}\boldsymbol{T}_{si}\cdot\left[\mathrm{j}wm_0\int_S\sum_m\sum_n\boldsymbol{J}(r')G_p\left(\frac{r}{r'}\right)\mathrm{d}S(r')\right]\mathrm{d}S_i +$$

$$\int_{si}\nabla\cdot\boldsymbol{T}_{si}\left[\frac{1}{\mathrm{j}we_0}\int_S\sum_m\sum_n\nabla'\cdot\boldsymbol{J}(r')G_p(r/r')\mathrm{d}S'\right]\mathrm{d}S_i \qquad i = 1,2,\cdots,N \quad (5)$$

After replacing the rooftop by an unit impulses, which occupies from $-\Delta s/2 \to \Delta s/2$ and have the same electric moment as rooftop; also replacing the $-\nabla'\cdot\boldsymbol{J}(s)$ by two impulses, one has amplitude–1 ($-\Delta s \to 0$);one has amplitude of +1 ($0 \to \Delta s$)(equal to the derivatives of rooftop function). The double integrals in Eq.(5) become integrals of the periodic Green function, because the impulses are constants within their defined domains .The expanded coefficients are then taken out of the integral symbol.



(a) Rooftop function (b) Razor function

Fig.5 Rooftop function and razor function

After taking inner product, the double-integrals yields complex members, and in turn, a set of algebraic equations for unknown expanding coefficients is obtained, write it in matrix form as

$$\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1(N_s+N_z)} \\ a_{21} & a_{22} & \cdots & a_{2(N_s+N_z)} \\ \vdots & \vdots & \cdots & \vdots \\ a_{N_s1} & a_{N_s2} & \cdots & a_{N_s(N_s+N_z)} \\ a_{(N_s+1)1} & a_{(N_s+1)2} & \cdots & a_{N_s(N_s+N_z)} \\ \vdots & \vdots & \vdots & \vdots \\ a_{(N_s+N_z)1} & a_{(N_s+N_z)2} & \cdots & a_{(N_s+N_z)(N_s+N_z)} \end{vmatrix} \begin{vmatrix} b_{s1} \\ b_{s2} \\ \vdots \\ b_{sN_s} \\ b_{z1} \\ \vdots \\ b_{zN_z} \end{vmatrix} = \begin{vmatrix} c_{s1} \\ c_{s2} \\ \vdots \\ c_{sN_s} \\ c_{z1} \\ \vdots \\ c_{zN_z} \end{vmatrix}$$

The column on right hand side is the inner product of test function with the given input field , it is a known column .By taking inverse of the square matrix $[\ ]^{-1}$, and using $[\ ]^{-1}$ right product with the two sides ,we get

$$\begin{vmatrix} b_{s1} \\ b_{s2} \\ \vdots \\ b_{sN_s} \\ b_{z1} \\ \vdots \\ b_{zN_z} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1(N_s+N_z)} \\ a_{21} & a_{22} & \cdots & a_{2(N_s+N_z)} \\ \vdots & \vdots & \cdots & \vdots \\ a_{N_s1} & a_{N_s2} & \cdots & a_{N_s(N_s+N_z)} \\ a_{(N_s+1)1} & a_{(N_s+1)2} & \cdots & a_{N_s(N_s+N_z)} \\ \vdots & \vdots & \vdots & \vdots \\ a_{(N_s+N_z)1} & a_{(N_s+N_z)2} & \cdots & a_{(N_s+N_z)(N_s+N_z)} \end{vmatrix}^{-1} \begin{vmatrix} c_{s1} \\ c_{s2} \\ \vdots \\ c_{sN_s} \\ c_{z1} \\ \vdots \\ c_{zN_z} \end{vmatrix} \tag{6}$$

The current distribution is obtained by substituting back the obtained expanding coefficients into the expanding equation.

1.3 The Acceleration of Periodic Green's Function

The periodic Green's function is an inherent slow convergence function .We adopt a technique to accelerate its convergence. Let [5,6]

$$G_p = (G_p - G^a) + F^{-1}(\tilde{G}^a)$$

where the G^a is a Green function slightly different with G_p ,let

$$G^a = \left\{ \mathrm{e}^{-\mathrm{j}k_0\left[(r_0 - r' - r_{mn})^2 + (z_0 - z' + t)^2\right]^{1/2}} \right\} \Big/ \left\{ \left[(r_0 - r' - r_{mn})^2 + (z_0 - z' + t)^2\right]^{1/2} \right\}$$

where t is a small number($t << \Delta z$).$\tilde{G}^a$ is the Fourier transformation of G^a; $F^{-1}(\tilde{G}^a)$ is the inverse transform of $\tilde{G}^a$. The $(G_p - G^a)$ convergent fast, and $\tilde{G}^a$, a transformation of a slow convergence function becomes a fast convergence one.

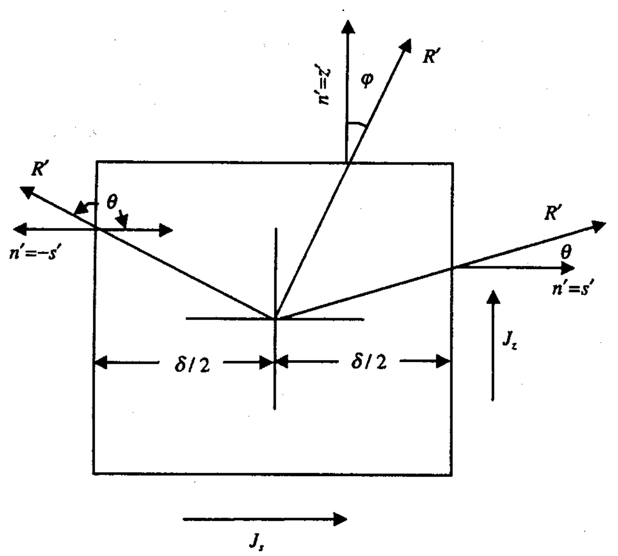


Fig.6 Calculation of singularity of Green function

1.4 Examination of the Adequate Truncated Term of Periodic Green's Function

After introducing the accelerating technique we examine how many terms should be taken in the summation in Eq.(5) to ensure the results accurate enough. By comparing the obtained data of matrix's element with m=8, n=8 and the data obtained with m=9, n=9, the difference between them are below 1%,show that the summation may be truncated at m=8, n=8.

1.5 Calculation of Singular Term of Green's Function

When the integral is proceeded to the current on the subdomains which the field point set on, as $\tilde{\boldsymbol{n}}' \to \tilde{\boldsymbol{n}}_0, z' \to z$, G_p will go to infinite. For calculating the contribution of the singular point, we take a small square area around the field point as Fig. 6 shows, exclude it from the above integral and write its contribution as $\bar{\bar{L}} \cdot \boldsymbol{J} / \mathrm{j}we_0$ [7,8]. Where $\bar{\bar{L}}$ is a dyadic and may be calculated by an integral around the square contour, through some mathematical operations, we get

$$\frac{\bar{\bar{L}} \cdot \boldsymbol{J}}{\mathrm{j}we_0} = \frac{1}{\mathrm{j}we_0 2\pi}\left[\int_{-d/2}^{d/2} \frac{s(d/2)J_s}{(d/2)^2 + z'^2}\mathrm{d}z' + \int_{-d/2}^{d/2} \frac{-s(d/2)J_s}{-\left[(d/2)^2 + z'^2\right]}\mathrm{d}z'\right] =$$

$$\frac{sd\,J_s}{\mathrm{j}we_0 2\pi}\int_{-d/2}^{d/2} \frac{1}{(d/2)^2 + z'^2}\mathrm{d}z' \qquad \boldsymbol{J} = sJ_s \tag{7}$$

similarly,

$$\frac{\bar{\bar{L}} \cdot \boldsymbol{J}}{\mathrm{j}we_0} = \frac{zd\,J_s}{\mathrm{j}we_0 2\pi}\int_{-d/2}^{d/2} \frac{1}{(d/2)^2 + s'^2}\mathrm{d}s' \qquad \boldsymbol{J} = zJ_s \tag{8}$$

1.6 Utilization of the Symmetry of the Structure

We complete the diagram for the double integral calculation. Calculating the matrix's elements is the most time consuming work. However, we can take the advantage of the symmetric relation of the structure to reduce the number of element to be calculated. After careful examination we discover more than half matrix elements have the same numerical value with some other elements. For an example, *c*=5 mm, *h*=8.2 mm the matrix has size (140×140)=19 600 elements, we have reduced the required calculation from 19 395 times to 5 000 times.

2 Reflection Coefficient

After obtaining the current distribution, the reflection coefficient may be calculated by substituting $\boldsymbol{J}(r')$ into Eq.(2). But we cannot calculate the far scattering field from an infinite structure.

The scatter field of the honeycomb consists of a fundamental mode and series of higher modes. All higher modes are evanescent modes, they decay rapidly with the distance from the surface, only the fundamental mode beers the reflection wave. The fundamental mode can be calculated by only one *Y* section directly.

The reflection coefficient and the reflected power are defined as

$$R = \frac{E^s{}_{\text{fundament}}}{E^i} \tag{9}$$

$$r = 20\lg|R| \tag{10}$$

3 Calculation Examples

Calculation of $r = 20\lg|R|$ for 8.2~12.0 GHz

c=2.75 mm, *h*=8.2 mm, *d*=0.1 mm, *t*=0.05 mm

impregnated material presented by Beijing Institute of Aeronautical Materials:

Tab.1 The electromagnetic parameter of the impregnated material

f/GHz	8.2	9.0	10.0	11.0	12.0
$(e_r^{'} - e_r^{''})$	21.66, 56.05	20.39, 51.83	19.07, 47.54	19.03, 43.50	18.60, 40.86
$(m_r^{'} - m_r^{''})$	0.96, 0.03	0.97, 0.03	0.98, 0.01	0.79, 0.02	0.97, 0.01

Fig.7 shows the reflected power versus frequency for the above honeycomb.Figs.(8~10) are the reflected power for various height, various impregnated thickness and various width separately.

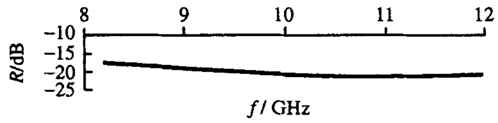


Fig.7 The reflected power versus frequency

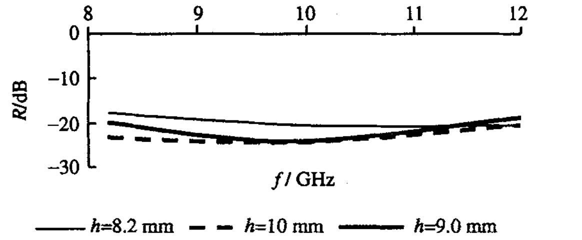


Fig.8 The reflected power versus frequency for various height

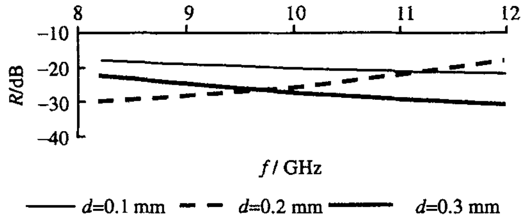


Fig.9 The reflected powers versus frequency for various impregnated thickness

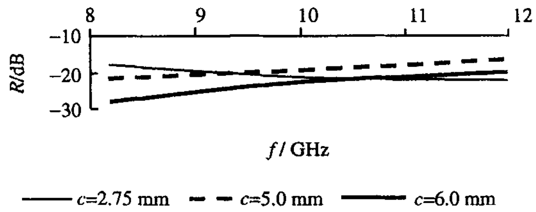


Fig.10 The reflected powers versus frequency for various width c

4 Conclusion

We proposed a complete diagram of MOM to analyze the reflection characteristics of the impregnated honeycomb. The method is shown to be highly accurate, efficient and easy to use. The numerical simulations shows that increasing height or impregnated thickness will improve the characteristics at the lower frequencies ,but make the behavior worse at higher frequencies .Increasing the width *c* to some extent is preferred in this case ,for it improves the all characteristics ,except at 12 GHz , and at the same time decreases the weight of the absorber slightly ,rather than increases the weight as increasing height and increasing impregnated thickness cases .These information may be very useful for honeycomb fabrication and application.

Reference

[1] Jorgenson R E, Mittra R. Scattering from structured slabs having two-dimensional periodicity[J] .IEEE Trans. Antenna and Propagation ,1991,39(2):152-157

[2] Jorgenson R E, Epp L, Mittra R. Determination of natural basis function set on thick, structured slabs using prony' s method[J].IEEE trans on Antenna and Propagation, 1991, 39(8):1 240-1 243

[3] Jorgenson R E, Mittra R. Efficient calculation of free space periodic Green function[J]. IEEE Trans. Antenna and Propaget,1991,AP38:633-642

[4] Glisson A W, Wilton D R. Simple and efficient numerical methods for problems of electromagnetic vadiation and scattering from sarface[J]. IEEE Trans Antennas Propagat,1980,AP-28:593-603

[5] Lampe R, Klock P, Maycs P. Integral transforms useful for the accelerated sumrnation of periodic, free space Green' s functions[J]. IEEE Trans Microwave Theory Tech.,1985,MTT-33:734-736

[6] Sinch S. Accelerationg the convergence of series representing the free space periodic green' s function[J]. IEEE Tans.AP, 1990,38(12):1 958-1 962

[7] Arthur D, Yaghjian. Electric dyadic green' s fanctions in the source region[J]. Proceedings of IEEE, 1980, 68(2):248-263

[8] Shuang-Wulee. Singularity in green' s function and its numerical evaluation[J]. IEEE Trans. On Antenna and Propagation,1980,30(3):311-317

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