Completely Generalized Nonlinear Implicit Quasi-Variational Inclusions for Fuzzy Mappings

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Abstract　A new class of completely generalized nonlinear implicit quasi-variational inclusions involving generalized m- accretive mappings for fuzzy mappings in q-uniformly smooth Banach space are introduced and studied. By using the Nadler' s theorem and the resolvent operator technique for generalized m- accretive mapping, some new iterative algorithms for finding the approximate solutions of this class of variational inclusions are constructed and the existence of solution for this kind of variational inclusion are proved. The iterative sequences generated by the algorithms converge to the exact solution of the quasi-variational inclusions.

Key words　variational inclusion;　generalized m- accretive mapping;　fuzzy mapping; resolvent operator;　iterative algorithm

Fuzzy映象的完全广义非线性隐拟变分包含

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【摘要】通过建立q-一致光滑Banach空间中一类新的涉及Fuzzy映象及广义m－增生映象的完全广义非线性隐-拟变分包含，利用Nadler定理及广义m-增生映象的解算子技巧，构造了新的迭代算法。由该算法得到了q-一致光滑Banach空间中这类完全广义非线性隐－拟变分包含的近似解并证明了该解的存在性。建立了由算法产生的迭代序列，得到了它收敛到变分包含的精确解。

关　键　词　变分包含;　广义m-增生映象;　Fuzzy映象;　解算子;　迭代算法

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1　Preliminaries

Let X be a real Banach space with dual space X^*, $\langle\cdot,\cdot\rangle$ be the dual pair between X and X^*. Let 2^X, $CB(X)$、$F(X)$ and $H(\cdot,\cdot)$ denote the family of all the nonempty subsets of X, the family of all the nonempty closed bounded subsets of X, the collection of all fuzzy mapping on X and the Hausdorff metric on $CB(X)$, respectively.

If F is a fuzzy mapping on X, then $F(x)$ (we denote it by F_x in the sequel) is a fuzzy set on X and $F_x(y)$ is the membership function of y in F_x，Let $M\in F(X)$, $q\in[0,1]$. Then the set $(M)_q=$

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$\{x\in H: M(x)\geq q\}$ is called a q-cut set of M.

Let $S,T: X\to F(X)$ be two fuzzy mappings satisfying the following condition(A):

(A)There exist two mappings $a,b: X\to[0,1]$ such that for all $x\in X$, we have

$$(S_x)_{a(x)}\in CB(X)$$

and

$$(T_x)_{b(x)}\in CB(X)$$

By using the fuzzy mappings S and T, we can define two set-valued mappings $\tilde{S}$ and $\tilde{T}$ as follows

$$\tilde{S}: X\to CB(X)\qquad x\mapsto (S_X)_{a(x)}$$

$$\tilde{T}: X\to CB(X)\qquad x\mapsto (T_X)_{b(x)}$$

where $\tilde{S}$ and $\tilde{T}$ are called the set-valued mappings induced by the fuzzy mappings S and T, respectively.

The generalized duality mapping $J_q: X\to 2^{X^*}$ is defined by

$$J_q(x)=\{f^*\in X^*:\langle x,f^*\rangle=\|f^*\|\ \|x\|,\|f^*\|=\|x\|^{q-1}\}\qquad \forall x\in X$$

where $q>1$ is a constant in particular, J_2 is the usual normalized duality mapping. It is known that

$$J_q(x)=\|x\|^{q-2}J_2(x)\qquad x\neq 0$$

where J_q is single-valued if X^* is strictly convex[1]. If $X=H$ is Hilbert space, then J_2 becomes the identity mapping of H. In the sequel, we shall denote the single-valued generalized duality mapping by J_q.

Definition A mapping $N: X\times X\to X$ is said to be

(1) Lipschitz continuous in the first argument, if there exists a constant $z>0$ such that

$$\|N(u.\cdot)-N(v,\cdot)\|\leq z\ \|u-v\|\qquad \forall u,v\in X$$

Similarly, we can define the Lipschitz continuous of $N(\cdot,\cdot)$ in the second argument;

(2) Strongly accretive in the first argument if there exists a constant $r>0$ and $j_q(u-v)\in J_q(u-v)$ such that

$$\langle N(u,\cdot)-N(v,\cdot), j_q(u-v)\rangle\geq r\|u-v\|^q\qquad \forall u,v\in X$$

Similarly, we can define the strongly accretive of $N(\cdot,\cdot)$ in the second argument.

Lemma 1[1] Let X be a real uniformly smooth Banach space. Then X is q-uniformly smooth if and only if there exists a constant $C_q>0$ such that for all $x,y\in X$

$$\|x+y\|^q\leq\|x\|^q+q\langle y,j_q(x)\rangle+C_q\|y\|^q$$

Lemma 2[2] Let $h: X\times X\to X^*$ be strictly monotone and $M: X\to 2^X$ be a generalized m- accretive mapping. Then the following conclusions hold

(1) $\langle x-y,h(u,v)\rangle\geq 0$, $\forall(y,v)\in Graph(M)$ implies that $(x,u)\in Graph(M)$, where, $Graph(M)=\{(x,u)\in X\times X: x\in Mu\}$;

(2) The inverse mapping $(I+lM)^{-1}$ is single-valued for any $l>0$.

Based on Lemma 2, we can define the resolvent operator for a generalized m- accretive mapping as follows:

$$J_r^M(z)=(I+rM)^{-1}(z)\qquad \forall z\in X$$

where $r>0$ is a constant and $h: X\times X\to X^*$ be strictly monotone mapping.

Lemma 3[2] Let $h: X\times X\to X^*$ be strongly monotone and Lipschtiz continuous with constant $a>0$ and $b>0$, respectively. Let $M: X\to 2^X$ be a generalized m- accretive mapping. Then the resolvent operator J_r^M for M is Lipschtiz continuous with constant ab^{-1}, i.e.

$$\|J_r^M(u)-J_r^M(v)\|\leq ab^{-1}\|u-v\|\qquad \forall u,v\in X$$

2 Viational Inclusions

Let X be a q-uniformly smooth Banach space. Given mappings $a,b: X\to[0,1]$, fuzzy mappings $S,T: X\to \mathrm{F}(X)$, single-valued mappings $h: X\times X\to X^*$, $N: X\times X\to X$ and $f,g: X\to X$. Suppose

$M: X \to 2^X$ such that for each fixed $t \in X$, $M(\cdot,t): X \to 2^X$ is a generalized m- accretive mapping.

Let' s consider the following problem:

Find $u \in X, S_u(w) \geq a(u)$ and $T_u(z) \geq b(u)$, such that

$$0 \in N(u,w) + M(f(u) - g(u), z) \tag{1}$$

which is called a completely generalized nonlinear implicit quasi-variational inclusion for a fuzzy mapping.

As examples, we now consider some particular variational inclusion for fuzzy mappings.

Example 1 If $F, G: X \to CB(X)$ are multi-valued mappings, by using F and G, we can define two fuzzy mappings as

$$S: X \to F(X) \qquad x \mapsto c_{F(x)}$$
$$T: X \to F(X) \qquad x \mapsto c_{G(x)}$$

where $c_{F(x)}$ and $c_{G(x)}$ are the characteristic functions of the set $F(x)$ and $G(x)$ respectively, taking $a(x) = 1, b(x) = 1, \quad \forall x \in X$, then formula (1) is equivalent to finding $u \in X, w \in Fu$ and $z \in Gu$ such that

$$0 \in N(u,w) + M(f(u) - g(u), z) \tag{2}$$

which is called a completely generalized nonlinear implicit quasi-variational inclusion for a set-valued mapping.

Example 2 If $M(x,t) = M(x)$ for all x, t in X, then formula (1) reduces to the following problem: Find $u \in X, S_U(w) \geq a(u)$ such that

$$0 \in N(u,w) + M(f(u) - g(u)) \tag{3}$$

This problem is called a completely generalized nonlinear implicit variational inclusion for a fuzzy mapping.

For appropriate and suitable choices of the mappings S, T, M, N, f, g, h, the functions a, b and the space X, the variational inclusion formula (1) includes a number of known classes of variational inclusion and variational inequalities as special cases.

3 Iterative Algorithm

Lemma 4 For given $u \in X, w \in \tilde{S}u$ and $z \in \tilde{T}u$, (u, w, z) is a solution of formula (1) if and only if

$$(f - g)(u) = J_r^{M(\cdot,z)}((f - g)(u) - rN(u,w))$$

where $J_r^{M(\cdot,z)} = (I + rM(\cdot,z))^{-1}$ and $r > 0$ is constant.

Proof This directly follows from the definition of $J_r^{M(\cdot,z)}$.

Based on Lemma 4 and Nadler[3], we can develop a new iterative algorithm for solving formula (1) as follows

Algorithm 1 For any given $u_0 \in X, w_0 \in \tilde{S}u_0$ and $z_0 \in \tilde{T}u_0$, we can get the iterative sequences $\{u_n\}, \{w_n\}$ and $\{z_n\}$ as follows:

$$\begin{cases} u_{n+1} = u_n - (f - g)(u_n) + J_r^{M(\cdot,z_n)}((f - g)(u_n) - rN(u_n, w_n)) \\ w_n \in \tilde{S}u_n, \ \|w_{n+1} - w_n\| \leq (1 + (1+n)^{-1})H(\tilde{S}u_{n+1}, \tilde{S}u_n) \qquad n = 0,1,2,\cdots \\ z_n \in \tilde{T}u_n, \ \|z_{n+1} - z_n\| \leq (1 + (1+n)^{-1})H(\tilde{T}z_{n+1}, \tilde{T}z_n) \end{cases} \tag{4}$$

From algorithm 4, we can get algorithms for solving formulae (2) and (3) as follows:

Algorithm 2 For any given $u_0 \in X, w_0 \in Fu_0$ and $z_0 \in Gu_0$, we can get the iterative sequences $\{u_n\}, \{w_n\}$ and $\{z_n\}$ as follows

$$\begin{cases} u_{n+1} = u_n - (f - g)(u_n) + J_r^{M(\cdot,z_n)}((f - g)(u_n) - rN(u_n, w_n) \\ w_n \in Fu_n, \ \|w_{n+1} - w_n\| \leq (1 + (1+n)^{-1})H(Fu_{n+1}, Fu_n) \qquad n = 0,1,2,\cdots \\ z_n \in Gu_n, \ \|z_{n+1} - z_n\| \leq (1 + (1+n)^{-1})H(Gz_{n+1}, Gz_n) \end{cases} \tag{5}$$

Algorithm 3 For any given $u_0 \in X$ and $w_0 \in \tilde{S}u_0$, we can get the iterative sequences $\{u_n\}$ and $\{w_n\}$ as follows

$$\begin{cases} u_{n+1} = u_n - (f-g)u_n + J_r^M((f-g)(u_n) - r N(u_n, w_n)) \\ w_n \in \tilde{S}u_n, \ \|w_{n+1} - w_n\| \le (1+(1+n)^{-1}) H(\tilde{S}u_{n+1}, \tilde{S}u_n) \end{cases} \quad n = 0,1,2,\cdots \tag{6}$$

4 Existence and Convergence

Theorem 1 Let X be a q-uniformly smooth Banach space and $h : X \times X \to X^*$ be strongly monotone and Lipschitz continuous with constants a, b. Let $S, T : X \to F(X)$ be fuzzy mappings satisfying the condition(A), and $\tilde{S}, \tilde{T} : X \to CB(X)$ be set-valued mappings induced by S, T, respectively. Suppose that $\tilde{S}$, $\tilde{T}$ are H-Lipschitz continuous with constants s, t, respectively, $f, g : X \to X$ are Lipschitz continuous with constants g, x and $f-g$ is strongly accretive with constant k. Let $N : X \times X \to X$ be Lipschitz continuous in the first and second arguments with constants z, e, respectively and strongly accretive in the first arguments with constant r. Let $M : X \times X \to 2^X$ be a set-valued mapping such that for each fixed $t \in X, M(\cdot, t)$ is a generalized m-accretive mapping. Suppose that there exist constants $r > 0$ and $d > 0$ such that, for each $x, y, v \in X$

$$\left\| J_r^{M(\cdot,x)}(v) - J_r^{M(\cdot,y)}(v) \right\| \le d \|x - y\| \tag{7}$$

$$q = (1+\frac{a}{b})[1 - qk + C_q(g+x)^q]^{\frac{1}{q}} + \frac{a}{b}(1 - rqr + C_q r^q z^q)^{\frac{1}{q}} + dt + \frac{a}{b} res < 1 \tag{8}$$

then the iterative sequences $\{u_n\}, \{w_n\}$ and $\{z_n\}$ generated by Algorithm 1 strongly converge to u^*, w^* and z^*, respectively, and (u^*, w^*, z^*) is a solution of formula (1).

Proof　From formulae (4)and(7) and Lemma 3, we have

$$\|u_{n+1} - u_n\| \le (1+ab^{-1})\|u_n - u_{n-1} - ((f-g)(u_n) - (f-g)(u_{n-1}))\| + d\|z_n - z_{n-1}\| + ab^{-1}\|u_n - u_{n-1} - r(N(u_n, w_n) - N(u_{n-1}, w_{n-1}))\| \tag{9}$$

since $f, g : X \to X$ are Lipschitz continuous and $f-g$ is strongly accretive, we obtain

$$\|u_n - u_{n-1} - ((f-g)(u_n) - (f-g)(u_{n-1}))\|^q \le [1 - qk + C_q(g+x)^q]\|u_n - u_{n-1}\|^q \tag{10}$$

Also since N is strongly accretive and Lipschitz continuous in the first and second arguments and $\tilde{S}$ is H-Lipschitz continuous, we have

$$\|(N(u_{n-1}, w_n) - N(u_{n-1}, w_{n-1}))\| \le e(1+\frac{1}{1+n})s\|u_n - u_{n-1}\| e(1+\frac{1}{1+n})s\|u_n - u_{n-1}\| \tag{11}$$

and

$$\|u_n - u_{n-1} - r(N(u_n, w_n) - N(u_{n-1}, w_{n-1}))\|^q \le (1 - rqr + C_q r^q z^q)\|u_n - u_{n-1}\|^q \tag{12}$$

by H-Lipschitz continuity of $\tilde{T}$, we know

$$\|z_n - z_{n-1}\| \le (1+\frac{1}{1+n})H(\tilde{T}u_n, \tilde{T}u_{n-1}) \le (1+\frac{1}{1+n})t\|u_n - u_{n-1}\| \tag{13}$$

it follows from formulae (9)~(12) that

$$\|u_{n+1} - u_n\| \le q_n \|u_n - u_{n-1}\| \tag{14}$$

where

$$q_n = (1+\frac{a}{b})[1 - qk + C_q(g+x)^q]^{\frac{1}{q}} + \frac{a}{b}(1 - rqr + C_q r^q z^q)^{\frac{1}{q}} + dt(1+\frac{1}{1+n}) + \frac{a}{b} res(1+\frac{1}{1+n})$$

It follows from formula (8) that $q_n \to q$ as $n \to \infty$. Since $0 \le q < 1$, we know that $q_n < 1$ for n sufficiently large. Thus, formula (14) implies that $\{u_n\}$ is a Cauchy sequence in X. Let $u_n \to u^*$ as $n \to \infty$. Now, formulae (11) and (13) imply that $\{w_n\}$ and $\{z_n\}$ are also Cauchy sequences. Let $w_n \to w^*$ and $z_n \to z^*$ as $n \to \infty$.

Furthermore, we have

$$d(w^*, \tilde{S}u^*) \le \|w^* - w_n\| + H(\tilde{S}u_n, \tilde{S}u^*) \le \|w^* - w_n\| + s\|u_n - u^*\| \to 0$$

which implies that $w^* \in \widetilde{S}u^*$. Similarly, we know that $z^* \in \widetilde{T}u^*$. Therefore, (u^*, w^*, z^*) is a solution of formula (1). The proof is completed.

From Theorem 1, we can obtain the following theorems.

Theorem 2 Let X, h, f, g, N, M be the same as in Theorem 1, and $F, G: X \to CB(X)$ be H-Lipschitz continuous with constants s, t, respectively. If the formulae (7) and (8) of Theorem 1 hold, then the iterative sequences $\{u_n\}, \{w_n\}$ and $\{z_n\}$ generated by Algorithm 2 strongly converge to u^*, w^* and z^*, respectively, and (u^*, w^*, z^*) is a solution of formula (2).

Theorem 3 Let X be a q-uniformly smooth Banach space and $h: X \times X \to X^*$ be strongly monotone Lipschitz continuous with constants a, b, respectively. Let $S: X \to \mathrm{F}(X)$ be a fuzzy mapping satisfying condition (A), $\widetilde{S}: X \to CB(X)$ be a set-valued mapping induced by S and $\widetilde{S}$ be H-Lipschitz continuous with constants s. Let $f, g: X \to X$ be Lipschitz continuous with constants g, x and $f - g$ be strongly accretive with constant k. Suppose that $N: X \times X \to X$ is Lipschitz continuous in the first and second arguments with constants z, e, respectively and strongly accretive in the first arguments with constant r. Let $M: X \to 2^X$ be a generalized m- accretive mapping. If there exists a constant $r > 0$ such that

$$q = (1 + \frac{a}{b})[1 - qk + C_q(g + x)^q]^{\frac{1}{q}} + \frac{a}{b}(1 - rqr + C_q r^q z^q)^{\frac{1}{q}} + dt + \frac{a}{b} res < 1$$

then the iterative sequences $\{u_n\}$ and $\{w_n\}$ generated by Algorithm 3 strongly converge to u^* and w^*, respectively, and (u^*, w^*) is a solution of formula (3).

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·科研成果介绍·

多功能网络服务器

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