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Nonlinear-Systems Model Identification with Additive-Multiplicative Fuzzy Neural Network

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Abstract A model identification approach of nonlinear systems where only the input-output data of the identified system are available is presented. To automatically acquire the fuzzy rule-base and the initial parameters of the fuzzy model, an unsupervised clustering method is used in structure identification. Based on the cluster result, a Fuzzy Neural Network (FNN) is constructed to match with it. The FNN is trained by its learning algorithm to obtain a precise fuzzy model and realize parameter identification. Finally, the effectiveness of the proposed technique is confirmed by the simulation results of two nonlinear systems.

Key words fuzzy neural network; structure identification; parameter identification; system identification

模糊神经网络用于非线性系统模型辨识

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【摘要】提出了一种非线性系统的模型辨识方法。在只有被辨识系统的输入输出数据的情况下，利用一种无监督的聚类算法来进行结构辨识，从而自动获得模糊规则库，并可以得到模糊系统的初始参数。在聚类的基础上，构造一个与之相匹配的模糊神经网络，用它的学习算法来训练网络得到一个精确的模糊模型，从而实现参数辨识。同时，证明了所构造的模糊神经网络具有通用逼近能力，这个能力在模糊建模和模糊控制方面非常有用。通过对两个非线性系统辨识的仿真结果验证了该方法的有效性。

关 键 词 模糊神经网络; 结构辨识; 参数辨识; 系统辨识

中图分类号 TN711.4; TP393 **文献标识码** A

In the last few years, based on neural network and fuzzy system, a lot of new system identification and control methods of nonlinear system have been proposed [1~5]. Despite the fact that these methods are effective in some application area, most of them are only used in parameter identification not in structure identification. The proposed model identification of nonlinear system is composed of two parts: structure identification and parameter identification. At the same time, this FNN has universal approximation capability, a property very useful in, e.g., modeling and control applications.

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1 Structure Identification

In this paper, we adopt the unsupervised clustering algorithm Refs.[6,7]. Those vectors with high relational grades will have the same characteristics, thus they can be grouped into a cluster.

The unsupervised algorithm can be described as follows.

Let $\boldsymbol{X} = \{\boldsymbol{x}^1, \boldsymbol{x}^2, \cdots, \boldsymbol{x}^p\}$ be a set of p vectors in a (n+1)-dimensional sample space, where $\boldsymbol{x}^k = \{\boldsymbol{x}_1^k, \boldsymbol{x}_2^k, \cdots, \boldsymbol{x}_n^k, \boldsymbol{x}_{n+1}^k\}$ is a vector. The preceding n scalars are input vectors of k th sample point, and the (n+1) th scalar is the corresponding output.

Step 1 Define p movable vectors $\boldsymbol{v}^k$ (k=1, 2, $\cdots$, p) and let $\boldsymbol{v}^k = \boldsymbol{x}^k$, that is, $\boldsymbol{x}^k$ is the initial value of $\boldsymbol{v}^k$.

Step 2 Calculate the relational grades between the reference vector $\boldsymbol{v}^k$ and the comparative vector $\boldsymbol{v}^l$ by

$$r_{kl} = \exp[-\|\boldsymbol{v}^k - \boldsymbol{v}^l\|^2/(2b^2)], \qquad k = 1,2,\cdots,p; \qquad l = 1,2,\cdots,p \tag{1}$$

where $\|\boldsymbol{v}^k - \boldsymbol{v}^l\|$ represents the Euclidean distance between $\boldsymbol{v}^k$ and $\boldsymbol{v}^l$; and b is the width of Guassian function.

Step 3 Modify the relational grades between the reference vector $\boldsymbol{v}^k$ and the comparative vector $\boldsymbol{v}^l$ by

$$r_{kl} = \begin{cases} 0 & , \text{ if } r_{kl} < x \\ r_{kl} & , \text{ otherwise} \end{cases} \tag{2}$$

where x is a small constant.

Step 4 Calculate $\boldsymbol{z}^k = \{\boldsymbol{z}_1^k, \boldsymbol{z}_2^k, \cdots, \boldsymbol{z}_{n+1}^k\}$ by

$$z^k = \sum_{l=1}^{p} r_{kl} v^l \Big/ \sum_{l=1}^{p} r_{kl} , \qquad k = 1,2,\cdots,p \tag{3}$$

Step 5 If all the vectors $\boldsymbol{z}^k$ are the same as $\boldsymbol{v}^k$, k =1, 2, $\cdots$, p, then go to Step 6; otherwise let $\boldsymbol{v}^k = \boldsymbol{z}^k$ and go to Step 2.

Step 6 Based on the final results $\boldsymbol{v}^k$, we can determine that the number of clusters is equal to the number of convergent vector, the original data with the same convergent vector are grouped into the same cluster, and the convergent vector is the cluster center.

When m clusters are obtained by this method for the given data set, and the corresponding cluster centers are c_j=(c_{1j}, c_{2j}, $\cdots$, c_{nj}, $c_{(n+1)j}$), j=1, 2, $\cdots$, m, the rule-base of an initial fuzzy model can be constructed as follows:

R^j : If $\boldsymbol{x}_1$ is $A_1^j(\boldsymbol{x}_1), \cdots$, and $\boldsymbol{x}_n$ is $A_n^j(\boldsymbol{x}_n)$ then

$$w_j, \quad j = 1,\ 2,\ \cdots,\ m \tag{4}$$

where m is the number of fuzzy rules, and n is the number of input variables. The membership function of the premise part in rules is Gaussian function (corresponded to the membership generation layer of Fig.1)

$$u_{ij} = \exp\left(-\frac{(\boldsymbol{x}_i - a_{ij})^2}{b_{ij}^2}\right) \qquad 1 \leq i \leq n;\ 1 \leq j \leq m \tag{5}$$

where a_{ij} is the center of the Guassian function, a_j=(a_{1j}, a_{2j}, $\cdots$, a_{nj})=(c_{1j}, c_{2j}, $\cdots$, c_{nj}),and b_{ij} is its width, b_j={b_{1j}, b_{2j}, $\cdots$, b_{nj} }. The real value w_j of the consequent part is expressed w_j=$c_{(n+1)j}$.

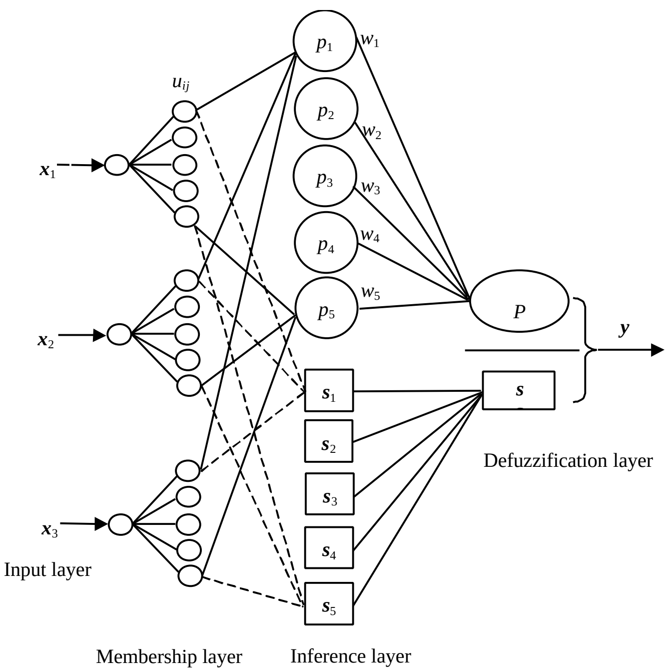

Fig.1 Architecture of AMFNN

2 Construction of Fuzzy Neural Network

To match reasoning principle with fuzzy model, this paper presents a new fuzzy neural network-Additive-Multiplicative Fuzzy Neural Network (AMFNN) by combining the additive inference and multiplicative inference into an integral whole, which is based on additive fuzzy system and multiplicative fuzzy system.

2.1 Architecture of AMFNN

AMFNN has four layers: input layer, membership generalization layer, inference layer and defuzzification layer (see Fig. 1).

The membership function is Eq. (5), where a_{ij} and b_{ij} are parameters corresponding to each node u_{ij} at the membership generation layer. In this layer, from the top down, u_{ij} can be specifically expressed as: u_{11}, u_{12}, $\cdots$, u_{1m}; u_{21}, u_{22}, $\cdots$, u_{2m}; $\cdots$; u_{n1}, u_{n2}, $\cdots$, u_{nm}, where n is the number of input variables, and m is the number of rules. Subscript of each a_{ij}, b_{ij} is similar to u_{ij}.

Multiplicative inference is paralleled to additive inference in the inference layer. The nodes of inference layer are divided into two kinds: one used in multiplicative inference, and the other used in additive inference.

The output of multiplicative inference node is an algebraic product of all its inputs:

$$p_j = u_{1j}u_{2j}\cdots u_{nj} = \prod_{i=1}^{n} u_{ij} \quad , \quad 1 \le j \le m \tag{6}$$

The output of additive inference node is an algebraic sum of all its inputs:

$$s_j = u_{1j} + u_{2j} + \cdots + u_{nj} = \sum_{i=1}^{n} u_{ij} \quad , \quad 1 \le j \le m \tag{7}$$

The final output of defuzzification layer is a ratio, the numerator of which is the weighted algebraic sum of every rule's output of multiplicative inference, and the denominator is the algebraic sum of every rule's output of additive inference.

Numerator

$$P = w_1 p_1 + w_2 p_2 + \cdots + w_m p_m \tag{8}$$

Denominator

$$S = s_1 + s_2 + \cdots + s_m \tag{9}$$

Ratio

$$\boldsymbol{y} = P / S \tag{10}$$

2.2 Learning Algorithm For AMFNN

According to Gradient Descent Method, the change of each AMFNN parameter is directly proportional to the negative derivative of Square Error Function with respect to it. So, a_{ij}, b_{ij}, w_j can be adjusted by using the following Eqs:

$$a_{ij}(n+1) - a_{ij}(n) = -h(\boldsymbol{y} - \boldsymbol{Y})\frac{w_j p_j \cdot S - P}{S^2}\frac{2(\boldsymbol{x}_i - a_{ij})}{b_{ij}^2} \tag{11}$$

$$b_{ij}(n+1) - b_{ij}(n) = -h(\boldsymbol{y} - \boldsymbol{Y})\frac{w_j p_j \cdot S - P}{S^2}\frac{2(\boldsymbol{x}_i - a_{ij})^2}{b_{ij}^3} \tag{12}$$

$$w_j(n+1) - w_j(n) = -h(\boldsymbol{y} - \boldsymbol{Y}) \cdot \frac{p_j}{S} \tag{13}$$

3 Approximation

Theorem 1 Let $f: U \subset R^n \to R$ be a continuous function defined on a compact U and $\boldsymbol{x} \in U$ is such that $\boldsymbol{x} = (\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_n)$. Consider the fuzzy rule type and the inference mechanism defined in Section 1 to construct the fuzzy neural network. There exists a set of fuzzy rules or, alternatively, a network, such that for any

$x \in U$ and $e > 0$ the following is satisfied

$$\sup\{| f(x) - y|| x \in U\} \le e \tag{14}$$

where y is the output of the network for input x. The proof is similar to Ref.[8] and omitted for brevity.

4 Numerical Examples

Example We consider a set of input-output data generated by the nonlinear system

$$y = \frac{\sin x_1 \sin x_2}{x_1 x_2} \tag{15}$$

with x_1, x_2 in the interval [–10, 10]. Each input component x_1 and x_2 is randomly chosen according to an uniform probability density function, thus 400 input-output pairs are generated.

In the structure identification step, the data points are clustered by the unsupervised clustering method. The cluster result shows that the data points are classified into 5 groups and there are 5 cluster centers. Consequently, we can construct a rule-base of the initial fuzzy model associated with the obtained five clusters. The parameter values of the premise and consequent fuzzy sets are listed in Tab.1.

Tab.1 Parameter value of the initial fuzzy model determined by the structure identification

j	Premise Part				Consequent Part
	a_{1j}	a_{2j}	b_{1j}	b_{2j}	w_j
1	–9.5554	–8.4924	–9.2051	–8.1013	–8.3525
2	–5.8707	–6.0978	–6.1056	–5.0993	–6.2236
3	1.3538	2.1216	1.2048	3.1050	2.2863
4	5.1562	7.5794	6.0998	7.2041	6.7139
5	9.5318	8.9234	9.2006	8.0930	9.1892

Tab.2 Final parameter values of the fuzzy model determined by the parameter identification

j	Premise Part				Consequent Part
	a_{1j}	a_{2j}	b_{1j}	b_{2j}	w_j
1	–9.5554	–8.4924	–8.2051	–9.1013	–9.3525
2	–5.8707	–6.0978	–6.1056	–4.0993	–5.2236
3	2.3538	1.1216	1.2048	2.1050	1.2863
4	5.1562	4.5794	6.0998	4.2041	5.7139
5	8.5318	9.9234	9.2006	7.0930	8.1892

In the parameter identification step, a fuzzy neural network with four layers is constructed for tuning the parameters of premise and consequent part to obtain a more precise fuzzy model. According to the number of input variables and the number of rules, the AMFNN has 5 neurons in input layer, 5×2 neuron in membership generation layer (in 5 groups, each group has 2 neuron), 5 neurons in inference layer. The learning algorithm above-mentioned in section 3 trains AMFNN. The final parameter values of the fuzzy model are listed in Tab. 2.

Testing samples are identified by obtained fuzzy model. The outputs of the original nonlinear system and the fuzzy model are plotted in Fig.2 and Fig.3. It is clear that the fuzzy model determined by the proposed approach has a very good approximation.

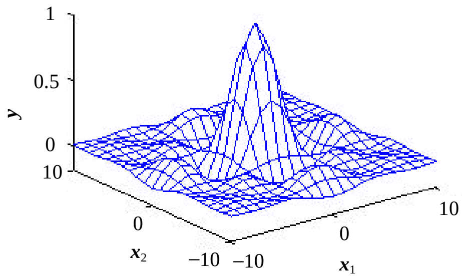

Fig. 2 Original input-output data set

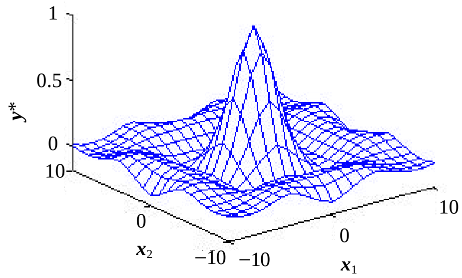

Fig.3 Output of the final fuzzy model

5 Conclusions

This paper presents a model identification approach of nonlinear systems. This approach can be used in structure identification and parameter identification. The AMFNN can dynamically identify the structure and parameter of systems according to the situation. Consequently, the proposed approach has adaptive capability to certain extent.

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