Study on Code Multi-Path Mitigation by Phase-Aided Smoothing Algorithm

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Abstract　With the progress of the differential technology, typical error source of global navigation satellite system (GNSS) receivers have been greatly reduced, while the multi-path ranging error remains the dominant one. To address this problem, the multi-path effects on code measurements and carrier-phase measurements are first analyzed in this paper. A low complexity code multi-path mitigation algorithm for the C/A code receivers based on the carrier-aided smoothing is then introduced and detailed. The code multi-path error is reduced from tens of meters to sub-meter and the C/A code GNSS receiver positioning accuracy is thus greatly improved. System simulation validates the proposed method.

Key words　carrier-aided;　code multi-path mitigation;　GNSS;　low complexity;　multi-path effect

载波相位辅助消除伪码多径误差的算法研究

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【摘要】　随着差分技术的发展，影响卫星导航接收机定位精度的典型误差源大多被消除，而多路径效应仍然是造成接收机定位误差的主要因素。该文分析了多路径效应对伪码测量量和载波相位测量量的影响。利用载波多路径误差源小于伪码多路径误差的性质，提出了一种针对伪码接收机的低复杂度的载波辅助伪码抗多径算法，并对算法进行了深入讨论和分析。该算法将接收机伪码测量量中的多路径误差由数十米减小到亚米级，大大提高了伪码接收机的定位精度。系统仿真证明了该算法的正确性。

关　键　词　载波辅助;　伪码抗多径;　卫星导航系统;　低复杂度;　多路径效应

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The C/A code GNSS receivers are widely employed nowadays. With the progress of the differential technology, typical error such as troposphere has been greatly reduced, while the multi-path ranging error remains the dominant one. Under extreme conditions, the multi-path introduced errors can be tens of meters [1]. Until now, the multi-path effect has been well studied by the scientists and researchers [1-2] and various algorithms have been proposed. The narrow correlator technology proposed by Ref.[3] narrows the correlator spacing to mitigate the code multi-path error. The pulse aperture correlation (PAC) technology proposed by Ref.[4] further reduced the multi-path error. Extra correlators and new code phase discriminators are introduced in these technologies to reduce errors. The strobe technology proposed by Ref.[5] can effectively mitigate the reflected signals delayed 30 meters behind the direct signal. And the multipath estimation delay lock loop (MEDLL) technology proposed by Ref.[6] employs multiple correlators to improve the tracking performance in the delay lock loop. All the technologies mentioned above are based on the correlator technologies and they utilizes the auto-correlation characteristics of the Gold code. The performances of these technologies are therefore limited by the correlator spacing and the sampling rate. The carrier-aided smoothing method which is easy to

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implement is introduced and detailed in this paper. The paper is organized as follows: the multi-path effect on the code and carrier-phase measurements are analyzed first, the carrier-aided smoothing algorithm is then analyzed and detailed, and the simulation result is given to validate the superiority of the algorithm.

1　Multi-Path Effect on Measurements

The GNSS is essentially a ranging system and the positioning principle is well illustrated in literatures. Two measurements are employed to measure the distance between the satellite and the receiver. The code measurement is used by the C/A code receiver for its simplicity and the carrier phase measurement is used by the high-end survey receiver for its high accuracy and the complexity of the ambiguity resolution problem.

1.1　Multi-Path Effects on Code Measurement

The received signal is expressed as:

$$s(t)=\sum_{i=0}^{n}\alpha_i c(t-\tau_i)\cos(\omega_c t+\theta_i) \qquad (1)$$

where n is the signal numbers and $i=0$ denotes the direct signal, $c(t)$ is the PN code, α_i and τ_i are the coefficient and time delay of the signal, ω_c and θ_i are the frequency and phase of the signal.

The input signal is de-spread and correlated with local reference signals. The delay locked loop is used to lock the signal. The early in-phase, late in-phase, early quadrature-phase and late quadrature-phase correlation signals are given as:

$$I_E=\sum_{i=0}^{n}\alpha_i R(\hat{\tau}_0-\tau_i+\tau_d)\cos(\theta_i-\hat{\theta}_0) \qquad (2)$$

$$I_L=\sum_{i=0}^{n}\alpha_i R(\hat{\tau}_0-\tau_i-\tau_d)\cos(\theta_i-\hat{\theta}_0) \qquad (3)$$

$$Q_E=\sum_{i=0}^{n}\alpha_i R(\hat{\tau}_0-\tau_i+\tau_d)\sin(\theta_i-\hat{\theta}_0) \qquad (4)$$

$$Q_L=\sum_{i=0}^{n}\alpha_i R(\hat{\tau}_0-\tau_i+\tau_d)\sin(\theta_i-\hat{\theta}_0) \qquad (5)$$

where $R(\cdot)$ is the autocorrelation function of the PN code, τ_d is the early-late spacing, $\hat{\tau}_0$ and $\hat{\theta}_0$ are the estimated time delay and carrier phase. The non-coherent early-minus-late discriminator is:

$$D_n=I_E^{\,2}+Q_E^{\,2}-I_L^{\,2}-Q_L^{\,2} \qquad (6)$$

In the presence of reflected signal, the code loop locks to the composite signal rather than the direct signal, and the code measurement error is then generated. When only one reflected signal is assumed,the discriminator outputs:

$$\begin{aligned}&a^2\left[R^2(\varepsilon+\tau_d-t)-R^2(\varepsilon-\tau_d-t)\right]=\\&\quad 2a\cos(\theta_m)R(\varepsilon-\tau_d)R(\varepsilon-\tau_d-t)+R^2(\varepsilon-\tau_d)-\\&\quad 2a\cos(\theta_m)R(\varepsilon+\tau_d)R(\varepsilon+\tau_d-t)-R^2(\varepsilon+\tau_d)\end{aligned} \qquad (7)$$

where $\varepsilon=\hat{\tau}_0-\tau_0$ is the code tracking error and $t=\tau_1-\tau_0$ is the time delay of the multi-path relative to the direct, $a=a_1/a_0$ is the ratio of the multi-path signal strength to the direct signal strength, $\theta_m=\theta_1-\theta_0$ is the phase of the multi-path relative to the direct. Some algebraic manipulation results in the following results [7]:

$$\varepsilon=\begin{cases}\dfrac{a}{1+a}t & 0\leqslant t\leqslant \tau_d(1+a)\\ a\tau_d & \tau_d(1+a)<t\leqslant 1+\tau_d(a-1)\\ \dfrac{a(s_2-2s_1+1)t-a[\tau_d(1-s_2)+s_2-2s_1+1]}{2(s_1-1)+a(s_2-2s_1+1)} & 1+\tau_d(a-1)<t\leqslant 1+\tau_d\left(1+a\dfrac{s_2-s_1}{s_1-1}\right)\\ \dfrac{a\tau_d(s_2-s_1)}{(s_1-1)} & 1+\tau_d\left(1+a\dfrac{s_2-s_1}{s_1-1}\right)<t\leqslant 2+\tau_d\left(-1+a\dfrac{s_2-s_1}{s_1-1}\right)\end{cases} \qquad (8)$$

where the amplitude of the side-lobe is $s_i(i=1,2,\cdots,2\ 022)\in\{-65,-1,663\}/1\ 023$. A graphical depiction for the case $\tau_d=0.5$, $a=0.5$ with PN code of GPS #PRN 1 is given in Fig.1.

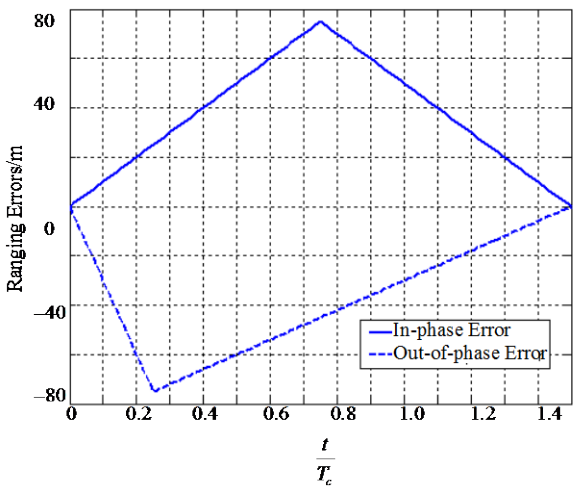

Fig.1　The code multi-path error envelop

As 1 chip error represents 300 meters ranging error, the maximum ranging error can be a magnitude of tens of meters which greatly degrades the receiver performance.

1.2 Multi-Path Effect on Carrier Measurement

Considering single reflected signal model and the arctan discriminator, the tracking error introduced by the multi-path effect is[7]:

$$\Delta\theta_0 = \arctan\left(\frac{\alpha R(\varepsilon - t)\sin(\theta_m)}{R(\varepsilon) + \alpha R(\varepsilon - t)\cos(\theta_m)}\right) \tag{9}$$

Where θ_m is assumed to be proportional to the carrier frequency, and the carrier multi-path envelop is given in Fig. 2.

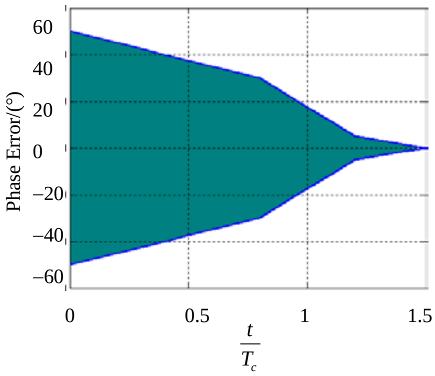

Fig.2 The carrier phase error envelop

As the maximum carrier phase error is about 50 degree, the ranging error is 2.8 cm for L1 carrier at frequency 1 575.42 MHz, and 3.6 cm for L2 carrier at 1 227.6 MHz.

2 Carrier-Aided Code Multi-Path Mitigating Algorithm

As it was analyzed in the previous section, the carrier phase multi-path error is two magnitudes smaller than the code error, so the code multi-path error can be greatly mitigated with the aid of the carrier phase measurements.

As illustrated in Fig. 3, the distances between the receiver and the satellite at the signal received time t_0, t_1, t_2 are ρ_0, ρ_1, ρ_2. The corresponding signal transmitted time is t_{s0}, t_{s1}, t_{s2}. The pseudo-range measured by the code phase measurements are $\hat{\rho}_0$, $\hat{\rho}_1$, $\hat{\rho}_2$.

$$\rho_i = c(t_i - t_{si}) \quad i = 1, 2, \cdots \tag{10}$$

where c is the speed of light. And the carrier received by the receiver is identical to the carrier transmitted by the satellite.

$$\int_{t_{s0}}^{t_{s1}} f_{\mathrm{T}}(t)\mathrm{d}t = \int_{t_0}^{t_1} (f_{\mathrm{T}}(t) + f_{\mathrm{d}}(t))\mathrm{d}t \tag{11}$$

where $f_{\mathrm{d}}(t)$ is the Doppler frequency and $f_{\mathrm{T}}(t)$ is the transmission frequency which is almost constant between t_{s0} and t_{s1}.

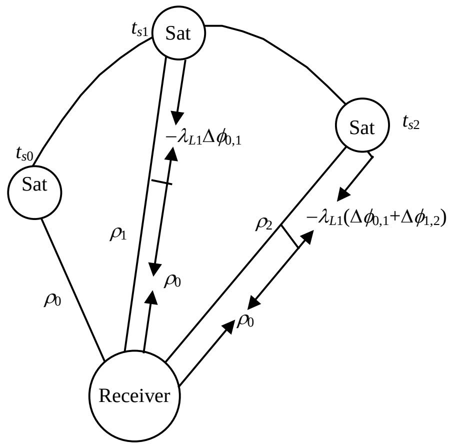

Fig.3 Principle of phase-aided smoothing algorithm

Eq. (11) can be rewritten as:

$$t_1 - t_{s1} - (t_0 - t_{s0}) = -\frac{1}{f_{\mathrm{T}}}\int_{t_0}^{t_1} f_{\mathrm{d}}(t)\mathrm{d}t \tag{12}$$

According to Eq.(10)～Eq.(12), the delta range between ρ_0 at time t_0 and ρ_1 at time t_1 can be measured by the carrier phase measurement.

$$\rho_1 - \rho_0 = -\lambda_{L1}\int_{t_0}^{t_1} f_{\mathrm{d}}(t)\mathrm{d}t = -\lambda_{L1}\Delta\phi_{01} \tag{13}$$

Based on the survey principle, averaging multiple measurements can effectively reduce the measurements variance.

$$\tilde{\rho}_0 = \frac{1}{2}(\hat{\rho}_0 + \hat{\rho}_1 + \lambda_{L1}\Delta\phi_{01}) \tag{14}$$

$$\tilde{\rho}_1 = \tilde{\rho}_0 - \lambda_{L1}\Delta\phi_{01} \tag{15}$$

So the smoothed pseudo-range at epoch (the pseudo-range extraction time) 1 is given as:

$$\tilde{\rho}_1 = \frac{1}{2}(\hat{\rho}_0 + \hat{\rho}_1 + \lambda_{L1}\Delta\phi_{0,1}) - \lambda_{L1}\Delta\phi_{0,1} = \frac{1}{2}\hat{\rho}_1 + \frac{1}{2}(\hat{\rho}_0 - \lambda_{L1}\Delta\phi_{0,1}) \tag{16}$$

similarly , the smoothed pseudo-range at epoch 2 is

$$\tilde{\rho}_2 = \frac{1}{3}(\hat{\rho}_0 + \hat{\rho}_1 + \lambda_{L1}\Delta\phi_{0,1} + \hat{\rho}_2 + \lambda_{L1}\Delta\phi_{0,2}) - \lambda_{L1}\Delta\phi_{0,2} \tag{17}$$

And the generalized expression is:

$$\tilde{\rho}_k = \frac{1}{k}\hat{\rho}_k + \frac{k-1}{k}\tilde{\rho}_{k-1} - \frac{k-1}{k}\lambda_{L1}\Delta_{k-1,k} \tag{18}$$

where k is the epoch count, $\hat{\rho}_k$ is the pseudo-range measured by code phase at the k th epoch, $\tilde{\rho}_{k-1}$ is the smoothed pseudo-range at the $(k-1)$th epoch, $-\lambda_{L1}\Delta\phi_{k-1,k}$ is the delta range between epoch $k-1$ and epoch k.

Suppose there is no multi-path effect at the first $k-1$ epochs, the smoothed pseudo-range $\tilde{\rho}_{k-1}$ is accurate. Even if $\hat{\rho}_k$ is contaminated by the multi-path effect, as the delta range $-\lambda_{L1}\Delta\phi_{k-1,k}$ is measured by carrier phase and the carrier phase multi-path error is of centimeter level, the overall ranging error in $\tilde{\rho}_k$ is greatly mitigated by the coefficient $1/k$ as illustrated in Eq. (18). As the pseudo-range is extracted several times in one second and $k-1$ epochs only takes a few seconds, the prerequisite for this code multi-path mitigation algorithm can be satisfied in most cases.

3 Simulation and Results

Simulations are carried out to validate the algorithm with the following parameters: PN code period 1 ms, code rate 1.023 MHz, intermediate frequency 4.309 MHz, sampling rate 16.384 MHz for the narrow correlator and 5.714 MHz for the proposed algorithm. For simplicity, the carrier-to-noise ratios are assumed to be typical (44 dB-Hz) and the same for all the satellites. Only one satellite is assumed to be affected by single reflected signal between epoch 400 and 450. Parameters of the reflected signal are set as follows: $\alpha=0.5$, $\tau=0.1$ chip, and $\theta=0.5\pi$. Simulation results are shown in Fig. 4. The red, black and blue line denotes the result of the least square algorithm, narrow correlator algorithm and the carrier-aided algorithm, respectively.

The carrier-aided algorithm outperforms the other two algorithms even under the environment without the multi-path effect because both the code multi-path effect and the Gaussian noise effect is mitigated by the smoothing process.

4 Conclusion

The multi-path effects on the code tracking loop and carrier tracking loop are analyzed. The carrier-aided code multi-path mitigation algorithm is detailed. It is superior to the correlator-based multi-path mitigation technologies and of very low complexity, thus it is a suitable solution for the low-cost C/A code GNSS receivers.

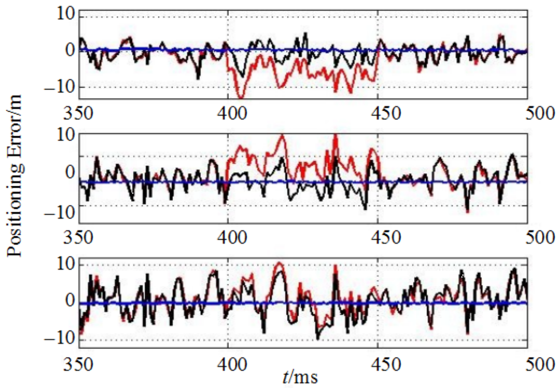

Fig.4 Positioning Results

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