变时滞随机大系统的输出反馈分散控制

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【摘要】研究了一类具有严反馈形式的变时滞随机非线性大系统，其互联项满足线性增长约束，选择了适合这类组合非线性系统的分散状态观测器，应用Backstepping方法，通过选取适当的四次型控制Lyapunov-Krasovskii泛函，并参照Lyapunov-Krasovskii泛函定理，设计了输出反馈分散控制器，使得其闭环系统的平衡点在概率意义下时滞无关渐近稳定。通过仿真实验，其结果表明了该控制算法的有效性。

关　键　词　Backstepping方法;　分散控制;　输出反馈;　随机大系统;　变时滞

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Decentralized Output Feedback Control for Stochastic Large-Scale Systems with Time-Varying Delays

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Abstract　The stochastic nonlinear large-scale systems with time-varying delays in the strict-feedback form is researched. The interconnections satisfy linear increase restriction. By using backstepping algorithms, decentralized output feedback controllers are designed through choosing appropriate state observers and employing guartic control Lyapunov-Krasovskii function. The numerical simulation shows that the closed-loop system is asymptotically stable and the effectiveness of our design is verified.

Key words　Backstepping algorithm;　decentralized control;　output feedback;　stochastic large-scale systems;　time-varying delays

实际工程系统中，由于传输、测量、反应等因素的影响，时滞现象很难避免，会影响系统的运行，很多学者注重研究时滞系统的稳定性[1-3]，时滞确定性大系统分散镇定的研究已有了许多成果[4-6]。针对带滞后状态级联的非线性大系统，文献[7]用分散状态反馈控制研究了干扰解耦问题，实际上，在文献[7]中考虑的系统只允许系统的部分状态带有时滞。时滞随机大系统更综合地反映随机性、时滞、分散化等性质，因而它的困难更多、难度更大。利用Backstepping方法构造时滞随机大系统控制器的过程中[8-9]，控制Lyapunov函数的选取也是一个难题，选取不当会造成时滞项在递推过程中发散[10]。

本文针对一类变时滞随机非线性大系统，设计状态观测器对系统未知状态进行估计，利用Backstepping方法构造性设计基于该观测器的输出反馈分散控制器。

1　问题描述

考虑一类由 N 个子系统组成的时滞随机非线性大系统，其第 i 个子系统具有以下形式：

$$\begin{cases} \mathrm{d}x_{ij} = x_{i,j+1}\mathrm{d}t + \phi_{ij}(\bar{x}_{ij})\mathrm{d}t + \varphi_{ij}(t, \boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_N, \\ \qquad \boldsymbol{x}_1(t-\tau_{ij1}), \boldsymbol{x}_2(t-\tau_{ij2}), \cdots, \boldsymbol{x}_N(t-\tau_{ijN})) \\ \qquad \mathrm{d}t + \boldsymbol{x}_{ij}^{\mathrm{T}}(y_i)\mathrm{d}w_i \\ \qquad j = 1, 2, \cdots, n_i - 1 \\ \mathrm{d}x_{in_i} = u_i\mathrm{d}t + \phi_{in_i}(\bar{x}_{in_i})\mathrm{d}t + \varphi_{in_i}(t, \boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_N, \\ \qquad \boldsymbol{x}_1(t-\tau_{in_i1}), \boldsymbol{x}_2(t-\tau_{in_i2}), \cdots, \boldsymbol{x}_N(t-\tau_{in_iN})) \\ \qquad \mathrm{d}t + \boldsymbol{g}_{in_i}^{\mathrm{T}}(y_i)\mathrm{d}w_i \\ y_i = \boldsymbol{x}_{i1} \end{cases} \tag{1}$$

式中　$i = 1, 2, \cdots, N$；$\boldsymbol{x}_i = [x_{i1}, x_{i2}, \cdots, x_{in_i}]^{\mathrm{T}} \in \boldsymbol{R}^{n_i}$ 为第 i 个子系统的状态；$\bar{\boldsymbol{x}}_{ij} = [x_{i1}, x_{i2}, \cdots, x_{ij}]^{\mathrm{T}}$；$j = 1, 2, \cdots, n_i$；$y_i \in \boldsymbol{R}$ 为第 i 个子系统的输出；$u_i \in \boldsymbol{R}$ 为第 i 个子系统的控制输入；非线性函数 $\phi_{ij}(\cdot)$ 和

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$\varphi_{ij}(\cdot)$ 均为局部Lipschitz连续，且满足 $\phi_{ij}(t,0,\cdots,0)=\varphi_{ij}(t,0,\cdots,0)=0$； $\tau_{ijk}(t)>0$ （$k=1,2,\cdots,N$）表示系统多重状态滞后，假定时变的时滞有界，且对时间的导数满足 $\dot{\tau}_{ijk}\leqslant\tau_{ijk}^*<1$，其中 τ_{ijk}^* 为正常数；$\boldsymbol{g}_{ij}(\cdot)$ 为非线性光滑函数，且有 $\boldsymbol{g}_{ij}(0)=\boldsymbol{0}$；随机噪声 w_i 是定义在概率空间上的 r_i 维独立的标准Wiener过程。

本文的研究基于如下假设：

假设 1　存在光滑函数 $\boldsymbol{\omega}_{ij}(\cdot)$ 使得 $\boldsymbol{g}_{ij}(y_i)=y_i\boldsymbol{\omega}_{ij}(y_i)$ 成立。

假设 2　存在正数 μ_{ijl} 使 $\left|\phi_{ij}(\bar{x}_{ij})-\phi_{ij}(\bar{\hat{x}}_{ij})\right|\leqslant\sum_{l=2}^{j}\mu_{ijl}\left|x_{il}-\hat{x}_{il}\right|$ 成立。其中，$\bar{\hat{\boldsymbol{x}}}_{ij}=[x_{i1},\ \hat{x}_{i2},\ \cdots,\hat{x}_{ij}]^{\mathrm{T}}$，$\hat{x}_{i2},\hat{x}_{i3},\cdots,\hat{x}_{in_i}$ 为式(2)观测器的状态(见下文)。可知存在正数 ρ_i，使得 $\|\boldsymbol{\phi}_i(x_i)-\boldsymbol{\phi}_i(\hat{x}_i)\|\leqslant\rho_i\|\boldsymbol{x}_i-\hat{\boldsymbol{x}}_i\|$ 成立。

假设 3　非线性函数 $\varphi_{ij}(\cdot)$ 满足下列不等式：

$$\left|\varphi_{ij}(t,\boldsymbol{x}_1,\boldsymbol{x}_2,\cdots,\boldsymbol{x}_N,\boldsymbol{x}_1(t-\tau_{ij1}),\boldsymbol{x}_2(t-\tau_{ij2}),\cdots,\boldsymbol{x}_N(t-\tau_{ijN}))\right|\leqslant\sum_{k=1}^{N}y_k(t-\tau_{ijk})\bar{\varphi}_{ijk}(y_k(t-\tau_{ijk}))$$

式中　$\bar{\varphi}_{ijk}(\cdot)$ 为已知正则非线性函数。

2　输出反馈分散控制器设计

2.1　状态观测器设计

由于式(1)系统的中间状态变量 $x_{i2},x_{i3},\cdots,x_{in_i}$ 不可直接量测，首先设计如下观测器进行状态估计：

$$\begin{cases}\dot{\hat{x}}_{ij}=\hat{x}_{i,j+1}+k_{ij}(x_{i1}-\hat{x}_{i1})+\phi_{ij}(\bar{\hat{\boldsymbol{x}}}_{ij}) & j=1,2,\cdots,n_i-1\\ \dot{\hat{x}}_{in_i}=u_i+k_{in_i}(x_{i1}-\hat{x}_{i1})+\phi_{in_i}(\bar{\hat{\boldsymbol{x}}}_{in_i})\end{cases}\tag{2}$$

式中　$k_{ij}>0$ 为观测器增益。令观测器误差为 $\tilde{\boldsymbol{x}}_i=\boldsymbol{x}_i-\hat{\boldsymbol{x}}_i$，则有：

$$\mathrm{d}\tilde{\boldsymbol{x}}_i=\boldsymbol{A}_i\tilde{\boldsymbol{x}}_i\mathrm{d}t+(\boldsymbol{\phi}_i(\boldsymbol{x}_i)-\boldsymbol{\phi}_i(\hat{\boldsymbol{x}}_i))\mathrm{d}t+\boldsymbol{\varphi}_i\mathrm{d}t+\boldsymbol{g}_i^{\mathrm{T}}(y_i)\mathrm{d}w_i\tag{3}$$

式中　$\boldsymbol{A}_i=\begin{pmatrix}-k_{i1} & \\ \vdots & \boldsymbol{I}\\ -k_{in_i} & 0\ \cdots\ 0\end{pmatrix}$。选择合适的 k_{ij}，使得 $\boldsymbol{A}_i$ 为Hurwitz矩阵。此时，对第 i 个子系统，式(1)非线性时滞随机大系统可以表示为如下由 $y_i,\hat{x}_{i2},\cdots,\hat{x}_{in_i}$ 描述的系统：

$$\begin{cases}\mathrm{d}\tilde{\boldsymbol{x}}_i=\boldsymbol{A}_i\tilde{\boldsymbol{x}}_i\mathrm{d}t+(\boldsymbol{\phi}_i(\boldsymbol{x}_i)-\boldsymbol{\phi}_i(\hat{\boldsymbol{x}}_i))\mathrm{d}t+\boldsymbol{\varphi}_i\mathrm{d}t+\boldsymbol{g}_i^{\mathrm{T}}(y_i)\mathrm{d}w_i\\ \mathrm{d}y_i=\left[\hat{x}_{i2}+\tilde{x}_{i2}+\phi_{i1}(x_{i1})+\varphi_{i1}\right]\mathrm{d}t+\boldsymbol{g}_{i1}^{\mathrm{T}}(y_i)\mathrm{d}w_i\\ \mathrm{d}\hat{x}_{ij}=\left[\hat{x}_{i,j+1}+k_{ij}(x_{i1}-\hat{x}_{i1})+\phi_{ij}(\bar{\hat{\boldsymbol{x}}}_{ij})\right]\mathrm{d}t\\ \qquad j=2,3,\cdots,n_i-1\\ \mathrm{d}\hat{x}_{in_i}=\left[u_i+k_{in_i}(x_{i1}-\hat{x}_{i1})+\phi_{in_i}(\bar{\hat{\boldsymbol{x}}}_{in_i})\right]\mathrm{d}t\end{cases}\tag{4}$$

因此，设计控制器使式(4)系统稳定，同样的控制器也可以使式(1)系统稳定。

2.2　输出反馈分散控制器设计过程

对于式(4)系统，选取下列坐标变换：

$$z_{i1}=x_{i1},z_{ij}=\hat{x}_{ij}-\alpha_{i,j-1}(\bar{\hat{\boldsymbol{x}}}_{i,j-1},y_i)$$
$$i=1,2,\cdots,N,\ j=2,3,\cdots,n_i\tag{5}$$

式中　函数 $\alpha_{ij}(\cdot,\cdot)$ 为虚拟控制，满足平衡条件 $\alpha_{ij}(0,0)=0$。虚拟控制的具体形式由式(7)、式(8)给出。根据式(5)和Itô微分法则[11]，式(4)系统可以变换为下列形式：

$$\begin{cases}\mathrm{d}z_{i1}=\left[\hat{x}_{i2}+\tilde{x}_{i2}+\phi_{i1}(x_{i1})+\varphi_{i1}\right]\mathrm{d}t+\boldsymbol{g}_{i1}^{\mathrm{T}}(y_i)\mathrm{d}w_i\\ \mathrm{d}z_{ij}=\left[z_{i,j+1}+\alpha_{ij}+k_{ij}\tilde{x}_{i1}+\phi_{ij}(\bar{\hat{\boldsymbol{x}}}_{ij})\right]\mathrm{d}t-\\ \quad\sum_{l=2}^{j-1}\dfrac{\partial\alpha_{i,j-1}}{\partial\hat{x}_{il}}\left[\hat{x}_{i,l+1}+k_{il}\tilde{x}_{i1}+\phi_{il}(\bar{\hat{\boldsymbol{x}}}_{il})\right]\mathrm{d}t-\\ \quad\dfrac{\partial\alpha_{i,j-1}}{\partial y_i}\left[\tilde{x}_{i2}+z_{i2}+\alpha_{i1}+\phi_{i1}(x_{i1})+\varphi_{i1}\right]\mathrm{d}t-\\ \quad\dfrac{1}{2}\dfrac{\partial^2\alpha_{i,j-1}}{\partial y_i^2}\boldsymbol{g}_{i1}^{\mathrm{T}}(y_i)\boldsymbol{g}_{i1}(y_i)\mathrm{d}t-\dfrac{\partial\alpha_{i,j-1}}{\partial y_i}\boldsymbol{g}_{i1}^{\mathrm{T}}(y_i)\mathrm{d}w_i\\ \hat{x}_{i,n_i+1}=u_i\qquad j=2,3,\cdots,n_i\end{cases}\tag{6}$$

对上述系统，选取虚拟控制律及控制律为：

$$\alpha_{i1}=-c_{i1}y_i-\phi_{i1}(x_{i1})-3n_i^{3/2}\varepsilon_{i3}^2\|\boldsymbol{P}_i\|^2y_i\|\boldsymbol{\omega}_i(y_i)\|^4-\frac{3(\eta_i^{4/3}+\delta_i^{4/3}+1)}{4}y_i-\frac{3}{2}y_i\boldsymbol{\omega}_{i1}^{\mathrm{T}}(y_i)\boldsymbol{\omega}_{i1}(y_i)-\frac{3\varepsilon_{i2}^2}{4}(n_i-1)y_i(\boldsymbol{\omega}_{i1}^{\mathrm{T}}(y_i)\boldsymbol{\omega}_{i1}(y_i))^2-\frac{1}{y_i^3}\sum_{j=1}^{n_i}\sum_{k=1}^{N}\left[\frac{\|\boldsymbol{P}_i\|^2}{1-\tau_{kji}^*}q_{kji}(y_i)+\frac{1}{2(1-\tau_{k1i}^*)}q_{k1i}(y_i)\right]\tag{7}$$

$$\alpha_{ij}=-c_{ij}z_{ij}-k_{ij}\tilde{x}_{i1}-\phi_{ij}(\bar{\hat{\boldsymbol{x}}}_{ij})+\sum_{l=2}^{j-1}\frac{\partial\alpha_{i,j-1}}{\partial\hat{x}_{il}}\left[\hat{x}_{i,l+1}+k_{il}\tilde{x}_{i1}+\phi_{il}(\bar{\hat{\boldsymbol{x}}}_{il})\right]+\frac{\partial\alpha_{i,j-1}}{\partial y_i}\left[z_{i2}+\alpha_{i1}+\phi_{i1}(x_{i1})\right]+\frac{1}{2}\frac{\partial^2\alpha_{i,j-1}}{\partial y_i^2}\boldsymbol{g}_{i1}^{\mathrm{T}}(y_i)\boldsymbol{g}_{i1}(y_i)-\frac{3}{4\varepsilon_{i2}^2}z_{ij}\left(\frac{\partial\alpha_{i,j-1}}{\partial y_i}\right)^4-\frac{3(\eta_{ij}^{4/3}+\delta_{i1}^{4/3})}{4}z_{ij}\left(\frac{\partial\alpha_{i,j-1}}{\partial y_i}\right)^{4/3}\qquad j=2,3,\cdots,n_i\tag{8}$$

$$u_i=\alpha_{in_i}\tag{9}$$

式中　c_{ij}、η_{ij}、δ_{ij}、ε_{i2}、ε_{i3} 为正常数；$\boldsymbol{P}_i$ 是正定矩阵，且满足 $\boldsymbol{A}_i^{\mathrm{T}}\boldsymbol{P}_i+\boldsymbol{P}_i\boldsymbol{A}_i=-\boldsymbol{I}$；光滑函数 $q_{ijk}(\cdot)$ 为局部Lipschitz连续函数，定义为 $q_{ijk}(y_i)=\dfrac{1}{2\delta_{ij}^4}y_i^4\times\|\bar{\varphi}_{ijk}(y_i)\|^4$。

由对式(4)系统定义的式(7)虚拟控制律、式(8)虚拟控制律和式(9)控制律，可以得到如下定理：

定理 1　式(1)非线性时滞随机大系统，在输出反馈式(7)虚拟控制律、式(8)虚拟控制律和式(9)控制律的作用下，其闭环系统的平衡点在概率意义下时滞无关渐近稳定。

证明　定义Lyapunov-Krasovskii泛函为：

$$V(\tilde{\boldsymbol{x}},\boldsymbol{y},\boldsymbol{z})=\sum_{i=1}^{N}\left[\frac{1}{2}(\tilde{\boldsymbol{x}}_i^{\mathrm{T}}\boldsymbol{P}_i\tilde{\boldsymbol{x}}_i)^2+\frac{1}{4}y_i^4+\frac{1}{4}\sum_{j=2}^{n_i}z_{ij}^4+H_i\right] \tag{10}$$

$$H_i=\sum_{j=1}^{n_i}\sum_{k=1}^{N}\frac{\|\boldsymbol{P}_i\|^2}{1-\tau_{kji}^*}\int_{t-\tau_{kji}(t)}^{t}q_{kji}(y_i(s))\mathrm{d}s+\sum_{k=1}^{N}\frac{n_i}{2(1-\tau_{k1i}^*)}\int_{t-\tau_{k1i}(t)}^{t}q_{k1i}(y_i(s))\mathrm{d}s \tag{11}$$

由Itô微分法则[11]，V 沿式(4)系统的微分生成元为：

$$\begin{aligned}\mathbf{L}V=&\sum_{i=1}^{N}\Big[-\tilde{\boldsymbol{x}}_i^{\mathrm{T}}\boldsymbol{P}_i\tilde{\boldsymbol{x}}_i\|\tilde{\boldsymbol{x}}_i\|^2+2\tilde{\boldsymbol{x}}_i^{\mathrm{T}}\boldsymbol{P}_i\tilde{\boldsymbol{x}}_i\tilde{\boldsymbol{x}}_i^{\mathrm{T}}\boldsymbol{P}_i(\boldsymbol{\phi}_i(\boldsymbol{x}_i)-\boldsymbol{\phi}_i(\hat{\boldsymbol{x}}_i)+\\&\boldsymbol{\varphi}_i))+2\mathrm{tr}\left\{\boldsymbol{g}_i(y_i)(2\boldsymbol{P}_i\tilde{\boldsymbol{x}}_i\tilde{\boldsymbol{x}}_i^{\mathrm{T}}\boldsymbol{P}_i+\tilde{\boldsymbol{x}}_i^{\mathrm{T}}\boldsymbol{P}_i\tilde{\boldsymbol{x}}_i\boldsymbol{P}_i)\boldsymbol{g}_i^{\mathrm{T}}(y_i)\right\}+\\&y_i^3(\hat{x}_{i2}+\tilde{x}_{i2}+\phi_{i1}(x_{i1})+\varphi_{i1})+\frac{3}{2}y_i^2\boldsymbol{g}_{i1}^{\mathrm{T}}(y_i)\boldsymbol{g}_{i1}(y_i)+\\&\sum_{j=2}^{n_i}z_{ij}^3\Big[z_{i,j+1}+\alpha_{ij}+k_{ij}\tilde{x}_{i1}+\phi_{ij}(\bar{\hat{\boldsymbol{x}}}_{ij})-\sum_{l=2}^{j-1}\frac{\partial\alpha_{i,j-1}}{\partial\hat{x}_{il}}(z_{i,l+1}+\\&\alpha_{il}+k_{il}\tilde{x}_{i1}+\phi_{il}(\bar{\hat{\boldsymbol{x}}}_{il})+\varphi_{il})-\frac{\partial\alpha_{i,j-1}}{\partial y_i}(\tilde{x}_{i2}+z_{i2}+\alpha_{i1}+\\&\phi_{i1}(x_{i1})+\varphi_{i1})-\frac{1}{2}\frac{\partial^2\alpha_{i,j-1}}{\partial y_i^2}\boldsymbol{g}_{i1}^{\mathrm{T}}(y_i)\boldsymbol{g}_{i1}(y_i)\Big]+\\&\frac{3}{2}\sum_{j=2}^{n_i}z_{ij}^2\left(\frac{\partial\alpha_{i,j-1}}{\partial y_i}\right)^2\boldsymbol{g}_{i1}^{\mathrm{T}}(y_i)\boldsymbol{g}_{i1}(y_i)+\dot{H}_i\Big]\end{aligned} \tag{12}$$

运用Young不等式[9]和本文的假设条件，可得：

$$\begin{aligned}&y_i^3\varphi_{i1}(t,\boldsymbol{x}_1,\boldsymbol{x}_2,\cdots,\boldsymbol{x}_N,\boldsymbol{x}_1(t-\tau_{i11}),\\&\boldsymbol{x}_1(t-\tau_{i12}),\cdots,\boldsymbol{x}_N(t-\tau_{i1N}))\leqslant\\&\frac{3\delta_{i1}^{4/3}}{4}y_i^4+\frac{1}{4\delta_{i1}^4}\sum_{k=1}^{N}y_k^4(t-\tau_{i1k})\|\bar{\varphi}_{i1k}(y_k(t-\tau_{i1k}))\|^4\end{aligned} \tag{13}$$

$$\begin{aligned}-\sum_{j=2}^{n_i}z_{ij}^3\frac{\partial\alpha_{i,j-1}}{\partial y_i}\varphi_{i1}\leqslant&\sum_{j=2}^{n_i}\frac{3\delta_{i1}^{4/3}}{4}z_{ij}^4\left(\frac{\partial\alpha_{i,j-1}}{\partial y_i}\right)^{4/3}+\\&\sum_{j=2}^{n_i}\sum_{k=1}^{N}\frac{1}{4\delta_{i1}^4}y_k^4(t-\tau_{i1k})\|\bar{\varphi}_{i1k}(y_k(t-\tau_{i1k}))\|^4\end{aligned} \tag{14}$$

$$\begin{aligned}2\tilde{\boldsymbol{x}}_i^{\mathrm{T}}\boldsymbol{P}_i\tilde{\boldsymbol{x}}_i\tilde{\boldsymbol{x}}_i^{\mathrm{T}}\boldsymbol{P}_i\varphi_i\leqslant&\|\boldsymbol{P}_i\|^2\sum_{j=1}^{n_i}\Bigg(\frac{3\delta_{ij}^{4/3}}{2}\|\tilde{\boldsymbol{x}}_i\|^4+\\&\sum_{k=1}^{N}\frac{1}{4\delta_{ij}^4}y_k^4(t-\tau_{ijk})\|\bar{\varphi}_{ijk}(y_k(t-\tau_{ijk}))\|^4\Bigg)\end{aligned} \tag{15}$$

运用文献[9]中的相关不等式并将式(13)～式(15)代入式(12)，整理可得：

$$\begin{aligned}\mathbf{L}V=&\sum_{i=1}^{N}\Bigg\{-\|\tilde{\boldsymbol{x}}_i\|^4(\lambda_{\min}(\boldsymbol{P}_i)-\|\boldsymbol{P}_i\|^2\times\\&\left(2\rho_i+\frac{3}{2}\sum_{j=1}^{n_i}\delta_{ij}^{4/3}+\frac{3n_i^{3/2}}{\varepsilon_{i3}^2}\right)-\sum_{j=1}^{n_i}\frac{1}{4\eta_{ij}^4}\Bigg)+\\&y_i^3\Big[\alpha_{i1}+\phi_{i1}(x_{i1})+3n_i^{3/2}\varepsilon_{i3}^2\|\boldsymbol{P}_i\|^2y_i\|\boldsymbol{\omega}_i(y_i)\|^4+\\&\frac{3(\eta_i^{4/3}+\delta_i^{4/3}+1)}{4}y_i+\frac{3}{2}y_i\boldsymbol{\omega}_{i1}^{\mathrm{T}}(y_i)\boldsymbol{\omega}_{i1}(y_i)+\\&\frac{3\varepsilon_{i2}^2}{4}(n_i-1)y_i(\boldsymbol{\omega}_{i1}^{\mathrm{T}}(y_i)\boldsymbol{\omega}_{i1}(y_i))^2\Big]+\sum_{j=2}^{n_i}z_{ij}^3[\alpha_{ij}+z_{ij}+\\&k_{ij}\tilde{x}_{i1}+\phi_{ij}(\bar{\hat{\boldsymbol{x}}}_{ij})-\sum_{l=2}^{j-1}\frac{\partial\alpha_{i,j-1}}{\partial\hat{x}_{il}}\left[\hat{x}_{i,l+1}+k_{il}\tilde{x}_{i1}+\phi_{il}(\bar{\hat{\boldsymbol{x}}}_{il})\right]-\\&\frac{\partial\alpha_{i,j-1}}{\partial y_i}\left[z_{i2}+\alpha_{i1}+\phi_{i1}(x_{i1})\right]-\frac{1}{2}\frac{\partial^2\alpha_{i,j-1}}{\partial y_i^2}\boldsymbol{g}_{i1}^{\mathrm{T}}(y_i)\boldsymbol{g}_{i1}(y_i)+\\&\frac{3(\eta_{ij}^{4/3}+\delta_{i1}^{4/3})}{4}z_{ij}\left(\frac{\partial\alpha_{i,j-1}}{\partial y_i}\right)^{4/3}+\frac{3}{4\varepsilon_{i2}^2}z_{ij}\left(\frac{\partial\alpha_{i,j-1}}{\partial y_i}\right)^4\Bigg]+\\&\|\boldsymbol{P}_i\|^2\sum_{j=1}^{n_i}\sum_{k=1}^{N}\frac{1}{2\delta_{ij}^4}y_k^4(t-\tau_{ijk}(t))\|\bar{\varphi}_{ijk}(y_k(t-\tau_{ijk}(t)))\|^4+\\&\sum_{j=1}^{n_i}\sum_{k=1}^{N}\frac{1}{4\delta_{i1}^4}y_k^4(t-\tau_{i1k}(t))\|\bar{\varphi}_{i1k}(y_k(t-\tau_{i1k}(t)))\|^4+\\&\|\boldsymbol{P}_i\|^2\sum_{j=1}^{n_i}\sum_{k=1}^{N}\left[\frac{1}{1-\tau_{kji}^*}q_{kji}(y_i(t))-q_{kji}(y_i(t-\tau_{kji}(t)))\right]+\\&\frac{n_i}{2}\sum_{k=1}^{N}\left[\frac{1}{1-\tau_{k1i}^*}q_{k1i}(y_i(t))-q_{k1i}(y_i(t-\tau_{k1i}(t)))\right]\Bigg\}\end{aligned} \tag{16}$$

选择适当的参数 ρ_i、η_{ij}、δ_{ij}、ε_{i3}，使得 $Q_i>0$ 成立，

$$Q_i\equiv\lambda_{\min}(\boldsymbol{P}_i)-\|\boldsymbol{P}_i\|^2\left(2\rho_i+\frac{3}{2}\sum_{j=1}^{n_i}\delta_{ij}^{4/3}+\frac{3n_i^{3/2}}{\varepsilon_{i3}^2}\right)-\sum_{j=1}^{n_i}\frac{1}{4\eta_{ij}^4}$$

并将设计的式(7)虚拟控制律、式(8)虚拟控制律和式(9)控制律代入式(16)，可得：

$$\mathbf{L}V\leqslant-\sum_{i=1}^{N}\left[Q_i\|\tilde{\boldsymbol{x}}_i\|^4+\sum_{j=1}^{n_i}c_{ij}z_{ij}^4\right] \tag{17}$$

根据Lyapunov-Krasovskii泛函定理[10]，可知式(1)系统在式(9)输出反馈控制律的作用下，其闭环系统的平衡点在概率意义下是时滞无关渐近稳定的。

证毕

2.3　仿真实验

考虑如下二阶互联系统：

$$\begin{cases} \mathrm{d}x_{11} = x_{12}\mathrm{d}t, \mathrm{d}x_{21} = x_{22}\mathrm{d}t, y_2 = x_{21}, y_1 = x_{11} \\ \mathrm{d}x_{12} = u_1\mathrm{d}t + x_{12}^2\mathrm{d}t + x_{21}(t-0.2\sin t)\mathrm{d}t + y_1\mathrm{d}w_1 \\ \mathrm{d}x_{22} = u_2\mathrm{d}t + x_{22}^2\mathrm{d}t + x_{11}(t-0.5\sin t)\mathrm{d}t + y_2\mathrm{d}w_2 \end{cases} \tag{18}$$

对式(18)随机大系统设计状态观测器为：

$$\dot{\hat{x}}_{11} = \hat{x}_{12} + k_{11}(x_{11} - \hat{x}_{11}), \dot{\hat{x}}_{12} = u_1 + x_{12}^2 + k_{12}(x_{11} - \hat{x}_{11})$$

$$\dot{\hat{x}}_{21} = \hat{x}_{22} + k_{21}(x_{21} - \hat{x}_{21}), \dot{\hat{x}}_{22} = u_2 + x_{22}^2 + k_{22}(x_{21} - \hat{x}_{21})$$

误差变量为：

$$z_{11} = y_1, z_{12} = \hat{x}_{12} - \alpha_{11}, z_{21} = y_2, z_{22} = \hat{x}_{22} - \alpha_{21}$$

根据推导过程，可知虚拟控制律及控制律为：

$$\alpha_{11} = Ay_1, \alpha_{21} = By_2 - \frac{y_1^4}{y_2^3}\left(\frac{1+\|P_2\|^2}{2\delta_{11}^4} + \frac{5\|P_2\|^2}{8\delta_{12}^4}\right)$$

$$u_1 = \alpha_{12} = -c_{12}z_{12} - x_{12}^2 - k_{12}(x_{11} - \hat{x}_{11}) + A(z_{12} + \alpha_{11}) - \frac{3(\eta_{12}^{4/3} + \delta_{11}^{4/3})}{4}z_{12}A^{4/3} - \frac{3}{4\varepsilon_{12}^2}z_{12}A^4$$

$$u_2 = \alpha_{22} = -c_{22}z_{22} - x_{22}^2 - k_{22}(x_{21} - \hat{x}_{21}) + C(z_{22} + \alpha_{21}) - \frac{3(\eta_{22}^{4/3} + \delta_{21}^{4/3})}{4}z_{22}C^{4/3} - \frac{3}{4\varepsilon_{22}^2}z_{22}C^4$$

式中

$$A = -c_{11} - 3\cdot 2^{3/2}\varepsilon_{13}^2\|P_1\|^2 - \frac{3(\eta_1^{4/3} + \delta_1^{4/3} + 1)}{4}$$

$$B = -c_{21} - 3\cdot 2^{3/2}\varepsilon_{23}^2\|P_2\|^2 - \frac{3(\eta_2^{4/3} + \delta_2^{4/3} + 1)}{4} - \left(\frac{1+\|P_2\|^2}{2\delta_{21}^4} + \frac{\|P_2\|^2}{\delta_{22}^4}\right)$$

$$C = B + \frac{3y_1^4}{y_2^4}\left(\frac{1+\|P_2\|^2}{2\delta_{11}^4} + \frac{5\|P_2\|^2}{8\delta_{12}^4}\right)$$

控制器增益取 $k_{11} = k_{12} = k_{21} = k_{22} = 5$，系统初值设为：$x_{11} = 0.5$，$x_{12} = -0.5$，$x_{21} = 1$，$x_{22} = -1$。参数选为：$\delta_{ij} = 0.1$，$\eta_{ij} = 10$，$\varepsilon_{ij} = 10$，$i = 1,2$，$j = 1,2$。

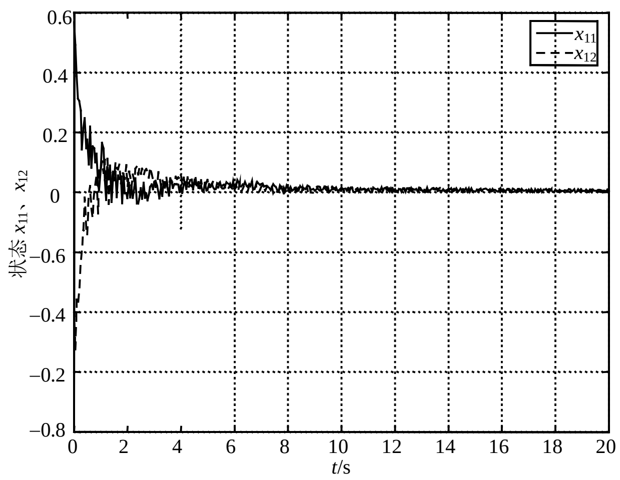

图1 子系统1状态曲线

通过MATLAB软件进行仿真实验，可得到闭环系统的状态随着时间变化的响应曲线如图1、图2所示。可以观察到闭环系统的状态随着 $t \to \infty$ 而快速收敛到零，说明所设计的分散控制器使得闭环系统的平衡点在概率意义下时滞无关渐近稳定。

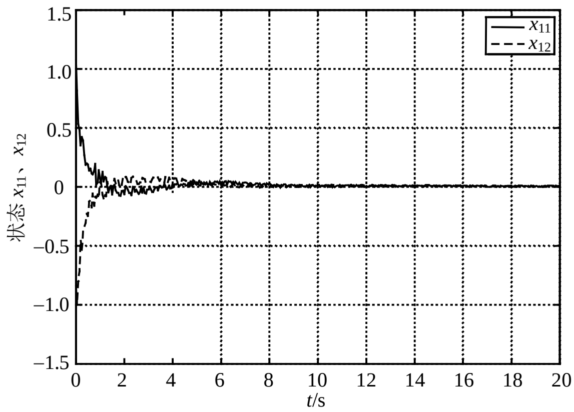

图2 子系统2状态曲线

5 结 论

本文研究了一类具有严反馈形式的互联随机变时滞系统输出反馈分散控制器的设计问题，结果表明利用Backstepping方法构造性设计的控制器使得其闭环系统的平衡点在概率意义下时滞无关渐近稳定，该结果丰富了随机大系统的控制理论，为进一步研究更具一般性的随机大系统控制问题提供了新的研究思路。

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