Impulsive Stabilization of Delayed Cellular Neural Networks via Partial States

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Abstract　The problem of impulsive stabilization of delayed cellular neural networks (DCNNs) via partial states is discussed. The time delay is allowed to be time-varying. By utilizing the piecewise linear property of the activation function of DCNNs and applying piecewise differential Lyapunov combined with Razumikhin- type analysis techniques, a sufficient condition for the existence of the impulsive control law via partial states is derived. The sufficient condition is given in terms of linear matrix inequalities concerning the interconnection matrices and the bounds of the impulsive intervals. By using this result, an impulsive stabilization scheme for a class of DCNNs is proposed. The impulsive stabilization scheme only utilizes the output of partial states of the controlled DCNN. A numerical example illustrates the efficiency of the proposed method.

Key words　cellular neural networks;　impulsive stabilization;　partial states;　time-varying delay

时滞细胞神经网络部分状态脉冲镇定

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【摘要】研究了具有变时滞的细胞神经网络的部分状态脉冲镇定问题。利用细胞神经网络激活函数的分段线性性，应用分段可微的Lyapunov函数，并结合Razumikhin型分析技术，得到了部分状态脉冲控制律存在的充分条件。该充分条件表示为基于互联矩阵和脉冲区间界的线性矩阵不等式。应用上述结果，对一类时滞细胞神经网络，提出了一种新的脉冲镇定方案。该脉冲镇定方案仅需利用部分状态的输出信息。最后，给出了一个数值例子说明了此方案的有效性。

关　键　词　细胞神经网络;　脉冲镇定;　部分状态;　变时滞

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Impulsive systems are a class of hybrid systems combining continuous evolution with instantaneous state jump, which are widely used in modeling real world evolutionary processes where the states undergo rapid changes. The impulsive control methods based on the stability theory of impulsive systems[1] have found important applications in the synchronization and control of complex dynamical systems. Compared with the continuous-time control, the impulsive control allows stabilization of systems only using small impulses generated by samples of the state variables at discrete time instants. This drastically reduces the amount of measured information transmitted from the system to the controller and increases the efficiency of bandwidth usage. However, in the most existing impulsive control results[2-8], the impulse control is exerted on all the state variables of the controlled system, which means that the full state information is required. There are many applications where only partial states are available or measurable. Thus, it is of importance to design an impulsive control law via partial states to stabilize the system under consideration.

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In Ref. [9], an impulsive control strategy via a single variable is proposed to synchronize two identical Chua's circuits. The choice of the impulsive instant depends on the monotonic property of the impulsively controlled state. In Ref.[10], the problem of impulsive synchronization of two identical Lur'e systems via partial states was studied. The new impulsive synchronization scheme only exerts the impulsive input on partial states of the driven system and is characterized by a set of conditions related to the impulsive interval bound, the impulsive magnitude and a coupling condition between them. From the results of Ref.[9-10], the key technique of designing impulsive control law via partial states lies in the choice of appropriate decomposition of the system: a stable subsystem and an unstable subsystem subject to impulsive input. For different systems, such decomposition may be different. In this paper, we will investigate the impulsive stabilization problem of delayed cellular neural networks (DCNNs) based on partial states. It is noted that cellular neural networks (CNNs) were introduced by Ref. [11-12] and they have been found important applications in image processing and solving nonlinear algebraic equations. When processing moving objects, one must introduce time-delays in the signal transmission among the cells. These lead to the model of DCNNs. Despite the CNN structure is simple because the activation function of the CNNs is a piecewise linear function, it should be pointed out that the CNNs may have complex dynamical behavior. For example, chaotic attractors were found in autonomous CNNs composed by three cells [13] and in DCNNs composed by two cells [14]. The main idea of this paper is to divide the network state variables into subgroups by utilizing the piecewise linear property of the activation function [15]. Then a piecewise differentiable Lyapunov function is introduced to analyze the stability of the DCNN with partial states subject to impulsive input. The existence conditions of the impulsive control law via partial states are expressed by a set of linear matrix inequalities (LMIs), which can be solved by the developed interior-point algorithm.

1 Problem Formulation

In the sequel, if not explicitly stated, matrices are assumed to have compatible dimensions. The notation $\boldsymbol{M} > (\geqslant, <, \leqslant) 0$ is used to denote a positive-definite (positive-semidefinite, negative, negative-semidefinite) matrix. $\boldsymbol{I}$ denotes an identity matrix of appropriate dimension. $\|\cdot\|$ denotes the Euclidean norm for vector or the spectral norm of matrices. N denotes the set of nonnegative integers, i.e., N ={0,1,2,$\cdots$}. For τ>0, PC ([$-\tau$,0],R^n) denotes the set of piecewise right continuous function: [$-\tau$,0]$\to$R^n with the norm defined by $\|\boldsymbol{\varphi}\|_\tau = \sup_{-\tau\leqslant\theta\leqslant 0}\|\boldsymbol{\varphi}(\theta)\|$. For σ >0, PC([$-\tau$,0],B$_\sigma$)= {$\boldsymbol{\varphi}\in$PC ([$-\tau$,0],R^n); $\|\boldsymbol{\varphi}\|_\tau<\sigma$}.

Consider the cellular neural networks with variable delay described by the following delay differential equations:

$$\begin{cases}\dot{\boldsymbol{x}}(t) = -\boldsymbol{x}(t) + \boldsymbol{A}\boldsymbol{f}(\boldsymbol{x}(t)) + \boldsymbol{B}\boldsymbol{f}(\boldsymbol{x}(t-\tau(t))) + \boldsymbol{u}\\ \boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{x}(t)\\ \boldsymbol{x}_{t_0} = \boldsymbol{\varphi}\end{cases} \tag{1}$$

where $\boldsymbol{x}=(x_1,x_2,\cdots,x_n)^{\mathrm{T}}$ is a real n-vector which denotes the state variables associated with the neurons, $A,B\in$ R$^{n\times n}$ are the connection weight matrix and the delayed connection weight matrix, respectively, $\boldsymbol{f}(\boldsymbol{x})=(f(x_1),f(x_2),\cdots,f(x_n))^{\mathrm{T}}$ is the neuron activation function given by $f(x_i)=0.5(|x_i+1|-|x_i-1|)$, $i = 1,2,\cdots,n$. $\boldsymbol{u}$ is a real constant input n-vector. The time-delay $\tau(t)$ is a bounded function, i.e., $0\leqslant\tau(t)\leqslant\tau$, where $\tau\geqslant 0$ is a constant. $\boldsymbol{y}\in$ R^p is the measure output, $\boldsymbol{C}\in$ R$^{p\times n}$ is a constant matrix. t_0 is the initial time and $\boldsymbol{\varphi}\in$PC ([$-\tau$, 0], R^n) is the state initial function.

It is known that at least one equilibrium point of the neural network (1) exists. Denote one of the equilibrium points by $\boldsymbol{x}^* = (x_1^*, x_2^*, \cdots, x_n^*)^{\mathrm{T}}$. For notational convenience, we will shift the equilibrium point $\boldsymbol{x}^*$ of DCNN (1) to the origin. If we let transformation $\tilde{\boldsymbol{x}}(t) = \boldsymbol{x}(t) - \boldsymbol{x}^*$, then we obtain the new description of DCNN Eq.(1) as

$$\begin{cases}\dot{\tilde{\boldsymbol{x}}}(t) = -\tilde{\boldsymbol{x}}(t) + \boldsymbol{A}\boldsymbol{g}(\tilde{\boldsymbol{x}}(t)) + \boldsymbol{B}\boldsymbol{g}(\tilde{\boldsymbol{x}}(t-\tau(t)))\\ \boldsymbol{y}(t) = \boldsymbol{C}(\tilde{\boldsymbol{x}}(t) + \boldsymbol{x}^*)\\ \tilde{\boldsymbol{x}}_{t_0} = \boldsymbol{\varphi} - \boldsymbol{x}^*\end{cases} \tag{2}$$

where $\tilde{\boldsymbol{x}} = (\tilde{x}_1, \tilde{x}_2, \cdots, \tilde{x}_n)^{\mathrm{T}}$ is the state vector of the new DCNN Eq.(2), $\boldsymbol{g}(\tilde{\boldsymbol{x}}) = \boldsymbol{f}(\tilde{\boldsymbol{x}} + \boldsymbol{x}^*) - \boldsymbol{f}(\boldsymbol{x}^*)$.

Set $N = \{1, 2, \cdots, n\}$. As in [15], we divide the index set N into three index subsets: $\tilde{N}_1 = \{i \in N;\ x_i^* > 1\}$, $\tilde{N}_2 = \{i \in N;\ x_i^* \leqslant 1\}$ and $\tilde{N}_3 = \{i \in N;\ x_i^* < 1\}$. We assume that $\tilde{N}_1 = \{i_1, i_2, \cdots, i_r\}$, $\tilde{N}_2 = \{i_{r+1}, i_{r+2}, \cdots, i_{r+m}\}$, $\tilde{N}_3 = \{i_{r+m}, \cdots, i_n\}$. For the simplification of notion, we rearrange the order of $\tilde{x}_i, \varphi_i, x_i^*$, and let $\boldsymbol{z}_j(t)=\tilde{\boldsymbol{x}}_{i_j}(t)$, $\boldsymbol{\varphi}_j^*(t)=\boldsymbol{\varphi}_{i_j}(t)$, $\boldsymbol{z}_j^*=\boldsymbol{x}_{i_j}^*$, $j\in N$. Let $N_1=\{j\in N;\ z_j^*>1\}$, $N_2=\{j\in N;\ |z_j^*|\leqslant 1\}$, $N_3=\{j\in N;\ z_j^*<-1\}$, then $N_1=\{1,2,\cdots,r\}$, $N_2=\{r+1, r+2,\cdots,r+m\}$, and $N_3=\{r+m+1, r+m+2,\cdots,n\}$. Set $\boldsymbol{z}^{(1)}(t)=(z_1(t), z_2(t),\cdots,z_r(t))^{\mathrm{T}}$, $\boldsymbol{z}^{(2)}(t)=(z_{r+1}(t), z_{r+2}(t),\cdots,z_{r+m}(t))^{\mathrm{T}}$, $\boldsymbol{z}^{(3)}(t)=(z_{r+m+1}(t),\cdots,z_n(t))^{\mathrm{T}}$, $\boldsymbol{z}(t)=(z_1(t), z_2(t),\cdots,z_n(t))^{\mathrm{T}}$. Then, Eq. (2) can be rewritten as

$$\begin{cases}\dot{\boldsymbol{z}}^{(i)}(t)=-\boldsymbol{z}^{(i)}(t)+\boldsymbol{A}_{i1}\boldsymbol{g}(\boldsymbol{z}^{(1)}(t))+\boldsymbol{A}_{i2}\boldsymbol{g}(\boldsymbol{z}^{(2)}(t))\\ \quad+\boldsymbol{A}_{i3}\boldsymbol{g}(\boldsymbol{z}^{(3)}(t))+\boldsymbol{B}_{i1}\boldsymbol{g}(\boldsymbol{z}^{(1)}(t-\tau(t)))\\ \quad+\boldsymbol{B}_{i2}\boldsymbol{g}(\boldsymbol{z}^{(2)}(t-\tau(t)))\\ \quad+\boldsymbol{B}_{i3}\boldsymbol{g}(\boldsymbol{z}^{(3)}(t-\tau(t))),\qquad i=1,2,3,\\ \boldsymbol{y}(t)=\sum_{i=1}^{3}\boldsymbol{C}_i(\boldsymbol{z}^{(i)}(t)+(\boldsymbol{z}^*)^{(i)}),\\ \boldsymbol{z}_{t_0}=\boldsymbol{\psi}=\boldsymbol{\varphi}^*-\boldsymbol{z}^*,\end{cases}\tag{3}$$

where $[\boldsymbol{g}^{\mathrm{T}}(\boldsymbol{z}^{(1)}(t))\ \boldsymbol{g}^{\mathrm{T}}(\boldsymbol{z}^{(2)}(t))\ \boldsymbol{g}^{\mathrm{T}}(\boldsymbol{z}^{(3)}(t))]^{\mathrm{T}}=\boldsymbol{f}(\boldsymbol{z}(t)+\boldsymbol{z}^*)-\boldsymbol{f}(\boldsymbol{z}^*)$.

In this paper, we assume that N_2 is not empty, $\boldsymbol{C}_1=0$ and $\boldsymbol{C}_3=0$. That is, for DCNN (3), only the output information of the state variable $\boldsymbol{z}^{(2)}(t)$ is available. To exponentially stabilize the zero equilibrium of DCNN (3), design an impulsive control law:

$$\Delta\boldsymbol{z}^{(2)}(t)=\boldsymbol{K}(\boldsymbol{y}(t^-)-\boldsymbol{C}_2(\boldsymbol{z}^*)^{(2)})\qquad t=t_k\tag{4}$$

where $\boldsymbol{K}$ is the impulsive gain matrix to be designed, $\{t_k\}$ is the impulse time sequence satisfying $0<t_1<t_2<\cdots<t_k<\cdots$ ($t_k\to\infty$ as $k\to\infty$), $\Delta\boldsymbol{z}^{(2)}(t)(t_k)=\boldsymbol{z}^{(2)}(t^+)-\boldsymbol{z}^{(2)}(t^-)$ describes the state jumping at impulse time $t=t_k$, and $\boldsymbol{z}^{(2)}(t_k^-)=\lim_{h\to 0-}\boldsymbol{z}^{(2)}(t_k+h)$, $\boldsymbol{z}^{(2)}(t_k^+)=\boldsymbol{z}^{(2)}(t_k)=\lim_{h\to 0+}\boldsymbol{z}^{(2)}(t_k+h)$, $k\in N$. Then we can obtain an impulsively controlled DCNN as Eq.(5).

$$\begin{cases}\dot{\boldsymbol{z}}^{(i)}(t)=-\boldsymbol{z}^{(i)}(t)+\boldsymbol{A}_{i1}\boldsymbol{g}(\boldsymbol{z}^{(1)}(t))+\boldsymbol{A}_{i2}\boldsymbol{g}(\boldsymbol{z}^{(2)}(t))+\\ \qquad\boldsymbol{A}_{i3}\boldsymbol{g}(\boldsymbol{z}^{(3)}(t))+\boldsymbol{B}_{i1}\boldsymbol{g}(\boldsymbol{z}^{(1)}(t-\tau(t)))+\\ \qquad\boldsymbol{B}_{i2}\boldsymbol{g}(\boldsymbol{z}^{(2)}(t-\tau(t)))+\\ \qquad\boldsymbol{B}_{i3}\boldsymbol{g}(\boldsymbol{z}^{(3)}(t-\tau(t)))\qquad i=1,2,3\\ \boldsymbol{y}(t)=\boldsymbol{K}\boldsymbol{C}_2\boldsymbol{z}^{(2)}(t)\\ \boldsymbol{z}_{t_0}=\boldsymbol{\psi}=\boldsymbol{\varphi}^*-\boldsymbol{z}^*\end{cases}\tag{5}$$

Remark 1 In system Eq.(5), only the state variable $\boldsymbol{z}^{(2)}(t)$ is subject to impulsive control. If $1\leqslant m\leqslant n-1$, we will call Eq.(4) a reduced-order impulsive control law of DCNN (1). In the case of $m=n$, the control law Eq.(4) becomes a full-order impulsive control law. The advantage of reduced-order impulsive control law over the full-order impulsive control law lies in the fact that it can effectively deal with the case when only partial states are available or measurable.

We define $\min_{i\in N_1}(z_i^*-1)=+\infty$ if N_1 is empty and $\min_{i\in N_3}(-1-z_i^*)=+\infty$ if N_3 is empty. Then $q=\min\{\min_{i\in N_1}(z_i^*-1),\ \min_{i\in N_3}(-1-z_i^*)\}>0$. It follows that if there exists some $T>t_0$ such that

$$|z_i(t)|<q,\ t\in[t_0-\tau,T]\qquad i\in N_1\cup N_3\tag{6}$$

then, $|z_i(t)+z_i^*|>1$, $\forall i\in N_1\cup N_3$, $t\in[t_0-\tau,T]$. Furthermore, if condition Eq.(6) holds, it is easy to verify that for $t\in[t_0-\tau,T]$, system Eq.(5) becomes

$$\begin{cases}\dot{\boldsymbol{z}}^{(i)}(t)=-\boldsymbol{z}^{(i)}(t)+\boldsymbol{A}_{i2}\boldsymbol{g}(\boldsymbol{z}^{(2)}(t))\\ \quad+\boldsymbol{B}_{i2}\boldsymbol{g}(\boldsymbol{z}^{(2)}(t-\tau(t)))\qquad t\neq t_k\qquad i=1,\ 2,\ 3\\ \boldsymbol{y}(t)=\boldsymbol{K}\boldsymbol{C}_2\boldsymbol{z}^{(2)}(t^-)\qquad t=t_k\\ \boldsymbol{z}_{t_0}=\boldsymbol{\psi}\end{cases}\tag{7}$$

As the stability of system Eq.(7) is equivalent to that of Eq.(5), we only need to study the impulsive stabilizability of system Eq.(7). For positive scalars β_1 and β_2 satisfying $\beta_1<\beta_2$, define $\mathrm{S}(\beta_1,\beta_2)=\{\{t_k\};\ \beta_1\leqslant t_k-t_{k-1}\leqslant\beta_2\}$. To establish sufficient conditions for exponential stability of the impulsive system Eq.(5), we introduce the following stability definition.

Definition 1 For given scalars β_1 and β_2 satisfying $\beta_1<\beta_2$, system Eq.(5) is said to be Uniformly Exponentially Stable (UES) over $\mathrm{S}(\beta_1,\beta_2)$, if for any given $\varepsilon>0$ and for any $\{t_k\}\in\mathrm{S}(\beta_1,\beta_2)$, there exist constants $\delta>0$ and $\gamma>0$ such that for any initial function $\boldsymbol{\psi}\in\mathrm{PC}([-\tau,0],\mathrm{B}_\delta)$, we have $\|\boldsymbol{z}(t,t_0,\boldsymbol{\psi})\|\leqslant\varepsilon\mathrm{e}^{-\gamma(t-t_0)}$, $\forall t\geqslant t_0$.

2 Main Results

In this section, we will combine piecewise differential Lyapunov techniques with the method of variation of parameters to investigate the exponential stability of the zero equilibrium of system Eq.(5). For this purpose, we introduce some piecewise differential functions.

For given impulse time sequence $\{t_k\}\in\mathrm{S}(\beta_1,\beta_2)$, we introduce the following two piecewise linear functions ρ, ν :$[t_0-\tau,+\infty]\to\mathrm{R}^+$ by $\rho(t)=(t_{k+1}-t)/(t_{k+1}-t_k)$ and $\nu(t)=1/(t_{k+1}-t_k)$, for $t\in[t_k,t_{k+1}]$, $k\in N$. It is easy to see that $\rho(t)\in[0,1]$ and $\nu(t)\in[1/\beta_2,1/\beta_1]$, for $t\geqslant t_0$. Then there exists $\rho_2(t)\in[0,1]$ such that $\nu(t)=(1-\rho_2(t))/\beta_1+\rho_2(t)/\beta_2$. Define $\rho_1(t)=\rho(t-\tau(t))$ if $t-\tau(t)\geqslant t_0$ and $\rho_1(t)=1$ if $t-\tau(t)<t_0$.

Theorem 1 Consider impulsive system Eq.(5).

Assume that $\{t_k\}\in S(\beta_1,\beta_2)$. If there exist matrices $\boldsymbol{P}_s>0$, diagonal matrices $\boldsymbol{D}_{lsij}>0$, $l=0,1,s,i,j=1,2$, matrix $\boldsymbol{Y}$, and positive scalars α, μ, λ_0, κ, such that the following matrix inequalities hold:

$$\begin{bmatrix} \boldsymbol{\Omega}_{si} & 0 & \boldsymbol{P}_s\boldsymbol{A}_{22}+\boldsymbol{D}_{0sij} & \boldsymbol{P}_s\boldsymbol{B}_{22} \\ 0 & -\alpha\boldsymbol{P}_j+\boldsymbol{D}_{1sij} & 0 & 0 \\ * & 0 & -2\boldsymbol{D}_{0sij} & 0 \\ * & 0 & 0 & -\boldsymbol{D}_{1sij} \end{bmatrix}<0 \quad s,i,j=1,2 \tag{8}$$

$$\begin{bmatrix} -\mu\boldsymbol{P}_1 & \boldsymbol{P}_2+\boldsymbol{C}_2^{\mathrm{T}}\boldsymbol{Y}^{\mathrm{T}} \\ * & -\boldsymbol{P}_2 \end{bmatrix}<0 \tag{9}$$

$$\lambda_0\boldsymbol{I}\leqslant\boldsymbol{P}_s\leqslant\kappa\lambda_0\boldsymbol{I},\quad s=1,2, \tag{10}$$

where $\boldsymbol{\Omega}_{si}=(-2+\alpha/\mu+\ln\mu/\beta_2)\boldsymbol{P}_s-(1/\beta_i)(\boldsymbol{P}_2-\boldsymbol{P}_1)$, then system Eq.(5) with $\boldsymbol{K}=\boldsymbol{P}_2^{-1}\boldsymbol{Y}$ is UES over $S(\beta_1,\beta_2)$. Moreover, for any given positive scalar:

$$\delta<\delta_1=\min_{j=1,3}[1+\sqrt{\kappa/\mu}(\|\boldsymbol{A}_{j2}\|+\|\boldsymbol{B}_{j2}\|)]^{-1}q \tag{11}$$

there exists scalar $\gamma\in(0,1)$ such that $\boldsymbol{\psi}\in PC([-\tau,0],B_\delta)$ implies $\|\boldsymbol{z}^{(2)}(t,t_0,\boldsymbol{\psi})\|<\delta\sqrt{\kappa/\mu}\mathrm{e}^{-\gamma(t-t_0)}$, $\|\boldsymbol{z}^{(j)}(t,t_0,\boldsymbol{\psi})\|<q\sqrt{\kappa/\mu}\mathrm{e}^{-\gamma(t-t_0)}$, $j=1,3,\ t\geqslant t_0$.

Proof　For given δ satisfying (11), by Eq.(8)～Eq.(9), there exist scalar $\mu_0\in(0,\mu)$ and sufficiently small positive scalars γ, ε_0, such that the following inequalities hold:

$$\boldsymbol{\Xi}_{sij}=\begin{bmatrix} \boldsymbol{\Omega}_{0si} & 0 & \boldsymbol{P}_s\boldsymbol{A}_{22}+\boldsymbol{D}_{0sij} & \boldsymbol{P}_s\boldsymbol{B}_{22} \\ 0 & -\alpha\mathrm{e}^{-2\gamma\tau}\boldsymbol{P}_j+\boldsymbol{D}_{1sij} & 0 & 0 \\ * & 0 & -2\boldsymbol{D}_{0sij} & 0 \\ * & 0 & 0 & -\boldsymbol{D}_{1sij} \end{bmatrix}<-\varepsilon_0 I \quad s,i,j=1,2 \tag{12}$$

$$\begin{bmatrix} -\mu_0\boldsymbol{P}_1 & \boldsymbol{P}_2+\boldsymbol{C}_2^{\mathrm{T}}\boldsymbol{Y}^{\mathrm{T}} \\ * & -\boldsymbol{P}_2 \end{bmatrix}<0 \tag{13}$$

$$\delta+\frac{\delta}{1-\gamma}\sqrt{\frac{\kappa}{\mu}}\max_{j=1,3}[(\|\boldsymbol{A}_{j2}\|+\|\boldsymbol{B}_{j2}\|\mathrm{e}^{\gamma\tau})]<q \tag{14}$$

where $\boldsymbol{\Omega}_{0si}=(-2+2\gamma+\alpha/\mu_0+\ln\mu_0/\beta_2)\boldsymbol{P}_s-(1/\beta_i)(\boldsymbol{P}_2-\boldsymbol{P}_1)$. For $s,i,j\in\{1,2\}$, define $\boldsymbol{\Psi}_s(t)=(1-\rho_1(t))[(1-\rho_2(t))\boldsymbol{\Psi}_{s11}+\rho_2(t)\boldsymbol{\Psi}_{s21}]+\rho_1(t)[(1-\rho_2(t))\boldsymbol{\Psi}_{s12}+\rho_2(t)\boldsymbol{\Psi}_{s22}]$, $\boldsymbol{\Omega}_s(t)=(-2+\alpha/\mu_0+\ln\mu_0/\beta_2)\boldsymbol{P}_s-\nu(t)(\boldsymbol{P}_2-\boldsymbol{P}_1)$, $\boldsymbol{\Phi}(t)=-\alpha\mathrm{e}^{-2\gamma\tau}((1-\rho_1(t))\boldsymbol{P}_1+\rho_1(t)\boldsymbol{P}_2)$:

$$\boldsymbol{\Gamma}_s(t)=\begin{bmatrix} \boldsymbol{\Omega}_s(t) & 0 & \boldsymbol{P}_s\boldsymbol{A}_{22} & \boldsymbol{P}_s\boldsymbol{B}_{22} \\ 0 & \boldsymbol{\Phi}(t) & 0 & 0 \\ * & 0 & 0 & 0 \\ * & 0 & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{\Psi}_s(t)=\begin{bmatrix} 0 & 0 & \boldsymbol{D}_{0sij} & 0 \\ 0 & \boldsymbol{D}_{1sij} & 0 & 0 \\ * & 0 & -2\boldsymbol{D}_{0sij} & 0 \\ 0 & 0 & 0 & -\boldsymbol{D}_{1sij} \end{bmatrix}$$

Then by by Eq.(12), we have:

$$\boldsymbol{\Xi}_s(t)=\boldsymbol{\Gamma}_s(t)+\boldsymbol{\Psi}_s(t)\leqslant-\varepsilon_0\boldsymbol{I} \tag{15}$$

For any given initial function $\boldsymbol{\psi}\in PC([-\tau,0],B_\delta)$, let $\boldsymbol{z}(t)=\boldsymbol{z}(t,t_0,\boldsymbol{\psi})$. For the simplification of notion, we denote $\boldsymbol{z}^{(2)}(t)$ by $w(t)$ and set $\boldsymbol{w}(t)=(w_1(t),w_2(t),\cdots,w_m(t))^{\mathrm{T}}$. Choose time-varying Lyapunov function $V(t)=\boldsymbol{w}^{\mathrm{T}}(t)\boldsymbol{P}(t)\boldsymbol{w}(t)$, where $\boldsymbol{P}(t)=(1-\rho(t))\boldsymbol{P}_1+\rho(t)\boldsymbol{P}_2$. Set $U(t)=\mathrm{e}^{2\gamma(t-t_0)}V(t)$ and $\varepsilon=\sqrt{\kappa/\mu_0}\delta$. Note that $\boldsymbol{z}^{(j)}(t)$ is continuous for $t\geqslant t_0$, $j=1,3$. Since $\|\boldsymbol{z}^{(j)}(t_0+\theta)\|<\delta<q$, $\theta\in[-\tau,0]$, $j=1,3$, it follows that there exists $T>t_0$ such that $\|\boldsymbol{z}^{(j)}(t)\|<q$ for $t\in[t_0-\tau,T]$, $j=1,3$. Set $T^*=\inf\{t>t_0,|\boldsymbol{z}^{(j)}(t)|<q,\ j=1,3\}$. We will prove $T^*=+\infty$. Otherwise, we will have $T^*<+\infty$, $\boldsymbol{z}^{(j)}(t)<q$ for $t\in[t_0-\tau,T^*]$, $j=1,3$, and either:

$$\boldsymbol{z}^{(1)}(T^*)=q \text{ or } \boldsymbol{z}^{(3)}(T^*)=q \tag{16}$$

We assume that $T^*\in[t_k,t_k+1]$ for some nonnegative integer k and set $t_i^*=\min\{t_i,T^*\}$. In the following, we will prove that:

$$U(t)<\lambda_0\varepsilon^2 \quad t\in[t_0-\tau,T^*] \tag{17}$$

To do this, we first prove that:

$$U(t)<\lambda_0\varepsilon^2 \quad t\in[t_0-\tau,t_1^*] \tag{18}$$

It is noticed that for $\theta\in[-\tau,0]$, by(10),

$$U(t_0+\theta)\leqslant V(t_0+\theta)=\boldsymbol{w}^{\mathrm{T}}(t_0+\theta)\boldsymbol{P}(t_0+\theta)\boldsymbol{w}(t_0+\theta)<\kappa\lambda_0\delta^2=\mu_0\lambda_0\varepsilon^2<\lambda_0\varepsilon^2$$

So it only needs to prove:

$$U(t)<\lambda_0\varepsilon^2 \quad t\in(t_0,t_1^*) \tag{19}$$

Suppose not, there exists some $t\in(t_0,t_1^*)$ such that $U(t)\geqslant\lambda_0\varepsilon^2$. Set $t^*=\inf\{t\in[t_0,t_1^*];U(t)\geqslant\lambda_0\varepsilon^2\}$, then $t^*\in(t_0,t_1^*)$ and $U(t^*)=\lambda_0\varepsilon^2$. Set $\bar{t}=\sup\{t\in[t_0,t^*];\ U(t)\leqslant\mu_0\lambda_0\varepsilon^2\}$, then $\bar{t}\in(t_0,t^*)$ and $U(\bar{t})=\mu_0\lambda_0\varepsilon^2$. For $t\in[\bar{t},t^*]$, $U(t)\geqslant\mu_0\lambda_0\varepsilon^2\geqslant\mu_0U(t+\theta)$, $\theta\in[-\tau,0]$. It is noted that for $t\in[t_0-\tau,T^*]$, system Eq.(5) becomes Eq.(7). It follows that for $t\in[\bar{t},t^*]$:

$$D^{+}U(t)\leqslant D^{+}U(t)+\alpha(\tfrac{1}{\mu_0}U(t)-U(t-\tau(t)))\leqslant \mathrm{e}^{2\gamma(t-t_0)}\Big[(2\gamma+\alpha/\mu_0)\boldsymbol{w}^{\mathrm{T}}(t)\boldsymbol{P}(t)\boldsymbol{w}(t)+2\boldsymbol{w}^{\mathrm{T}}(t)\boldsymbol{P}(t)(-\boldsymbol{w}(t)+\boldsymbol{A}_{22}\boldsymbol{g}(\boldsymbol{w}(t))+\boldsymbol{B}_{22}\boldsymbol{g}(\boldsymbol{w}(t-\tau(t)))))-\nu(t)\boldsymbol{w}^{\mathrm{T}}(t)(\boldsymbol{P}_2-\boldsymbol{P}_1)\boldsymbol{w}(t)-\alpha\mathrm{e}^{-2\gamma\tau}\boldsymbol{w}^{\mathrm{T}}(t-\tau(t))\boldsymbol{P}(t-\tau(t))\boldsymbol{w}(t-\tau(t))\Big]=\mu_1U(t)+2\mathrm{e}^{2\gamma(t-t_0)}\boldsymbol{\xi}^{\mathrm{T}}(t)((1-\rho(t))\boldsymbol{\Gamma}_1(t)+\rho(t)\boldsymbol{\Gamma}_2(t))\boldsymbol{\xi}(t) \quad (20)$$

where $\mu_1=-\ln\mu_0/\beta_2$, $\boldsymbol{\xi}^{\mathrm{T}}(t)=[\boldsymbol{w}^{\mathrm{T}}(t)\ \ \boldsymbol{w}^{\mathrm{T}}(t-\tau(t))\ \ \boldsymbol{g}^{\mathrm{T}}(\boldsymbol{w}(t))\ \ \boldsymbol{g}^{\mathrm{T}}(\boldsymbol{w}(t-\tau(t)))]$.

On the other hand, by the property of the activation function $f(\cdot)$, it is easy to see that for any $h\in\{1,2,\cdots,m\}$:

$$0\leqslant w_h(t)g(w_h(t))-g^2(w_h(t)) \quad (21)$$

$$0\leqslant w_h^2(t-\tau(t))-g^2(w_h(t-\tau(t))) \quad (22)$$

Set $D_{lsij}=\mathrm{diag}(d_{lsij}^1,d_{lsij}^2,\cdots,d_{lsij}^m)$, then $d_{lsij}^h\geqslant 0$, l=0,1,s, i,j=1,2, h=1,2,$\cdots$,m. Let:

$$\upsilon_{ls}^h=\tilde{\rho}_1(t)\Big[\tilde{\rho}_2(t)d_{ls11}^h+\rho_2(t)d_{ls21}^h\Big]+\rho_1(t)\Big[\tilde{\rho}_2(t)d_{ls12}^h+\rho_2(t)d_{ls22}^h\Big]$$

where $\tilde{\rho}_i(t)=1-\rho_i(t)$, s,i=1,2. Using Eq.(21)～Eq.(22), we have:

$$0\leqslant \mathrm{e}^{2\gamma(t-t_0)}\sum_{h=1}^{m}2\Big[\tilde{\rho}(t)\upsilon_{01}^h(t)+\rho(t)\upsilon_{02}^h(t)\Big]\times\Big[w_h(t)g(w_h(t))-g^2(w_h(t))\Big]+\mathrm{e}^{2\gamma(t-t_0)}\sum_{h=1}^{m}2\Big[\tilde{\rho}(t)\upsilon_{11}^h(t)+\rho(t)\upsilon_{12}^h(t)\Big]\times\Big[w_h^2(t-\tau(t))-g^2(w_h(t-\tau(t)))\Big]=\mathrm{e}^{2\gamma(t-t_0)}\boldsymbol{\xi}^T(t)\Big[\tilde{\rho}(t)\boldsymbol{\Psi}_1(t)+\rho_1(t)\boldsymbol{\Psi}_2(t)\Big]\boldsymbol{\xi}(t) \quad (23)$$

where $\tilde{\rho}(t)=1-\rho(t)$. Adding the right side of Eq.(23) to Eq.(20) gives:

$$D^{+}U(t)\leqslant \mu_1U(t)+\mathrm{e}^{2\gamma(t-t_0)}\boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{\Xi}(t)\boldsymbol{\xi}(t) \quad (24)$$

where $\boldsymbol{\Xi}(t)=(1-\rho(t))\boldsymbol{\Xi}_1(t)+\rho(t)\boldsymbol{\Xi}_2(t)$. By Eq.(15), we have $\boldsymbol{\Xi}(t)\leqslant-\varepsilon_0\boldsymbol{I}$. Thus, by Eg.(24) and noticing that $V(t)\leqslant\kappa\lambda_0\|\boldsymbol{w}(t)\|^2$, we obtain:

$$D^{+}U(t)\leqslant(\mu_1-\varepsilon_0/(\kappa\lambda_0))U(t) \qquad t\in[\bar{t},t^*]$$

which leads to:

$$U(t^*)\leqslant\exp((\mu_1-\varepsilon_0/(\kappa\lambda_0))\beta_2)U(\bar{t})<\lambda_0\varepsilon^2$$

This is a contradiction, so Eq.(18) holds. Now we assume that for some $l\in N$ satisfying $t_l<T^*$,

$$U(t)<\lambda_0\varepsilon^2 \qquad t\in[t_0-\tau,t_l] \quad (25)$$

we will prove that $U(t)<\lambda_0\varepsilon^2$ for $t\in[t_l,t_{l+1}]$. If not, there exists some $t\in[t_l,t_{l+1}]$ such that $U(t)\geqslant\lambda_0\varepsilon^2$. Then, set $\tilde{t}=\inf\{t\in[t_l,t_{l+1}];U(t)\geqslant\lambda_0\varepsilon^2\}$. By Eq.(13) with $\boldsymbol{Y}=\boldsymbol{P}_2\boldsymbol{K}$ and using Schur complement, we have $(\boldsymbol{I}+\boldsymbol{K}\boldsymbol{C}_2)^{\mathrm{T}}\boldsymbol{P}_2(\boldsymbol{I}+\boldsymbol{K}\boldsymbol{C}_2)\leqslant\mu_0\boldsymbol{P}_1$. It follows from $\rho(t_l)=1$ and $\rho(t_l^-)=0$ that:

$$U(t_l)=\mathrm{e}^{2\gamma(t_l-t_0)}\boldsymbol{w}^{\mathrm{T}}(t_l)\boldsymbol{P}_2\boldsymbol{w}(t_l)=\mathrm{e}^{2\gamma(t_l-t_0)}\boldsymbol{w}^T(t_l^-)(\boldsymbol{I}+\boldsymbol{K}\boldsymbol{C}_2)^T\boldsymbol{P}_2(\boldsymbol{I}+\boldsymbol{K}\boldsymbol{C}_2)\boldsymbol{w}(t_l^-)=\mu_0\mathrm{e}^{2\gamma(t_l-t_0)}\boldsymbol{w}^{\mathrm{T}}(t_l^-)\boldsymbol{P}_1\boldsymbol{w}(t_l^-)=\mu_0U(t_l^-).$$

Then by Eq.(25), we obtain $U(t_l)<\mu_0\lambda_0\varepsilon^2$. Thus $\tilde{t}\in(t_l,t_{l+1})$ and $U(\tilde{t})=\lambda_0\varepsilon^2$. Set $\bar{t}=\sup\{t\in[t_l,\tilde{t}];U(t)\leqslant\mu_0\lambda_0\varepsilon^2\}$. then $U(\bar{t})=\mu_0\lambda_0\varepsilon^2$. Moreover, we have $U(t)\geqslant\mu_0\lambda_0\varepsilon^2\geqslant\mu_0U(t+\theta)$, for $t\in[\bar{t},\tilde{t}]$ and $\theta\in[-\tau,0]$. Then applying the same argument of the proof on the interval $[t_0,t_1]$, we obtain:

$$U(\tilde{t})\leqslant\exp((\mu_1-\varepsilon_0/(\kappa\lambda_0))\beta_2)U(\bar{t})<\lambda_0\varepsilon^2$$

which yields a contradiction. Therefore, $U(t)<\lambda_0\varepsilon^2$ for $t\in[t_l,t_{l+1}]$.

By mathematical induction Eq.(25) holds for any $l\in N$. That is Eq.(17) holds, which implies that:

$$\|\boldsymbol{w}(t)\|<\varepsilon\mathrm{e}^{-\gamma(t-t_0)} \qquad t\in[t_0,T^*) \quad (26)$$

By the method of variation of parameters, for $t\in[t_0,T^*]$, we have:

$$\boldsymbol{z}^{(j)}(t)=\boldsymbol{z}^{(j)}(t_0)\mathrm{e}^{-(t-t_0)}+\int_{t_0}^{t}\mathrm{e}^{-(t-s)}\times\Big[\boldsymbol{A}_{j2}\boldsymbol{g}(\boldsymbol{w}(s))+\boldsymbol{B}_{j2}\boldsymbol{g}(\boldsymbol{w}(s-\tau(s)))\Big]\mathrm{d}s$$

j=1,3. Applying Eq.(26) to the above inequality and using Eq.(14), we get:

$$\|\boldsymbol{z}^{(j)}(t)\|<\delta\mathrm{e}^{-(t-t_0)}+\tfrac{\varepsilon}{1-\gamma}(\|\boldsymbol{A}_{j2}\|+\|\boldsymbol{B}_{j2}\|\mathrm{e}^{\gamma\tau})\times(\mathrm{e}^{-\gamma(t-t_0)}-\mathrm{e}^{-(t-t_0)})<\delta\Big[1+\tfrac{\sqrt{\kappa/\mu}}{1-\gamma}(\|\boldsymbol{A}_{j2}\|+\|\boldsymbol{B}_{j2}\|\mathrm{e}^{\gamma\tau})\Big]\mathrm{e}^{-\gamma(t-t_0)}<q\mathrm{e}^{-\gamma(t-t_0)},\ t\in[t_0,T^*) \qquad j=1,3 \quad (27)$$

By the continuity of $\boldsymbol{z}^{(j)}(t)$, the above inequality implies that $\|\boldsymbol{z}^{(j)}(T^*)\|<q$ for j=1,3, which contradicts Eq.(16). Therefore, $T^*=+\infty$. Combining Eg.(26) and Eq.(27) yields the exponent estimates of $\boldsymbol{z}^{(j)}(t)$, j=1,2,3. Then the proof is complete.

It is obvious that if Eq.(8)～Eq.(9) are feasible, then there exists positive scalars λ_0 and κ such that Eq.(8)～Eq.(10) are feasible. Thus, we have the following corollary.

Corollary 1 Consider impulsive system Eq.(5). Assume that $\{t_k\}\in S(\beta_1,\beta_2)$. If there exist matrices $\boldsymbol{P}_s>0$, diagonal matrices $\boldsymbol{D}_{lsij}>0$, l=0,1,s,i, j=1,2, matrix $\boldsymbol{Y}$, and positive scalars α, μ, λ_0, κ, such that the matrix inequalities Eq.(8)～Eq.(9) hold, then system Eq.(5) with $\boldsymbol{K}=\boldsymbol{P}^{-1}\boldsymbol{Y}$ is UES over $S(\beta_1,\beta_2)$.

Remark 2 Theorem 1 proposes a reduced-order impulsive control scheme for DCNNs (1). When the lower bound bound β_1 and the upper bound β_2 of the impulse intervals are known, the impulsive gain matrix $\boldsymbol{K}$ can be derived by solving the matrix inequalities Eq.(8)～Eq.(9). Moreover, the impulsive gain matrix $\boldsymbol{K}$ is robust in the sense that the impulsive control law stabilize the zero equilibrium of system Eq.(5) for any $\{t_k\}\in S(\beta_1,\beta_2)$.

Remark 3 When $q=\infty$, the conditions in Theorem 1 guarantee the zero equilibrium of system Eq.(5) to be globally exponentially stable. When $q<\infty$, Theorem 1 indicates that a subset of the domain of attraction of the zero equilibrium of system Eq.(5) is $PC([-\tau,0];B_\delta)$.

3 Numerical Example

In this section, we present one example to illustrate the effectiveness of our results.

Example 1: Consider the DCNN (1) with

$$A=\begin{bmatrix}0.5 & 0.1 & 0.1 & 0.2\\ 0 & -0.8 & 0.3 & 0.2\\ 0.1 & 0.2 & 0.6 & 0.7\\ -0.2 & 0.2 & 0.1 & 0.8\end{bmatrix},$$

$$\boldsymbol{B}=\begin{bmatrix}0.8 & 0 & 0.1 & 0.1\\ 0.1 & 0.2 & 0.1 & 0.2\\ 0.1 & 0 & 0.6 & 0.3\\ 0 & 0.1 & 0.2 & 0.5\end{bmatrix},\ \boldsymbol{u}=\begin{bmatrix}1.01\\ 1.22\\ 0.76\\ -0.61\end{bmatrix}$$

One can verify that DCNN (1) has an isolated equilibrium point $\boldsymbol{x}^*=(2,0.5,-0.3,-2.05)^{\mathrm{T}}$. According to the equilibrium point x^*, the index set $N=\{1,2,3,4\}$ can be divided into three index subsets: $N_1=\{1\}$, $N_2=\{2,3\}$, and $N_3=\{4\}$. Moreover, for some $T>0$, DCNN (1) on the interval $[-\tau,T]$ can be rewritten in the form of Eq.(7) with $\boldsymbol{A}_{12}=[0.1\ \ 0.1]$, $\boldsymbol{B}_{12}=[0\ \ 0.1]$, $\boldsymbol{A}_{32}=[0.2\ \ 0.1]$, $\boldsymbol{B}_{32}=[0.1\ 0.2]$, and

$$\boldsymbol{A}_{22}=\begin{bmatrix}-0.8 & 0.3\\ 0.2 & 0.6\end{bmatrix},\ \boldsymbol{B}_{22}=\begin{bmatrix}0.2 & 0.1\\ 0 & 0.6\end{bmatrix}$$

To better illustrate the stability of $\boldsymbol{x}^*$, we have computed numerically the solutions of DCNN (1) with $\tau=1$ starting from the neighborhood of $\boldsymbol{x}^*$, as shown in Fig. 1. One can see that all these trajectories go away from $\boldsymbol{x}^*$. This shows that the equilibrium $\boldsymbol{x}^*$ is unstable.

Now we assume that the system's output is $\boldsymbol{y}(t)=\boldsymbol{C}_2\bar{\boldsymbol{x}}(t)$, where $\boldsymbol{C}_2=[0\ 1]$, $\bar{\boldsymbol{x}}(t)=(x_2(t),x_3(t))^{\mathrm{T}}$. We will design an impulsive control law with the form of

$$w(t_k)=(\boldsymbol{I}+\boldsymbol{K}\boldsymbol{C}_2)(\boldsymbol{w}(t_k^-)-\bar{\boldsymbol{x}}^*)+\bar{\boldsymbol{x}}^* \qquad (28)$$

where $\{t_k\}\in S(0.4,\ 0.5)$, and $\bar{\boldsymbol{x}}^*=(0.5,-0.3)^{\mathrm{T}}$. to stabilize the equilibrium $\boldsymbol{x}^*$. Applying Theorem 1 with the choice of $\alpha=\mu=0.88$, it has been found that the LMIs Eq.(8)～Eq.(9) are feasible and the derived impulsive gain matrix $\boldsymbol{K}=(-0.128\ 2,-1.000\ 4)^{\mathrm{T}}$. The simulations of system (1) under the impulsive control law Eq.(28) with $t_k-t_{k-1}=0.45$ and $\tau(t)=1$ are given in Fig. 2. One can see that under the reduced-order impulsive control law Eq.(28), the trajectories starting from the neighborhood of $\boldsymbol{x}^*$ converge to $\boldsymbol{x}^*$.

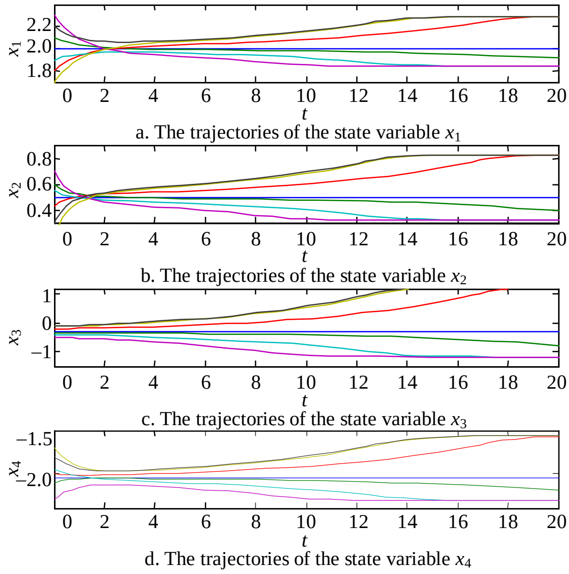

Fig.1 Trajectories of the DCNN (1) starting from the neighborhood of $\boldsymbol{x}^*$

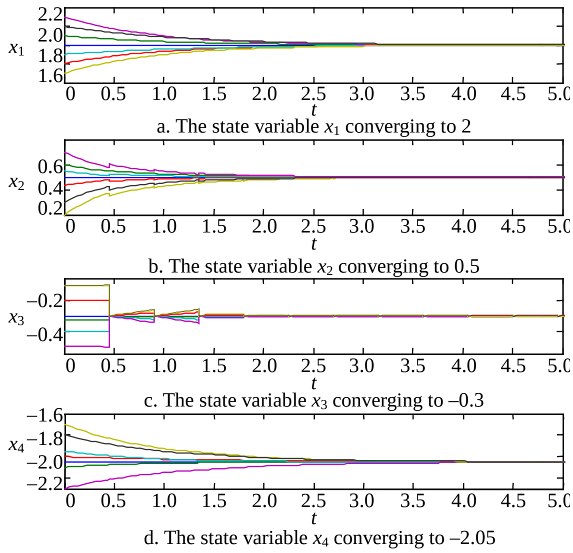

Fig.2 Trajectories of the DCNN (1) under impulsive control Eq. (28)

4 Conclusions

This paper has investigated the impulsive stabilization problem for DCNNs via partial states. A piecewise differential Lyapunov function has been introduced to derive the sufficient condition for exponential impulsive stabilization via partial states. The sufficient condition is expressed in terms of linear matrix inequalities concerning the interconnected matrices and the bounds of the impulsive intervals. With the help of LMI solver, it is easy to check the existence of the impulsive control law via partial states for DCNNs. An example has demonstrated the effectiveness of the proposed impulsive stabilization scheme.

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