Performance Analysis for BPSK Alamouti's Scheme with Distributed Transmit Antennas

LI Zhi-gang and TANG You-xi

(National Key Laboratory of Science and Technology on Communications, University of Electronic Science and Technology of China　Chengdu　611731)

Abstract　An approximate expression of bit error rate (BER) is derived for a circular cell distributed antenna system with space-time block coding based on Alamouti's scheme and utilizing binary phase shift keying (BPSK) modulation. The analysis is carried out on considering the effects of path loss, log-normal shadowing, multipath fading and background noise. A simulation is carried out under the condition of two transmit and one received antennas, single-path Rayleigh fading channels, antenna spacing of 200 m, and standard deviation of shadowing of 8 dB. The simulation results show that, the numerical results match the simulation results well, and at the BER of 3×10^{-1}, the proposed method is superior by about 1.4 dB to the traditional methods in bit signal-to-noise ratio.

Key words　bit error rate;　distributed antenna systems;　large-scale fading;　space-time block coding

分布式发射天线下BPSK Alamouti方案性能分析

李志刚，唐友喜

(电子科技大学通信抗干扰技术国家级重点实验室　成都　611731)

【摘要】在分布式发射天线系统中，针对圆形小区下基于Alamouti空时分组编码和采用BPSK调制的情况，推导了误比特性能的近似解析表达式。理论推导过程考虑了路径损耗、阴影衰落、单径瑞利衰落和背景噪声的影响。在2发1收、单径瑞利衰落信道、200 m发射天线间距以及8 dB阴影标准差的条件下进行仿真，结果表明，所提方法的数值分析结果和计算机仿真结果是相吻合的，且在比特误码率为3×10^{-1}时，与传统方法相比，该文方法在性能上有约1.4 dB的比特信噪比增益。

关　键　词　比特误码率;　分布式天线系统;　大尺度衰落;　空时分组码

中图分类号　TN911　　**文献标识码**　A　　doi:10.3969/j.issn.1001-0548.2011.05.006

Apart from the in-depth research on co-located antenna systems (CAS), distributed antenna systems (DAS) have been attracting worldwide interest and are expected to be one of the promising techniques in the next generation wireless communication systems[1-3]. Space-time block coding (STBC), which was first proposed by Alamouti[4] for systems with two transmit antennas, is a superb candidate for next-generation wireless systems.

However, there is little information on the analysis of exact or approximate expressions of bit error rate (BER) for space-time block coding with distributed transmit antennas.

In Ref. [5], a tight closed-form upper bound was presented for the BER of space-time block coding in Rician and log-normal fading channel, respectively. An expression of BER over small-scale fading and large-scale fading was given in Ref. [2], but it was an approximate expression in high signal-to-noise ratio (SNR) region.

This letter derives an approximate BER for arbitrary SNR for DAS with space-time block coded based on Alamouti's scheme and utilizes binary phase shift keying (BPSK) modulation over composite exponential/log-normal fading.

Received date: 2010 – 01 – 18;　Revised date: 2011 – 03 – 07

收稿日期：2010 – 01 – 18;　修回日期：2011 – 03 – 07

Foundation item: This work was supported by the National 863 High Technology Development Project (2009AA01Z236), National Natural Science Foundation of China (60902027, 60832007 and 60901018), the Funds (9140A21030209DZ02), and the Fundamental Research Funds for the Central Universities (ZYGX2009J008 and ZYGX2009J010).

基金项目：国家863计划(2009AA01Z236)；国家自然科学基金(60902027, 60832007, 60901018)；基金项目(9140A21030209DZ02)；中央高校基本科研业务费专项资金(ZYGX2009J008, ZYGX2009J010)

Biography: LI Zhi-gang was born in 1982. He is Ph. D student. His research interest is in wireless communication technology.

作者简介：李志刚(1982 –)，男，博士生，主要从事无线通信方面的研究.

1 System Model

1.1 Transmitter

For DAS, two transmit antennas connected to a single base station by fiber optic or coaxial cables are located at different places of a cell. For practical reason, the mobile station (MS) is assumed to have a single antenna. The separation distance between the antennas is denoted by D ($>>\lambda$, where λ is the wavelength). The separation distance between the MS and the kth transmit antenna (TX_k) is denoted by d_k (k=1,2). We assume a system with ideal synchronization. We also assume a noise-limited environment. This assumption holds for an isolated cell or a multicell system with large frequency reuse distance where the interference is small compared with the thermal noise[6].

1.2 Receiver

The output of the STBC decoder can be given by Ref. [4]:

$$R = \sqrt{P^t / 2}\|\boldsymbol{h}\| X + W \tag{1}$$

where P^t is the total transmitted power at the base station, $\boldsymbol{h}$ is a channel vector of size 2×1, $\|\boldsymbol{h}\|$ is Euclidean norm of $\boldsymbol{h}$, X represents the transmitted symbol, and W is additive complex Gaussian noise with zero mean and variance N_0. The coefficient $\sqrt{P^t/2}$ ensures that the total transmitted power is P^t.

In DAS, the transmit antennas are spaced by large distance. Hence, the channel model involves not only small-scale fading but also large-scale fading. Therefore, the channel vector can be expressed as:

$$\boldsymbol{h} = [h_1\sqrt{S_1}, h_2\sqrt{S_2}]^{\mathrm{T}} \tag{2}$$

where h_k represents composite multipath/shadowing fading, and S_k is a factor that captures the effects of path loss for the TX_k-MS link (k=1,2).

We model h_k as a complex composite exponential/log-normal random variable. The composite exponential/log-normal probability density function (PDF) of SNR γ_k can be presented as[7]:

$$P_{\gamma_k}(\gamma_k) = \int_0^\infty \frac{1}{\Omega_k}\exp\left(-\frac{\gamma_k}{\Omega_k}\right)\frac{\xi}{\sqrt{2\pi}\sigma_k\Omega_k}\times \exp\left[-\frac{(10\lg\Omega_k - \mu_k)^2}{2\sigma_k^2}\right]\mathrm{d}\Omega_k \tag{3}$$

where Ω_k is the average received power, $\xi = 10/\ln 10$, $\mu_k(\mathrm{dB}) = 10\lg\overline{\gamma}_k$ and $\sigma_k(\mathrm{dB})$ are the mean and standard deviation of $10\lg\Omega_k$, respectively, and $\overline{\gamma}_k$ is the average SNR per bit.

We model S_k as follows[8]:

$$S_k = \begin{cases} \left(\dfrac{\lambda}{4\pi}\right)^2 d_k^{-2} & d_k \leqslant \Upsilon \\ (h_b h_s)^2 d_k^{-4} & d_k > \Upsilon \end{cases} \tag{4}$$

where λ is the wavelength, h_b is the transmit antenna height, h_s is the MS antenna height, and $\Upsilon = 4\pi h_b h_s/\lambda$ denotes the break point.

2 BER Analysis

According to the described channel model, the instantaneous power received from TX_k can be expressed as:

$$P_k^r = \frac{1}{2}P^t|h_k|^2 S_k \tag{5}$$

Then, the total received power is $P^r = \sum_{k=1}^{2} P_k^r$. Under the assumption of BPSK modulation, the BER, conditioned on the mean SNR $\overline{\gamma}_1 = P^t S_1/2N_0$ and $\overline{\gamma}_2 = P^t S_2/2N_0$, can be expressed as:

$$P_e(\overline{\gamma}_1,\overline{\gamma}_2) = \int_0^\infty\int_0^\infty Q(\sqrt{2\gamma})f(\gamma_1,\gamma_2\,|\,\overline{\gamma}_1,\overline{\gamma}_2)\mathrm{d}\gamma_1\mathrm{d}\gamma_2 \tag{6}$$

where $Q(\cdot)$ is the Gaussian probability function[9], $\gamma = \sum_{k=1}^{2}\gamma_k$ is the total instantaneously received SNR, $\gamma_k = P_k^r/N_0$ represents the instantaneous SNR received from TX_k, and $f(\gamma_1,\gamma_2\,|\,\overline{\gamma}_1,\overline{\gamma}_2)$ is the joint conditional PDF of γ_1 and γ_2. According to the given channel model, γ_1 and γ_2, conditioned on $\overline{\gamma}_1$ and $\overline{\gamma}_2$, are independent and identically distributed composite exponential/log-normal random variables. Hence,

$$f(\gamma_1,\gamma_2\,|\,\overline{\gamma}_1,\overline{\gamma}_2) = P_{\gamma_1}(\gamma_1|\overline{\gamma}_1)P_{\gamma_2}(\gamma_2\,|\,\overline{\gamma}_2) \tag{7}$$

where $P_{\gamma_k}(\gamma_k\,|\,\overline{\gamma}_k)$ is the conditional PDF of γ_k in Eq. (3). Using Craig's formula for the Gaussian Q-function[9], $Q(\sqrt{2\gamma})$ can be expressed as:

$$Q(\sqrt{2\gamma}) = \frac{1}{\pi}\int_0^{\pi/2}\exp\left(-\frac{\gamma}{\sin^2\theta}\right)\mathrm{d}\theta = \frac{1}{\pi}\int_0^{\pi/2}\prod_{k=1}^{2}\exp\left(-\frac{\gamma_k}{\sin^2\theta}\right)\mathrm{d}\theta \tag{8}$$

where θ is an integral parameter. Then, it follows from Eq. (6) that:

$$P_e(\bar{\gamma}_1,\bar{\gamma}_2)=\int_0^\infty\int_0^\infty\left[\frac{1}{\pi}\int_0^{\pi/2}\prod_{k=1}^{2}\exp\left(-\frac{\gamma_k}{\sin^2\theta}\right)\mathrm{d}\theta\times\prod_{k=1}^{2}P_{\gamma_k}(\gamma_k\,|\,\bar{\gamma}_k)\,\mathrm{d}\gamma_1\mathrm{d}\gamma_2\right] \quad (9)$$

After exchanging the integral order of θ and γ_k in Eq. (9), we have:

$$P_e(\bar{\gamma}_1,\bar{\gamma}_2)=\frac{1}{\pi}\int_0^{\pi/2}\left\{\prod_{k=1}^{2}\int_0^\infty\left[\exp\left(-\frac{\gamma_k}{\sin^2\theta}\right)\times P_{\gamma_k}(\gamma_k\,|\,\bar{\gamma}_k)\right]\mathrm{d}\gamma_k\right\}\mathrm{d}\theta \quad (10)$$

Based on the moment generating function (MGF) of composite exponential/log-normal PDF[7], we have:

$$\int_0^\infty\left[\exp\left(-\frac{\gamma_k}{\sin^2\theta}\right)P_{\gamma_k}(\gamma_k\,|\,\bar{\gamma}_k)\right]\mathrm{d}\gamma_k=\frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}\left(1+\frac{1}{\sin^2\theta}10^{(x\sqrt{2}\sigma_k+\mu_k)/10}\right)^{-1}\mathrm{e}^{-x^2}\mathrm{d}x \quad (11)$$

It can be easily derived that:

$$10^{(x\sqrt{2}\sigma_k+\mu_k)/10}=\bar{\gamma}_k\times 10^{x\sqrt{2}\sigma_k/10} \quad (12)$$

Substituting Eq. (11) and Eq. (12) into Eq. (10) yields:

$$P_e(\bar{\gamma}_1,\bar{\gamma}_2)=\frac{1}{\pi}\int_0^{\pi/2}\left\{\prod_{k=1}^{2}\left[\frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}\left(1+\frac{\bar{\gamma}_k}{\sin^2\theta}10^{x\sqrt{2}\sigma_k/10}\right)^{-1}\mathrm{e}^{-x^2}\mathrm{d}x\right]\right\}\mathrm{d}\theta \quad (13)$$

The inner integral can be computed efficiently using a Gauss-Hermite quadrature integration[7,10], that is:

$$\frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}\left(1+\frac{\bar{\gamma}_k}{\sin^2\theta}10^{x\sqrt{2}\sigma_k/10}\right)^{-1}\mathrm{e}^{-x^2}\mathrm{d}x=\frac{1}{\sqrt{\pi}}\sum_{i=1}^{n}w_i\frac{\sin^2\theta}{\sin^2\theta+\bar{\gamma}_k\times 10^{x_i\sqrt{2}\sigma_k/10}} \quad (14)$$

where $x_i(i=1,2,\cdots,n)$ are the zeros of the n-order Hermite polynomial $He_n(x)$ and $w_i(i=1,2,\cdots,n)$ are weight factors tabulated in Table 25.10 of Ref. [10] for values of n from 2 to 20. To get numerical results in section IV, of Ref. [10] we can choose n=20 to compute BER as the approximate results. Then, we have:

$$P_e(\bar{\gamma}_1,\bar{\gamma}_2)=\frac{1}{\pi^2}\int_0^{\pi/2}\sum_{i=1}^{n}w_i\frac{\sin^2\theta}{\sin^2\theta+\bar{\gamma}_1 10^{x_i\sqrt{2}\sigma_1/10}}\times\sum_{i=1}^{n}w_i\frac{\sin^2\theta}{\sin^2\theta+\bar{\gamma}_2 10^{x_i\sqrt{2}\sigma_1/10}}\mathrm{d}\theta \quad (15)$$

If $\bar{\gamma}_1\neq\bar{\gamma}_2$, we spread the product of two polynomials in Eq. (15) as follows:

$$\left(\sum_{i=1}^{n}w_ia_i\right)\left(\sum_{i=1}^{n}w_ib_i\right)=\sum_{i=1}^{n}w_i^2a_ib_i+\sum_{1\leqslant i<j\leqslant n}w_iw_j(a_ib_j+a_jb_i) \quad (16)$$

Since

$$\frac{\sin^2\theta}{\sin^2\theta+c_1}\frac{\sin^2\theta}{\sin^2\theta+c_2}=1-\frac{c_1}{\sin^2\theta+c_1}-\frac{c_2}{\sin^2\theta+c_2}+\frac{c_1c_2}{c_2-c_1}\left(\frac{1}{\sin^2\theta+c_1}-\frac{1}{\sin^2\theta+c_2}\right) \quad (17)$$

and

$$\frac{1}{\pi}\int_0^{\pi/2}\frac{c}{\sin^2\theta+c}\mathrm{d}\theta=\frac{1}{2}\sqrt{\frac{c}{1+c}} \quad (18)$$

we have:

$$\frac{1}{\pi}\int_0^{\pi/2}\frac{\sin^2\theta}{\sin^2\theta+c_1}\frac{\sin^2\theta}{\sin^2\theta+c_2}\mathrm{d}\theta=\frac{1}{2}\frac{c_2\psi(c_2)-c_1\psi(c_1)}{c_2-c_1} \quad (19)$$

where $c_1\neq c_2$, and ψ is a function of c, i.e. $\psi(c)=1-\sqrt{\dfrac{c}{1+c}}$. Letting $c_k^{(i)}=\bar{\gamma}_k 10^{x_i\sqrt{2}\sigma_k/10}$ $(k=1,2,\cdots,n)$ and using Eq. (19), $P_e(\bar{\gamma}_1,\bar{\gamma}_2)$ can be expressed as:

$$P_e(\bar{\gamma}_1,\bar{\gamma}_2)=\frac{1}{2\pi}\left\{\sum_{i=1}^{n}w_i^2\frac{c_2^{(i)}\psi(c_2^{(i)})-c_1^{(i)}\psi(c_1^{(i)})}{c_2^{(i)}-c_1^{(i)}}+\sum_{1\leqslant i<j\leqslant n}w_iw_j\left[\frac{c_2^{(j)}\psi(c_2^{(j)})-c_1^{(i)}\psi(c_1^{(i)})}{c_2^{(j)}-c_1^{(i)}}+\frac{c_2^{(i)}\psi(c_2^{(i)})-c_1^{(j)}\psi(c_1^{(j)})}{c_2^{(i)}-c_1^{(j)}}\right]\right\} \quad (20)$$

If $\bar{\gamma}_1=\bar{\gamma}_2$, we spread the product of two polynomials in Eq. (15) as follows:

$$\left(\sum_{i=1}^{n}w_ia_i\right)^2=\sum_{i=1}^{n}w_i^2a_i^2+\sum_{i\neq j}w_iw_ja_ia_j \quad (21)$$

According to Ref. [7], we have:

$$\varphi(c)\triangleq\frac{1}{\pi}\int_0^{\pi/2}\left(\frac{\sin^2\theta}{\sin^2\theta+c}\right)^2\mathrm{d}\theta=\frac{1}{2}\psi(c)\left[1-\frac{1}{2}(\psi(c)-1)(\psi(c)-2)\right] \quad (22)$$

Hence, $P_e(\bar{\gamma}_1,\bar{\gamma}_2)$ can be expressed as:

$$P_e(\bar{\gamma}_1,\bar{\gamma}_2)=\frac{1}{2\pi}\left\{\sum_{i=1}^{n}w_i^2\varphi(c_1^i)+\sum_{i\neq j}^{n}w_iw_j\frac{c_1^{(j)}\psi(c_1^{(j)})-c_1^{(i)}\psi(c_1^{(i)})}{c_1^{(j)}-c_1^{(i)}}\right\} \quad (23)$$

3 Numerical and Simulation Results

Numerical results are provided to demonstrate the analysis developed in this letter and to compare it with simulation results. The approximate results are also compared with the ones obtained by Ref. [2]. The basic simulation parameters are tabulated in Table 1 as follows.

Table 1 Simulation parameters

parameter vitem	value
Total transmit power P^t /mW	50
Transmit antenna height h_b /m	5
MS antenna height h_s /m	1.5
Wavelength λ /m	0.3
Radius of Cell R/m	250
Antenna Spacing D/m	200
Channel Bandwidth B/MHz	20

The total SNR is $\gamma=\sum_{i=1}^{2}\gamma_k=\sum_{i=1}^{2}P_k^r/N_0$ and noise power of receiver N_0 [8] is -174 dBm+10lg$B$.

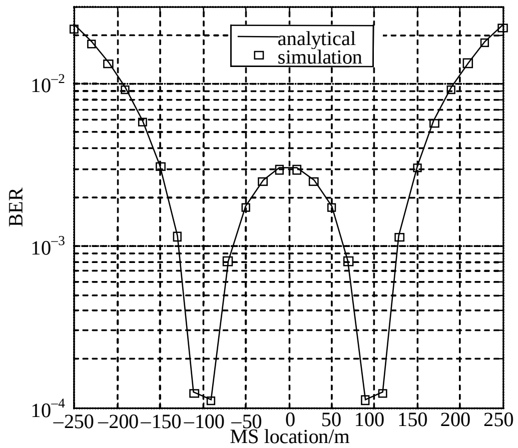

Fig. 1 BER performance of distributed Alamouti STBC versus MS location with $\sigma_k=8$ dB along a line through the center of circular cell and TX$_k$

Fig. 1 shows the BER performance of distributed Alamouti STBC versus MS location with $\sigma_k=8$ dB along a line through the center of circular cell and TX$_k$, where the transmit power of each transmit antenna is 25 mW. Curves are obtained both by analysis and simulation. From Fig. 1 we can observe that, theoretical analysis and simulation results match well for MS along the line. Furthermore, it can be obtained that the closer the distance between MS and transmit antenna, the better the BER performance.

Fig. 2 shows the BER performance of distributed Alamouti STBC versus the SNR at $d_1=100$ m and $d_2=110$ m, with $\sigma_k=4$ dB and 8 dB, respectively. Curves are obtained both by analysis and simulation. It can be seen that analysis results agree well with theoretical ones. Compared with the approximate BER under high SNR condition in Ref. [2], the presented analysis is more accurate for arbitrary SNR.

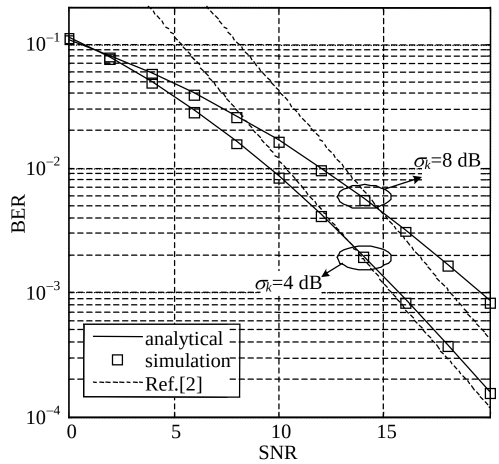

Fig. 2 BER performance of distributed Alamouti STBC with BPSK modulation versus SNR

4 Conclusions

This paper analyzed the BER performance for STBC with distributed transmit antennas, where Alamouti's scheme was employed. For BPSK modulation, an efficiently approximate expression of BER was derived. Comparison of analytical and simulation results validated the presented expression. For further work, we will generalize the BER analysis to higher order modulations such as the MPSK, MQAM or consider the scenario where there are some spatial correlations between two transmission channels.

参 考 文 献

[1] CASTANHEIRA D, GAMEIRO A. Distributed antenna system capacity scaling[J]. IEEE Wireless Communications, 2010, 17(3): 68-75.

[2] SHEN Y, TANG Y, KONG T, et al. Optimal antenna location for STBC-OFDM downlink with distributed transmit antennas in linear cells[J]. IEEE Communications Letters, 2007, 11(5): 387-389.

(下转第675页)

[3] HAN K Y, LEE S W, LIM J S, et al. Time-variant fading channel estimation by extended RLS[J]. IEICE Transactions on Communications, 2004, E87-B(6): 1715-1718.
[4] 张桦，朱近康．基于隐式训练和加权最小二乘的迭代信道估计[J]．华中科技大学学报(自然科学版)，2008，36(4): 13-16.
ZHANG Hua, ZHU Jin-kang. Iterative channel estimation using superimposed training and weighted- least-square method[J]. Huazhong University of Science & Technilogy (Natural Science Edition), 2008, 36(4): 13-16.
[5] STOJANOVIC M. Retrofocusing techniques for high rate acoustic communications[J]. Journal of the Acoustical Society of America, 2005, 117(3): 1173-1185.
[6] DE BEEK V J J, EDFORS O, SANDELL M, et al. On channel estimation in OFDM systems[C]//The 1995 IEEE 45th Vehicular Technology Conference. Chicago: IEEE Press, 1995.
[7] KANG Y, KIM K, PARK H. Efficient DFT-based channel estimation for OFDM systems on multi-path channels[J]. IET Communications, 2007, 1(2): 197-202.
[8] 程履帮．OFDMA系统中基于LMMSE信道估计算法的改进及其性能分析[J]．电子学报, 2008, 36(9): 1782-1785.
CHENG Lü-bang. An improved LMMSE channel estimation method and its performance analysis[J]. Acta Electronica Sinica, 2008, 36(9): 1782-1785.
[9] 郭业才．自适应盲均衡技术[M]．合肥：合肥工业出版社, 2007.
GUO Ye-cai. Adaptive blind equalization technique[M]. Hefei: Hefei Industry Press, 2007.
[10] SAYED A H, KAILATH T. A state-space approach to adaptive RLS filtering[J]. IEEE Signal Processing Magazine, 1994, 11(3): 18-60.

编 辑 税 红

(上接第670页)

[3] ZHOU S, ZHOU M, XU X, et al. Distributed wireless communication systems: a new architecture for future public wireless access[J]. IEEE Communications Magazine, 2003, 41(3): 108-113.
[4] ALAMOUTI S M. A simple transmit diversity technique for wireless communications[J]. IEEE Journal on Select Areas in Communications, 1998, 16(8): 1451-1458.
[5] PAPADOPOULOS G D, KARAGIANNIDIS G K, PAVLIDOU F N. Performance bounds of space-time block coding in rician and log-normal fading channels[J]. IET Communications, 2007, 1(1): 86-91.
[6] CATREUX S, DRIESSEN P F, GREENSTEIN L J. Data throughputs using multiple-input multiple-output (MIMO) techniques in a noise-limited cellular environment[J]. IEEE Transactions on Wireless Communications, 2002, 1(2): 226-235.
[7] SIMON M K, ALOUINI M S. Digital communications over fading channels: a unified approach to performance analysis[M]. New York: Wiley, 2000.
[8] ANDREWS J G, GHOSH A, MUHAMED R. Fundamentals of WiMAX: Understanding broadband wireless networking[M]. Upper Saddle River, NJ: Prentice Hall, 2007.
[9] LEVER K V. New derivation of Craig's formula for the Gaussian probability function[J]. Electronics Letters, 1998, 34(19): 1821-1822.
[10] ABRAMOWITZ M, STEGUN I A. Handbook of mathematical function with formular, graphs, and mathematical tables[M]. New York: Dover Press, 1972.

编 辑 张 俊