Very Expressive Fuzzy Description Logics over Lattices for the Semantic Web

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Abstract　In this paper we present a very expressive fuzzy description logic framework, L-SROIQ(D), based on certainty lattices, which is a fuzzy extension of the description logic SROIQ(D) (theoretical basis of the ontology language OWL 2). Some logical properties of L-SROIQ(D) are researched and the decidability of L-SROIQ(D) is proved in case of linearly ordered lattices.
Key words　certainty lattices;　description logics;　fuzzy description logics;　fuzzy logics;　semantic Web

面向语义Web的Expressive格值描述逻辑

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【摘要】提出了一种基于可信度格的Expressive模糊描述逻辑框架，对描述逻辑SROIQ(D)进行了模糊化扩充，建立了一种面向语义Web的基于格的模糊描述逻辑L-SROIQ(D)，给出了L-SROIQ(D)的语法、语义和逻辑性质，以及建立了一个线序格下的将模糊描述逻辑L-SROIQ(D)转换为经典描述逻辑SROIQ(D)的推理算法，从而证明了线序格下L-SROIQ(D)的可满足性推理是可判定的。
关　键　词　可信度格;　描述逻辑;　模糊描述逻辑;　模糊逻辑;　语义Web
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The Semantic Web[1] has recently attracted much attention, both from academia and industry, and is widely regarded as the next step in the evolution of the World Wide Web. It aims at an extension of the current Web by standards and technologies that help machines to understand the information on the Web so that they can support richer discovery, data integration, navigation, and automation of tasks. The main ideas behind it are to add a machine-understandable "meaning" to web pages, to use ontologies for a precise definition of shared terms in web resources, to use KR technology for automated reasoning from web resources, and to apply cooperative agent technology for processing the information of the Web[2]. This vision has led to the introduction of a stack of new generation ontology definition languages.

An ontology is defined as an explicit and formal specification of a shared conceptualization[3], which means that ontologies represent the concepts and the relationships in a domain promoting interrelation with other models and automatic processing. Ontologies allow to add semantics to data, making knowledge maintenance, information integration as well as the reuse of components easier[4]. The current standard language for ontology creation is the Web Ontology Language (OWL), which comprises three sublanguages of increasing expressive power: OWL Lite, OWL DL and OWL Full. An extension of OWL is

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OWL 2[5-6].

Description Logics (DLs for short)[7] are a family of logic-based knowledge representation formalisms that are tailored towards representing the terminological knowledge of an application domain in a structured and formally well-understood way. DLs have proved to be very useful as ontology languages[8]. For example, OWL Lite, OWL DL and OWL 2 are essentially very expressive DLs with an RDF syntax. More specifically, OWL Lite, OWL DL and OWL 2 are nearly equivalents to the DLs SHIF(D), SHOIN(D), and SROIQ(D), respectively[4, 9-11].

Nevertheless, it has been widely pointed out that classical ontology languages and DLs are not appropriate to deal with imprecise and vague knowledge, which is inherent to several real world domains[2,4,12]. The rising popularity of DLs and their use, and the need to deal with uncertainty and vagueness, both especially in the Semantic Web, is increasingly attracting the attention of many researchers and practitioners towards DLs able to cope with uncertainty and vagueness[2]. Especially, there are many works attempted to extend the DLs using fuzzy set theory[13], that is, fuzzy DLs[4,9-11,14-19] are presented based on fuzzy set theory. For a more detailed survey on fuzzy DLs the reader is referred to Ref. [2].

The above-mentioned fuzzy DLs can only address quantitative reasoning (by relying e.g. on [0, 1]), but can not address qualitative uncertainty reasoning (by relying e.g. on {false, likelyfalse, unknown, likelytrue, true}, in increasing order)[20]. In order to address qualitative uncertainty reasoning, Ref. [20] extends DLs allowing to express that a sentence is not just true or false like in classical DLs, but certain to some degree, which is taken from a certainty lattice, and presents fuzzy description logic over lattices L-ALC. The adopted approach is more general than the fuzzy logic based approach[17], as the adopted approach subsumes the fuzzy logic based approach (just take the lattice over the real unit interval [0, 1] with order ≤) and four-valued DLs such as Ref. [21] and Ref. [22], but is orthogonal to almost all other approaches. Subsequently, Ref. [23] extend L-ALC and present fuzzy description logic over lattices with number restrictions L-ACLN. Recently, Ref. [24] consider a fuzzy extension of SHIN based on certainty lattices, present fuzzy description logic over lattices L-SHIN.

Obviously, we know that classical description logics (denoted by DLs) ALC[25], ALCN[7], SHIN[16] and SROIQ(D)[9-10], fuzzy description logics based on fuzzy logic (denoted by f-DLs) f-ALC[17], f-ALCN[26], f-SHIN[16] and f-SROIQ(D)[9-10], and fuzzy description logics over lattices (denoted by L-DLs) L-ALC[20], L-ALCN[23] and L-SHIN[24] have relationships depicted in Fig. 1:

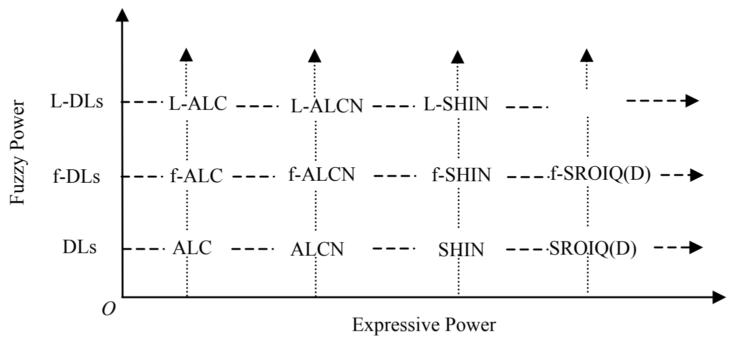


Fig. 1 DLs, f-DLs and L-DLs

From Fig. 1, it is naturally known that we need a novel fuzzy description logic L-SROIQ(D), which is the extension of L-SHIN and f-SROIQ(D). In the current paper, we extend the results obtained in Ref. [24] for L-SHIN and in Ref. [9] and Ref. [10] for f-SROIQ(D), thus creating L-SROIQ(D). In this paper we consider a fuzzy extension of SROIQ(D) based on certainty lattices, present the very expressive fuzzy description logic over lattices L-SROIQ(D), and give its syntax and semantics.

1 Preliminaries

The current section provides some background. Section 1.1 describes SROIQ(D), the DL which stands behind the Web ontology language OWL 2. Section 1.2 recalls some basic notions of certainty lattices. See especially Ref. [4,9-10,20,27] for further details and background.

1.1 The description logic SROIQ(D)

SROIQ(D)[9-10] extends ALC standard DL[25] with transitive roles, complex role axioms, nominals, inverse roles, qualified number restrictions, and concrete domain.

First, we present the syntax of SROIQ(D).

A concrete domain[28-29] is a pair $\langle \Delta_D, \Phi_D \rangle$, where Δ_D is a concrete interpretation domain and Φ_D is a set of domain predicates d with a predefined arity n and an interpretation $d_D \subseteq \Delta_D{}^n$. For simplicity we assume arity 1.

As usual, SROIQ(D) assumes three alphabets of symbols, for concepts, roles, and individuals. In DLs, complex concepts and roles can be built using different concept and role constructors. In SROIQ(D), the concepts (denoted C or D) and abstract roles (R) can be built inductively from atomic concepts (A), atomic roles (R_A), top concept $\top$, bottom concept $\bot$, named individuals (o_i), simple roles (S, which will be defined below), universal role (U), and concrete roles (T). Concretely, SROIQ(D)-concepts are composed inductive according to the following abstract syntax:

$C, D \rightarrow \top |\ \bot |\ A |\ \neg C |\ C \sqcap D |\ C \sqcup D |\ \exists R.C |\ \forall R.C |\ \exists T.d |\ \forall T.d |\ \{o_1, o_2, \cdots, o_m\} | \geqslant n\ S.C | \leqslant n\ S.C | \geqslant n\ T.d | \leqslant n\ T.d |\ \exists S.\text{Self}$, where n, m denote natural numbers ($n \geqslant 0$, $m > 0$).

SROIQ(D)-roles are composed inductive according to the following abstract syntax:

$R \rightarrow R_A |\ R^- |\ U |\ T$, where R^- denotes inverse role.

A Knowledge Base (KB) comprises two parts: the intensional knowledge, i.e., general knowledge about the application domain (a Terminological Box or TBox (TB) and a Role Box or RBox (RB)), and the extensional knowledge, i.e., particular knowledge about some specific situation (an Assertional Box or ABox (AB) with statements about individuals).

An ABox (AB) consists of a finite set of assertions about individuals: concept assertions a:C, abstract role assertions (a, b):R, negated abstract role assertions (a, b):$\neg R$, concrete role assertions (a, v):T, negated concrete role assertions (a, v):$\neg T$, inequality assertions $a \neq b$, and equality assertions $a = b$.

A TBox (TB) consists of a finite set of general concept inclusion (GCI) axioms $C \sqsubseteq D$ (C is more specific than D).

Let w be a role chain (a finite string of roles not including the universal role U). An RBox RB consists of a finite set of role axioms: role inclusion axioms (RIAs) $w \sqsubseteq R$, transitive role axioms trans(R), disjoint role axioms dis(S_1, S_2), reflexive role axioms ref(R), irreflexive role axioms irr(S), symmetric role axioms sym(R), and asymmetric role axioms asy(S).

Now we will introduce some definitions which will be useful for imposing some limitations in the language. A strict partial order $\prec$ on a set A is an irreflexive and transitive relation on A. A strict partial order $\prec$ on the set of roles is called a regular order if it also satisfies $R_1 \prec R_2 \Leftrightarrow R_2^- \prec R_1$, for all roles R_1 and R_2.

In order to guarantee the decidability of the logic, there are some restrictions in the use of roles. Given a regular order $\prec$, every role axiom cannot contain U and every RIA should be $\prec$–regular. A RIA $w \sqsubseteq R$ is $\prec$–regular if R is atomic and: 1) $w = RR$, or 2) $w = R^-$, or 3) $w = S_1, S_2, \cdots, S_n$ and $S_i \prec R$ for all $i = 1, 2, \cdots, n$, or 4) $w = RS_1, S_2, \cdots, S_n$ and $S_i \prec R$ for all $i = 1, 2, \cdots, n$, or 5) $w = S_1, S_2, \cdots, S_n R$ and $S_i \prec R$ for all $i = 1, 2, \cdots, n$.

Note that, in order to prove decidability of the reasoning, roles are assumed to be simple in some concept constructors (local reflexivity, at-least and at-most number restrictions) and role axioms (disjoint, irreflexive and asymmetric role axioms)[27]. Simple roles are inductively defined as follows: 1) R_A is simple if it does not occur on the right side of a RIA, 2) R^- is simple if R is, 3) if R occurs on the right side of a RIA, R is simple if, for each $w \sqsubseteq R$, $w = S$ for a simple role S.

Let us now turn to the semantics of SROIQ(D).

An interpretation I with respect to a concrete domain D is a pair $I = (\Delta^I, \bullet^I)$, where Δ^I is a non-empty domain of the interpretation disjoint with Δ_D, and $\bullet^I$ is an interpretation function that maps each atomic concept A to a set $A^I \subseteq \Delta^I$, each abstract atomic role R_A to a binary relation $R_A{}^I \subseteq \Delta^I \times \Delta^I$, each concrete role T to a binary relation $T^I \subseteq \Delta^I \times \Delta_D$, each abstract individual name a to an element $a^I \in \Delta^I$, each concrete individual name v to an element $v_D \in \Delta_D$, and each n-ary concrete predicate d to a set $d_D \subseteq \Delta_D^n$. The interpretations of complex concepts and roles are shown as follows:

1) $\top^I=\Delta^I$, $\bot^I=\phi$, $(\neg C)^I=\Delta^I \backslash C^I$, $(C \sqcap D)^I=C^I \cap D^I$, $(C \sqcup D)^I=C^I \cup D^I$;

2) $(\exists R.C)^I=\{a \in \Delta^I |\ \exists b \in \Delta^I,\ (a,\ b) \in R^I \wedge b \in C^I\}$, $(\forall R.C)^I=\{a \in \Delta^I |\ \forall b \in \Delta^I, (a, b) \in R^I \rightarrow b \in C^I\}$;

3) $(\exists T.d)^I=\{a \in \Delta^I |\ \exists v \in \Delta_D,\ (a,\ v) \in T^I \wedge v \in d_D\}$, $(\forall T.d)^I=\{a \in \Delta^I |\ \forall v \in \Delta_D, (a, v) \in T^I \rightarrow v \in d_D\}$;

4) $\{o_1, o_2, \cdots, o_m\}^I=\{o_1^I, o_2^I, \cdots, o_m^I\}$, $(\exists S.\mathrm{Self})^I=\{a \in \Delta^I | (a, a) \in S^I\}$;

5) $(\geqslant n\ S.C)^{MI}=\{a \in \Delta^I |\ \#\{b \in \Delta^I |\ (a,\ b) \in S^I \wedge b \in C^I\} \geqslant n\}$;

6) $(\leqslant n\ S.C)^{MI}=\{a \in \Delta^I |\ \#\{b \in \Delta^I |\ (a,\ b) \in S^I \wedge b \in C^I\} \leqslant n\}$;

7) $(\geqslant n\ T.d)^{MI}=\{a \in \Delta^I |\ \#\{v \in \Delta_D |\ (a,\ v) \in T^I \wedge v \in d_D\} \geqslant n\}$;

8) $(\leqslant n\ T.d)^{MI}=\{a \in \Delta^I |\ \#\{v \in \Delta_D |\ (a,\ v) \in T^I \wedge v \in d_D\} \leqslant n\}$;

9) $(R^-)^I=\{(b, a) \in \Delta^I \times \Delta^I | (a, b) \in R^I\}$, $U^I=\Delta^I \times \Delta^I$.

where $\#X$ denotes the cardinality of the set X.

Let o be the standard composition of relations. An interpretation I satisfies (is a model of):

1) $a{:}C$ iff $a^I \in C^I$, $(a, b){:}R$ iff $(a^I, b^I) \in R^I$, $(a, b){:}\neg R$ iff $(a^I, b^I) \notin R^I$;

2) $(a, v){:}T$ iff $(a^I, v_D) \in T^I$, $(a, v){:}\neg T$ iff $(a^I, v_D) \notin T^I$;

3) $a \neq b$ iff $a^I \neq b^I$, $a=b$ iff $a^I=b^I$, $C \sqsubseteq D$ iff $C^I \subseteq D^I$, $R_1, R_2, \cdots, R_n \sqsubseteq R$ iff $R_1^I o, R_2^I o, \cdots, R_n^I o \subseteq R^I$;

4) trans(R) iff $(a, b) \in R^I$ and $(b, c) \in R^I$ imply $(a, c) \in R^I$, $\forall a, b, c \in \Delta^I$;

5) dis(S_1, S_2) iff $S_1^I \cap S_2^I=\phi$, ref(R) iff $\forall a \in \Delta^I$, $(a, a) \in R^I$, irr(S) iff $\forall a \in \Delta^I$, $(a, a) \notin S^I$;

6) sym(R) iff $(a, b) \in R^I$ implies $(b, a) \in R^I$, $\forall a, b \in \Delta^I$;

7) asy(S) iff $(a, b) \in S^I$ implies $(b, a) \notin S^I$, $\forall a, b \in \Delta^I$;

8) a knowledge base KB=⟨AB, TB, RB⟩ iff it satisfies each element in AB, TB and RB.

A DL not only stores axioms and assertions, but also offers some reasoning services, such as KB satisfiability, concept satisfiability or subsumption. However, if a DL is closed under negation, most of the basic reasoning tasks are reducible to KB satisfiability, so it is usually the only task considered[4,30].

1.2 Certainty lattices

Let $L=\langle \mathrm{CV}, \preccurlyeq \rangle$ be a certainty lattice (a complete lattice)[20,24], where CV is a set of certainty values and $\preccurlyeq$ is a partial order over CV. Let $\oplus$ and $\otimes$ be the join and meet operators induced by $\preccurlyeq$, respectively. Let f and t be the least and greatest element in CV, respectively. We also assume that there is a function from CV to CV, called negation function (denoted by $\neg$) that is anti-monotone w.r.t. $\preccurlyeq$ and satisfies $\neg\neg\alpha=\alpha$, $\forall \alpha \in \mathrm{CV}$. The main idea is that an assertion $a{:}C$, rather being interpreted as either true or false, will be mapped into a certainty value c in CV. The intended meaning is that c indicates to which extend (how certain it is that) "a is a C".

Typical certainty lattices are (given a set of real values CV, consider $L_{\mathrm{CV}}=\langle \mathrm{CV}, \preccurlyeq \rangle$) as follows.

Classical 0-1: $L_{\{0,1\}}$ corresponds to the classical truth-space, where 0 stands for 'false', while 1 stands for 'true'.

Fuzzy: $L_{[0,1]}$, which relies on the unit real interval, is quite frequently used as certainty lattice. In $L_{[0,1]}$, $\neg\alpha=1-\alpha$ is quite typical.

Four-valued: Another frequent certainty lattice is Belnap's FOUR[31], where CV is $\{f, t, u, i\}$ with $f \preccurlyeq u \preccurlyeq t$ and $f \preccurlyeq i \preccurlyeq t$. Here, u stands for 'unknown', whereas i stands for inconsistency. We denote the lattice as L_B. Additionally, besides $\neg f=t$, we have $\neg u=u$ and $\neg i=i$.

Many-valued: $L=\left\langle \{0, \dfrac{1}{n-1}, \cdots, \dfrac{n-2}{n-1}, 1\}, \leqslant \right\rangle$, n positive integer. A special case is L_4, where CV is $\{f, lf, lt, t\}$ with $f \leqslant lf \leqslant lt \leqslant t$. Here, lf stands for 'likely false', whereas lt stands for 'likely true'. Besides $\neg f=t$, we have $\neg lf=lt$.

Belief-Doubt: A further popular lattice allows us to reason about belief and doubt. Indeed, the idea is to take any lattice L, and to consider the cartesian product $L \times L$. For any pair $(b, d) \in L \times L$, b indicates the degree of believing a reasoning agent has about a sentence s, while d indicates the degree of doubting the agent has about s. The order on $L \times L$ is determined by $(b, d) \preccurlyeq (b', d')$ iff $b \preccurlyeq b'$ and $d \preccurlyeq d'$, i.e., belief goes up, while doubt goes down. The minimal element is (f, t) (no belief, maximal doubt), while the maximal element is (t, f) (maximal belief, no doubt). Negation is given by $\neg(b,$

d)=(d, b) (exchange belief with doubt).

2 Fuzzy description logic over lattices L-SROIQ(D)

In this section we define the fuzzy DL over lattices L-SROIQ(D), which is a fuzzy extension of the DL SROIQ(D) where concepts denote fuzzy sets of individuals, roles denote fuzzy binary relations, and axioms are extended to the fuzzy case in such a way that some of them hold to a degree in a certainty lattice L.

2.1 Syntax and semantics

A fuzzy concrete domain D is a pair $\langle \Delta_D, \Phi_D \rangle$, where Δ_D is a concrete interpretation domain, Φ_D is a set of fuzzy concrete predicates d with an arity n and an interpretation d_D: $\Delta_D^n \to$ CV, which is an n-ary fuzzy relation over Δ_D, and L=⟨CV, $\preccurlyeq$⟩ is a certainty lattice. For simplicity we assume arity 1.

Similarly, as in SROIQ(D) and f-SROIQ(D)[9-10], L-SROIQ(D) assumes three alphabets of symbols, for concepts, roles and individuals.

Let n, m be natural numbers ($n \geqslant 0$, $m>0$) and $\alpha_i \in$CV. The concepts (denoted C or D) of L-SROIQ(D) can be built inductively from atomic concepts (A), top concept $\top$, bottom concept $\perp$, named individuals (o_i), abstract roles (R), concrete roles (T), simple roles (S, which will be defined below), and fuzzy concrete predicates (d) as:

$C, D \to \top | \perp | A | \neg C | C \sqcap D | C \sqcup D | \exists R.C | \forall R.C | \exists T.d | \forall T.d | \{\alpha_1/o_1, \alpha_2/o_2, \cdots, \alpha_m/o_m\} | \geqslant m\ S.C | \leqslant n\ S.C | \geqslant m\ T.d | \leqslant n\ T.d | \exists S.\mathrm{Self} | [C \succcurlyeq \alpha] | [C \preccurlyeq \beta]$.

The abstract roles (denoted R) of L-SROIQ(D) can be built inductively according to the following syntax rule:

$R \to R_A | R^- | U | [R \succcurlyeq \alpha]$.

Concrete roles are denoted T and cannot be complex.

Abstract individuals are denoted a, $b \in \Delta^I$, and concrete individuals are denoted $v \in \Delta_D$.

Note that we will not allow modified concepts and modified roles in our L-SROIQ(D), since the definition of the modifier under lattices is unclear at present. However, indeed, this is definitely a point that has to be addressed in forthcoming works.

In the rest of the paper, we will assume $\bowtie \in \{\succcurlyeq, \succ, \preccurlyeq, \prec\}$, α, β, $\gamma \in$T, $\alpha \neq f$, $\beta \neq t$. The symmetric $\bowtie^-$ and the negation $\neg\bowtie$ of an operator $\bowtie$ are defined as in table 1:

Table 1 The definition of $\bowtie$、$\bowtie^-$、$\neg\bowtie$

$\bowtie$	$\bowtie^-$	$\neg\bowtie$
$\succcurlyeq$	$\preccurlyeq$	$\prec$
$\succ$	$\prec$	$\preccurlyeq$
$\preccurlyeq$	$\succcurlyeq$	$\succ$
$\prec$	$\succ$	$\succcurlyeq$

An L-SROIQ(D) knowledge base KB comprises a fuzzy ABox (AB), a fuzzy TBox (TB) and a fuzzy RBox (RB).

A fuzzy ABox (AB) consists of a finite set of fuzzy assertions. A fuzzy assertion can be an inequality assertion $\langle a \neq b \rangle$, an equality assertion $\langle a=b \rangle$ or a constraint on the certainty value of a concept or role assertion, namely, an expression of the form $\langle \psi \succcurlyeq \alpha \rangle$, $\langle \psi \succ \beta \rangle$, $\langle \psi \preccurlyeq \beta \rangle$ or $\langle \psi \prec \alpha \rangle$, where ψ is of the form $a{:}C$, $(a, b){:}R$, $(a, b){:}\neg R$, $(a, v){:}T$ or $(a, v){:}\neg T$.

A fuzzy TBox (TB) consists of fuzzy GCIs, which constrain the certainty value of a GCI, i.e., they are expressions of the form $\langle C \sqsubseteq D \succcurlyeq \alpha \rangle$ or $\langle C \sqsubseteq D \succ \beta \rangle$.

A fuzzy RBox (RB) consists of a finite set of role axioms, which can be fuzzy RIAs $\langle w \sqsubseteq R \succcurlyeq \alpha \rangle$, $\langle w \sqsubseteq R \succ \beta \rangle$ for a role chain $w=R_1, R_2, \cdots, R_n$, or $\langle T_1 \sqsubseteq T_2 \succcurlyeq \alpha \rangle$, $\langle T_1 \sqsubseteq T_2 \succ \beta \rangle$, or any other of the role axioms from the crisp case: transitive role axioms trans(R), disjoint role axioms dis(S_1, S_2), dis(T_1, T_2), reflexive role axioms ref(R), irreflexive role axioms irr(S), symmetric role axioms sym(R) or asymmetric role axioms asy(S).

We are ready now to formally define simple roles.

Simple roles are defined as in the crisp SROIQ(D) and the fuzzy f-SROIQ(D)[9-10], 1) R_A is simple if it does not occur on the right side of a RIA, 2) R^- is simple if R is, 3) if R occurs on the right side of a RIA, R is simple if, for each $\langle w \sqsubseteq R \rhd \gamma \rangle$, $w=S$ for a simple role S.

Note that concrete roles are always simple.

A fuzzy axiom is positive (denoted $\langle \tau \rhd \alpha \rangle$) if it is of the form $\langle \tau \succcurlyeq \alpha \rangle$ or $\langle \tau \succ \beta \rangle$, and negative (denoted $\langle \tau \lhd \alpha \rangle$) if it is of the form $\langle \tau \preccurlyeq \beta \rangle$ or $\langle \tau \prec \alpha \rangle$. $\langle \tau = \alpha \rangle$ is equivalent to the pair of axioms $\langle \tau \succcurlyeq \alpha \rangle$ and $\langle \tau \preccurlyeq \alpha \rangle$. If nothing is specified we assume that a fuzzy axiom is true with degree t, so we can use the abbreviation: $\tau \equiv \langle \tau \succcurlyeq t \rangle$.

As in the f-SROIQ(D) DL[9-10], negative fuzzy GCIs or RIAs are not allowed in L-SROIQ(D), because they correspond to negated GCIs and RIAs respectively, which are not part of crisp SROIQ(D).

Now we will introduce some definitions which will be useful to impose some limitations in the expressivity of the language.

A strict partial order $\prec$ on a set S is an irreflexive and transitive relation on S. A strict partial order $\prec$ on the set of roles is called a regular order if it also satisfies $R_1 \prec R_2 \Leftrightarrow R_2^- \prec R_1$, for all roles R_1 and R_2.

A RIA $\langle w \sqsubseteq R \rhd \gamma \rangle$ is $\prec$–regular if R is atomic and: 1) $w=RR$, or 2) $w=R^-$, or 3) $w=S_1, S_2, \cdots, S_n$ and $S_i \prec R$ for all $i=1, 2, \cdots, n$, or 4) $w=RS_1, S_2, \cdots, S_n$ and $S_i \prec R$ for all $i=1, 2, \cdots, n$, or 5) $w=S_1, S_2, \cdots, S_nR$ and $S_i \prec R$ for all $i=1, 2, \cdots, n$.

As in the crisp SROIQ(D) and the fuzzy f-SROIQ(D)[9-10], there are some restrictions in the use of roles, in order to guarantee the decidability of the logic.

Firstly, some concept constructors require simple roles: non-concrete qualified number restrictions and local reflexivity. Some role axioms also require simple roles: disjoint, irreflexive and asymmetric role axioms. Role axioms cannot contain the universal role U. Finally, every RIA should be $\prec$–regular for a given regular order $\prec$.

Let us now turn to the semantics of L-SROIQ(D).

For a certainty lattice $L=\langle \mathrm{CV}, \preccurlyeq \rangle$, an L-interpretation I with respect to a fuzzy concrete domain D is a pair $(\Delta^I, \bullet^I)$ consisting of a non empty set Δ^I disjoint with Δ_D and a fuzzy interpretation function $\bullet^I$ mapping:

1) Every abstract individual a onto an element a^I of Δ^I.

2) Every concrete individual v onto an element v_D of Δ_D.

3) Every concept C onto a function C^I: $\Delta^I \to \mathrm{CV}$.

4) Every abstract role R onto a function R^I: $\Delta^I \times \Delta^I \to \mathrm{CV}$.

5) Every concrete role T onto a function T^I: $\Delta^I \times \Delta_D \to \mathrm{CV}$.

6) Every n-ary concrete fuzzy predicate d onto the fuzzy relation d_D: $\Delta_D^{\,n} \to \mathrm{CV}$.

The complete set of semantics of L-SROIQ(D)-concepts and L-SROIQ(D)-roles is depicted as follows:

1) $\top^I(a)=t$;

2) $\bot^I(a)=f$;

3) $(C \sqcap D)^I(a)=C^I(a) \otimes D^I(a)$;

4) $(C \sqcup D)^I(a)=C^I(a) \oplus D^I(a)$;

5) $(\neg C)^I(a)=\neg C^I(a)$;

6) $(\forall R.C)^I(a)=\otimes_{d' \in \Delta^I} \{\neg R^I(a, d') \oplus C^I(d')\}$;

7) $(\exists R.C)^I(a)=\oplus_{d' \in \Delta^I} \{R^I(a, d') \otimes C^I(d')\}$;

8) $(\forall T.d)^I(a)=\otimes_{v \in \Delta_D} \{\neg T^I(a, v) \oplus d_D(v)\}$;

9) $(\exists T.d)^I(a)=\oplus_{v \in \Delta_D} \{T^I(a, v) \otimes d_D(v)\}$;

10) $(\{\alpha_1/o_1, \alpha_2/o_1, \cdots, \alpha_m/o_m\})^I(a)=\oplus_{i \mid d=o_i^I} \alpha_i$;

11) $(\geqslant m \ S.C)^I(a)=\oplus_{\substack{d_1, d_2, \cdots, d_m \in \Delta^I \\ |\{d_1, d_2, \cdots, d_m\}|=m}} \{\otimes_{i=1}^{m} \{S^I(a, d_i) \otimes C^I(d_i)\}\}$;

12) $(\leqslant n \ S.C)^I(a) =(\neg(\geqslant n+1 \ S.C))^I(a)=\otimes_{\substack{d_1, d_2, \cdots, d_{n+1} \in \Delta^I \\ |\{d_1, d_2, \cdots, d_{n+1}\}|=n+1}} \{\oplus_{i=1}^{n+1} \{\neg S^I(a, d_i) \oplus C^I(d_i)\}\}$;

13) $(\geqslant m \ T.d)^I(a)=\oplus_{\substack{v_1, v_2, \cdots, v_m \in \Delta_D \\ |\{v_1, v_2, \cdots, v_m\}|=m}} \{\otimes_{i=1}^{m} \{T^I(a, v_i) \otimes d_D(v_i)\}\}$;

14) $(\leqslant n \ T.d)^I(a) =(\neg(\geqslant n+1 \ T.d))^I(a)=\otimes_{\substack{v_1, v_2, \cdots, v_{n+1} \in \Delta_D \\ |\{v_1, v_2, \cdots, v_{n+1}\}|=n+1}} \{\oplus_{i=1}^{n+1} \{\neg T^I(a, v_i) \oplus d_D(v_i)\}\}$;

15) $(\exists S.\mathrm{Self})^I(a)=S^I(a, a)$;

16) $([C \succcurlyeq \alpha])^I(a)=t$ if $C^I(a) \succcurlyeq \alpha$, f otherwise;

17) $([C \preccurlyeq \beta])^I(a)=t$ if $C^I(a) \preccurlyeq \beta$, f otherwise;

18) $(R^-)^I(a, b)=R^I(b, a)$;

19) $U^I(a, b)=t$;

20) $([R \succcurlyeq \alpha])^I(a, b)=t$ if $R^I(a, b) \succcurlyeq \alpha$, f otherwise.

Obviously, C^I denotes the membership function of the fuzzy concept C w.r.t. the L-interpretation I. $C^I(a)$ gives us the degree of certainty of being the individual a an element of the fuzzy concept C under I. As in the crisp SROIQ(D) and the fuzzy f-SROIQ(D)[9-10], we do not impose Unique Name Assumption, that is, two individual names (or nominals) might refer to the same individual.

The L-interpretation is extended to fuzzy axioms (or assertions) as follows:

1) $(a{:}C)^I=C^I(a^I)$;

2) $((a, b){:}R)^I=R^I(a^I, b^I)$;

3) $((a, b){:}\neg R)^I=\neg R^I(a^I, b^I)$;

4) $((a, v){:}T)^I=T^I(a^I, v_D)$;

5) $((a, v){:}\neg T)^I=\neg T^I(a^I, v_D)$;

6) $(C\sqsubseteq D)^I=\otimes_{a\in\Delta^I}\{\neg C^I(d)\oplus D^I(a)\}$;

7) $(R_1, R_1,\cdots, R_n\sqsubseteq R)^I=\otimes_{d_1,d_2,\cdots,d_{n+1}\in\Delta^I}\{(\neg R_1{}^I(d_1, d_2))\oplus\cdots\oplus(\neg R_n{}^I(d_n, d_{n+1}))\oplus R^I(d_1, d_{n+1})\}$;

8) $(T_1\sqsubseteq T_2)^I=\otimes_{a\in\Delta^I,v\in\Delta_D}\{\neg T_1{}^I(a, v)\oplus T_2{}^I(a, v)\}$.

An L-interpretation $I=(\Delta^I, \bullet^I)$ satisfies (is a model of, denoted $I\vDash$):

1) $\langle a{:}C \bowtie\gamma\rangle$ iff $(a{:}C)^I\bowtie\gamma$;

2) $\langle(a, b){:}R \bowtie\gamma\rangle$ iff $((a, b){:}R)^I\bowtie\gamma$;

3) $\langle(a, b){:}\neg R \bowtie\gamma\rangle$ iff $((a, b){:}\neg R)^I\bowtie\gamma$;

4) $\langle(a, v){:}T \bowtie\gamma\rangle$ iff $((a, v){:}T)^I\bowtie\gamma$;

5) $\langle(a, v){:}\neg T \bowtie\gamma\rangle$ iff $((a, v){:}\neg T)^I\bowtie\gamma$;

6) $\langle a\neq b\rangle$ iff $a^I\neq b^I$;

7) $\langle a=b\rangle$ iff $a^I=b^I$;

8) $\langle C\sqsubseteq D \rhd\gamma\rangle$ iff $(C\sqsubseteq D)^I\rhd\gamma$;

9) $\langle R_1, R_2,\cdots, R_n\sqsubseteq R \rhd\gamma\rangle$ iff $(R_1, R_2,\cdots, R_n\sqsubseteq R)^I\rhd\gamma$;

10) $\langle T_1\sqsubseteq T_2 \rhd\gamma\rangle$ iff $(T_1\sqsubseteq T_2)^I\rhd\gamma$;

11) trans(R) iff $\forall a, c\in\Delta^I$, $R^I(a, c)\succcurlyeq\oplus_{b\in\Delta^I}\{R^I(a, b)\otimes R^I(b, c)\}$;

12) dis(S_1, S_2) iff $\forall a, b\in\Delta^I$, $S_1{}^I(a, b)=f$ or $S_2{}^I(a, b)=f$;

13) dis(T_1, T_2) iff $\forall a\in\Delta^I$, $v\in\Delta_D$, $T_1{}^I(a, v)=f$ or $T_2{}^I(a, v)=f$;

14) ref(R) iff $\forall a\in\Delta^I$, $R^I(a, a)=t$;

15) irr(S) iff $\forall a\in\Delta^I$, $S^I(a, a)=f$;

16) sym(R) iff $\forall a, b\in\Delta^I$, $R^I(a, b)=R^I(b, a)$;

17) asy(S) iff $\forall a, b\in\Delta^I$, if $S^I(a, b)\succ f$ then $S^I(b, a)=f$;

Now, we will define the reasoning problems of the L-SROIQ(D) DL.

We will say that two fuzzy concepts C and D are said to be equivalent (denoted by $C\equiv D$) when $C^I=D^I$ for all L-interpretation I. Two fuzzy assertions ζ_1 and ζ_2 are said to be equivalent (denoted by $\zeta_1\equiv\zeta_2$) iff they are satisfied by the same set of L-interpretations.

An L-interpretation I satisfies an L-SROIQ(D) knowledge base KB=⟨AB, TB, RB⟩ (resp., an ABox AB, a TBox TB, an RBox RB) if it satisfies each element in KB (resp., AB, TB, RB); in this case, we say that I is a model of KB (resp., AB, TB, RB). An L-SROIQ(D) knowledge base KB is satisfiable (unsatisfiable) iff there exists (does not exist) an L-interpretation I which satisfies all elements in KB. An L-SROIQ(D)-concept C is satisfiable (unsatisfiable) w.r.t. an RBox RB and a TBox TB (resp., a knowledge base KB) iff there exists (does not exist) some model I of RB and TB (resp., KB) for which there is some $a\in\Delta^I$ such that $C^I(a)=\alpha$, and $\alpha\neq f$. In this case, C is called α-satisfiable w.r.t. RB and TB (resp., KB). Let C and D be two L-SROIQ(D)-concepts. We say that $\langle C\sqsubseteq D \rhd\gamma\rangle$ w.r.t. RB and TB if for every model I of RB and TB it holds that $(C\sqsubseteq D)^I\rhd\gamma$. Furthermore, an L-SROIQ(D) ABox AB is consistent w.r.t. RB and TB if there exists a model I of RB and TB that is also a model of AB. Moreover, given a fuzzy concept axiom or a fuzzy assertion $\varphi\in\{\langle C\sqsubseteq D \rhd\gamma\rangle, \langle\psi \bowtie\gamma\rangle\}$, where ψ is of the form $a{:}C$, $(a, b){:}R$, $(a, b){:}\neg R$, $(a, v){:}T$ or $(a, v){:}\neg T$, an L-SROIQ(D) knowledge base KB entails φ, written KB$\vDash\varphi$, iff all models of KB also satisfy φ.

Finally, given a fuzzy knowledge base KB and an assertion ψ, it is interest to compute ψ's best lower and upper certainty-value bounds. We define the greatest lower bound of ψ w.r.t. KB (denoted by glb(KB, ψ)) to be $\oplus\{\alpha|\ \text{KB}\vDash\langle\psi\succcurlyeq\alpha\rangle\}$, where $\oplus\phi=f$. Similarly, we define the least upper bound of ψ w.r.t. KB (denoted by lub(KB, ψ)) to be $\otimes\{\beta|\ \text{KB}\vDash\langle\psi\preccurlyeq\beta\rangle\}$, where $\otimes\phi=t$. Determing the glb and lub is called the Best Certainty-Value Bound (BCVB) problem.

In the rest of the paper we will only consider fuzzy knowledge base KB satisfiability, since (as in the crisp case and the fuzzy case) many other reasoning problems can be reduced to this problem[9-10,16-17,24].

As in the f-SROIQ(D) DL[9-10], in order to manage correctly infima and superma in the reasoning, we also need to define the notion of witnessed interpretations. An L-interpretation I is witnessed iff it verifies:

1) for all $a\in\Delta^I$, there is $b\in\Delta^I$ such that $(\exists R.C)^I(a)=R^I(a, b)\otimes C^I(b)$, and

2) for all $a\in\Delta^I$, there is $v\in\Delta_D$ such that $(\exists T.d)^I(a)=T^I(a, v)\otimes d_D(v)$, and

3) for all $a\in\Delta^I$, there is $b\in\Delta^I$ such that $(\forall R.C)^I(a)=\neg R^I(a, b)\oplus C^I(b)$, and

4) for all $a\in\Delta^I$, there is $v\in\Delta_D$ such that $(\forall T.d)^I(a)=\neg T^I(a, b)\oplus d_D(v)$, and

5) there is $a\in\Delta^I$ such that $(C\sqsubseteq D)^I=\neg C^I(a)\oplus D^I(a)$, and

6) there are $a_1, a_2,\cdots, a_{n+1}\in\Delta^I$ such that $(R_1, R_2, \cdots,$

$R_n \sqsubseteq R)^I = (\neg R_1{}^I(a_1, a_2)) \oplus \cdots \oplus (\neg R^I(a_n, a_{n+1})) \oplus R^I(a_1, a_{n+1})$, and

7) there are $a \in \Delta^I$, $v \in \Delta_D$ such that $(T_1 \sqsubseteq T_2)^I = (\neg T_1{}^I(a, v)) \oplus T_2{}^I(a, v)$, and

8) if $I \vDash \mathrm{trans}(R)$, for all $a, c \in \Delta^I$, there is $b \in \Delta^I$ such that $\oplus_{b' \in \Delta^I} \{R^I(a, b') \otimes R^I(b', c)\} = R^I(a, b) \otimes R^I(b, c)$.

Now we shortly discuss the fuzzy nominals of our L-SROIQ(D).

In fuzzy DL literature, there are proposals for crisp nominals[18,32] and fuzzy nomimals[4,9-10,33]. In the current paper we use fuzzy nomimals. Moreover, the semantics of the fuzzy nomimals are based on certainty lattices. Recall that it are defined as:

$(\{\alpha_1/o_1, \alpha_2/o_2, \cdots, \alpha_m/o_m\})^I(a) = \oplus_{i|d=o_i^I} \alpha_i$

Obviously, the semantics of the fuzzy nomimals presented in this paper is an extension of that of fuzzy nomimals[4,9-10,33]. Since we are not imposing unique name assumption, it is possible that $a=(o_i)^I$ for more than one o_i. This is the reason why we need to compute the supremum over the α_i associated to these named individuals o_i. And, of course, if $\forall i \in \{1, 2, \cdots, m\}$, $d \neq (o_i)^I$, then $(\{\alpha_1/o_1, \alpha_2/o_2, \cdots, \alpha_m/o_m\})^I(d) = \oplus \phi = f$.

Finally, we discuss the fuzzy concrete domains of our L-SROIQ(D).

Let us recall the notion of the fuzzy concrete domains in the f-SROIQ(D) DL[9-10] firstly. A fuzzy concrete domain D is a pair $\langle \Delta_D, \Phi_D \rangle$, where Δ_D is a concrete interpretation domain, Φ_D is a set of fuzzy concrete predicates d with an arity n and an interpretation d_D: $\Delta_D{}^n \to [0, 1]$, which is an n-ary fuzzy relation over Δ_D[18,34].

On the other hand, concerning non crisp fuzzy domain predicates, we recall that in fuzzy set theory and practice there are many membership functions for fuzzy sets membership specification. However, the triangular, the trapezoidal, the left shoulder function and the right shoulder function are simple, yet are most frequently used to specify membership degrees (see Ref. [9-10,18] for more details). For example, Ref. [9] and Ref. [10] restrict them to the trapezoidal membership function trap: $\mathbb{Q} \cap [k_1, k_2] \to [0, 1]$ which is defined as follows:

1) $\mathrm{trap}_{k_1,k_2}(x; q_1, q_2, q_3, q_4) = (x-q_1)/(q_2-q_1)$, if $x \in [q_1, q_2]$;

2) $\mathrm{trap}_{k_1,k_2}(x; q_1, q_2, q_3, q_4) = 1$, if $x \in [q_2, q_3]$;

3) $\mathrm{trap}_{k_1,k_2}(x; q_1, q_2, q_3, q_4) = (q_4-x)/(q_4-q_3)$, if $x \in [q_3, q_4]$;

4) $\mathrm{trap}_{k_1,k_2}(x; q_1, q_2, q_3, q_4) = 0$, if $x \in [k_1, q_1] \cup [q_4, k_2]$.

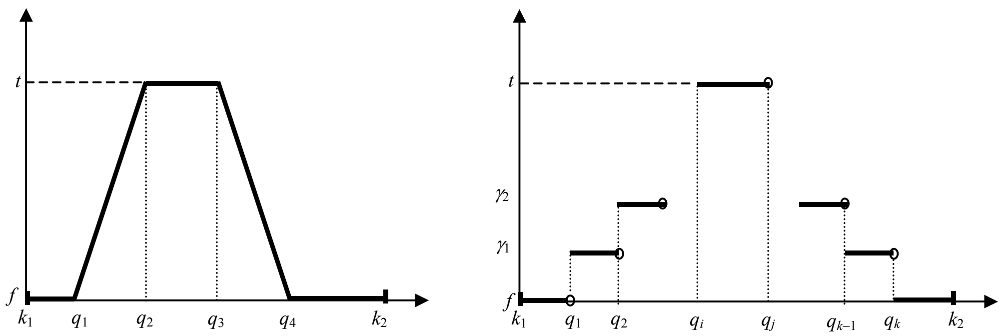
In fact, the trapezoidal membership function can be used to represent other popular membership functions such as the triangular $\mathrm{tri}_{k_1,k_2}(x; q_1, q_2, q_3)$, the left shoulder function $L_{k_1,k_2}(x; q_1, q_2)$ and the right shoulder function $R_{k_1,k_2}(x; q_1, q_2)$ as $\mathrm{trap}_{k_1,k_2}(x; q_1, q_2, q_2, q_3)$, $\mathrm{trap}_{k_1,k_2}(x; k_1, k_1, q_1, q_2)$ and $\mathrm{trap}_{k_1,k_2}(x; q_1, q_2, k_2, k_2)$ respectively.

Now, in our L-SROIQ(D), a fuzzy concrete domain D is a pair $\langle \Delta_D, \Phi_D \rangle$, where Δ_D is a concrete interpretation domain, Φ_D is a set of fuzzy concrete predicates d with an arity n and an interpretation d_D: $\Delta_D{}^n \to \mathrm{CV}$, which is an n-ary fuzzy relation over Δ_D, $L = \langle \mathrm{CV}, \preccurlyeq \rangle$ be a certainty lattice. Obviously, we need to extend the interpretation d_D: $\Delta_D{}^n \to [0, 1]$ of f-SROIQ(D) to d_D: $\Delta_D{}^n \to \mathrm{CV}$ of L-SROIQ(D). That is, the domain of the interpretation d_D of L-SROIQ(D) is the set of certainty values CV (not the real unit interval [0, 1]).

In order to define the fuzzy concrete domains and provide reasoning preserving procedure for L-SROIQ (D), in what follows, we assume the lattice L is a linear order.

If the set of certainty values CV is continuous and infinite (e.g. the lattice over the real unit interval [0, 1] with order $\leqslant$), we may define a similar trapezoidal membership function $\mathrm{trap}_{k_1,k_2}(x; q_1, q_2, q_3, q_4)$ as in Fig. 2a. If the set of certainty values CV is discrete and finite (e.g. $\mathsf{L}_{\{0,1\}}$, Belnap's FOUR and L_4 (see Section 2.2), or the certainty lattice $L = \langle \mathrm{CV}, \preccurlyeq \rangle$ defined in Example 1), we may define the membership function $f_{k_1,k_2}(x; q_1, q_2, \cdots, q_{k-1}, q_k)$ as in Fig. 2b.

In the rest of this work we will restrict ourselves to the discrete membership function $f_{k_1,k_2}(x; q_1, q_2, \cdots, q_{k-1}, q_k)$. Hence, we assume a unique fuzzy predicate $d = f_{k_1,k_2}(q_1, q_2, \cdots, q_{k-1}, q_k)$.



a. Continuous function　　b. Discrete function

Fig. 2　Membership functions defined in $[k_1, k_2]$

2.2　Logical properties

It can be easily shown that L-SROIQ(D) is a sound extension of f-SROIQ(D), in the sense that L-interpretations coincide with fuzzy interpretations if we restrict the degree of certainty to the lattice over the real unit interval [0, 1] with order $\leqslant$. In the following, we discuss some properties of our DL L-SROIQ(D).

The first ones are straightforward: $\neg\top\equiv\bot$, $\neg\bot\equiv\top$, $C\sqcap\top\equiv C$, $C\sqcup\bot\equiv C$, $C\sqcap\bot\equiv\bot$, $C\sqcup\top\equiv\top$, $\exists R.\bot\equiv\bot$ and $\forall R.\top\equiv\top$, where C is a fuzzy, R is an abstract fuzzy role.

The definitions of the operators over lattices (see Section 1.2) imply that the following properties hold:

Proposition 1　Let C, C_1, C_2, C_3 and D be five L-SROIQ(D) concepts, R L-SROIQ(D) abstract role and S simple L-SROIQ(D) abstract role. Then

1) $\neg\neg C\equiv C$, $C\sqcap C\equiv C$, $C\sqcup C\equiv C$;

2) $\neg(C\sqcap D)\equiv\neg C\sqcup\neg D$, $\neg(C\sqcup D)\equiv\neg C\sqcap\neg D$;

3) $\neg(\forall R.C)\equiv\exists R.\neg C$, $\neg(\exists R.C)\equiv\forall R.\neg C$;

4) $(\leqslant n\ S.C)\equiv(\neg(\geqslant n+1\ S.C))$, $(\geqslant m\ S.C)\equiv(\neg(\leqslant m-1\ S.C))$.

Proof This is easily obtained from the definitions of semantics of L-SROIQ(D) concepts (see Section 1.1).

Please note that the properties 1)～3) are satisfied in L-SHIN[24]. From Proposition 1 we know that these properties are also satisfied in L-SROIQ(D). On the other hand, From Proposition 1 we also know that it would be possible to transform fuzzy concept expressions into a semantically equivalent Negation Normal Form (NNF), which is obtained by using the equivalences of Proposition 1 to push negation in front of atomic concepts, fuzzy nominals and local reflexivity concept ($\exists S$.Self).

Proposition 2　For a certainty lattice $L=\langle \mathrm{CV}, \preccurlyeq\rangle$, $\gamma\in\mathrm{CV}$, $\rhd\in\{\succcurlyeq, \succ\}$ and $\lhd\in\{\preccurlyeq, \prec\}$, the following properties are verified:

1) $\langle a{:}\neg C \rhd \gamma\rangle\equiv\langle a{:}C \rhd^{-}\neg\gamma\rangle$;

2) $\langle a{:}\neg C \lhd \gamma\rangle\equiv\langle a{:}C \lhd^{-}\neg\gamma\rangle$;

3) $\langle (a, b){:}\neg R \rhd \gamma\rangle\equiv\langle (a, b){:}R \rhd^{-}\neg\gamma\rangle$;

4) $\langle (a, b){:}\neg R \lhd \gamma\rangle\equiv\langle (a, b){:}R \lhd^{-}\neg\gamma\rangle$;

5) $\langle (a, v){:}\neg T \rhd \gamma\rangle\equiv\langle (a, v){:}T \rhd^{-}\neg\gamma\rangle$;

6) $\langle (a, v){:}\neg T \lhd \gamma\rangle\equiv\langle (a, v){:}T \lhd^{-}\neg\gamma\rangle$.

Proof　To be omitted.

Obviously, we can assume that negated role assertions of the form $\langle (a, b){:}\neg R \rhd \gamma\rangle$ or $\langle (a, b){:}\neg R \lhd \gamma\rangle$ do not appear in the fuzzy knowledge base KB (and similarly for concrete roles) due to the equivalences 3)～6) of Proposition 2.

Similarly as in L-SHIN[24], we have the following properties about entailment in our fuzzy DL L-SROIQ(D).

$\mathrm{KB}\vDash\langle\psi\succcurlyeq\alpha\rangle$ iff $\mathrm{glb}(\mathrm{KB}, \psi)\succcurlyeq\alpha$, and similarly $\mathrm{KB}\vDash\langle\psi\preccurlyeq\beta\rangle$ iff $\mathrm{lub}(\mathrm{KB}, \psi)\preccurlyeq\beta$. Furthermore, from $\langle a{:}C\preccurlyeq\beta\rangle$ iff $\langle a{:}\neg C\succcurlyeq\neg\beta\rangle$ (see 1) of Proposition 2), it follows $\mathrm{lub}(\mathrm{KB}, a{:}C)=\neg\mathrm{glb}(\mathrm{KB}, a{:}\neg C)$. From $\langle (a, b){:}R\preccurlyeq\beta\rangle$ iff $\langle (a, b){:}\neg R\succcurlyeq\neg\beta\rangle$ (see 3) of Proposition 2), it follows $\mathrm{lub}(\mathrm{KB}, (a, b){:}R)=\neg\mathrm{glb}(\mathrm{KB}, (a, b){:}\neg R)$. Similarly, from $\langle (a, v){:}T\preccurlyeq\beta\rangle$ iff $\langle (a, v){:}\neg T\succcurlyeq\neg\beta\rangle$ (see 5) of Proposition 2), it follows $\mathrm{lub}(\mathrm{KB}, (a, v){:}T)=\neg\mathrm{glb}(\mathrm{KB}, (a, v){:}\neg T)$. Therefore, lub can be determined through glb and vice versa in L-SROIQ(D).

L-SROIQ(D) allows some sort of modus ponens over concepts and roles, even with the new semantics of fuzzy GCIs:

Proposition 3　For a linear ordered certainty lattice $L=\langle \mathrm{CV}, \preccurlyeq\rangle$, $\alpha, \beta\in\mathrm{CV}$, $\rhd\in\{\succcurlyeq, \succ\}$, $\beta \,+\!\rhd\, \neg\alpha$

(where $+\succcurlyeq = \succ$, $+\succ = \succcurlyeq$), the following properties are verified:

1) $\langle a{:}C \rhd \alpha\rangle$ and $\langle C\sqsubseteq D \rhd \beta\rangle$ imply $\langle a{:}D \rhd \beta\rangle$.

2) $\langle (a, b){:}R \rhd \alpha\rangle$ and $\langle R\sqsubseteq R' \rhd \beta\rangle$ imply $\langle (a, b){:}R' \rhd \beta\rangle$.

3) $\langle (a, b){:}R \rhd \alpha\rangle$ and $\langle a{:}\forall R.C \rhd \beta\rangle$ imply $\langle b{:}C \rhd \beta\rangle$.

Proof To be omitted.

Finally, we will provide the property about soundness of the semantics.

Our fuzzy semantics based on certainty lattices is sound w.r.t. crisp semantics. In fact, let KB=⟨AB, TB, RB⟩ be an L-SROIQ(D) knowledge base. Let us consider the following transformation #(·) of fuzzy assertions (resp. fuzzy axioms) into assertions (resp. axioms), where #(·) takes the "crisp" assertional (resp. terminological) part of a fuzzy assertion (resp. fuzzy axiom):

$\#(\langle\psi \rhd \gamma\rangle) \to \psi$;

$\#(a{:}C \lhd \gamma) \to a{:}\neg C$;

$\#((a, b){:}R \lhd \gamma) \to (a, b){:}\neg R$;

$\#((a, v){:}T \lhd \gamma) \to (a, v){:}\neg T$;

$\#((a, b){:}\neg R \lhd \gamma) \to (a, b){:}R$;

$\#((a, v){:}\neg T \lhd \gamma) \to (a, v){:}T$;

$\#(\langle C\sqsubseteq D \rhd \gamma\rangle) \to C\sqsubseteq D$;

$\#(\langle w\sqsubseteq R \rhd \gamma\rangle) \to w\sqsubseteq R$.

We extend #(·) to fuzzy knowledge base KB=⟨AB, TB, RB⟩ as follows: $\#(\mathrm{KB})=\{\#(\zeta)\mid \zeta\in \mathrm{AB}\} \cup \{\#(\zeta)\mid \zeta\in \mathrm{TB}\} \cup \{\#(\zeta)\mid \zeta\in \mathrm{RB}\}$. It can be shown that

Proposition 4 For a certainty lattice $L=\langle \mathrm{CV}, \preccurlyeq\rangle$, $\gamma\in \mathrm{CV}$, $\rhd\in\{\succcurlyeq, \succ\}$ and $\lhd\in\{\preccurlyeq, \prec\}$, let KB=⟨AB, TB, RB⟩ be a fuzzy knowledge base and let ζ be a fuzzy assertion (or a fuzzy axiom), i.e., ζ is an expression of the form $\langle\psi \rhd \gamma\rangle$,$\langle\psi \lhd \gamma\rangle$, $\langle C\sqsubseteq D \rhd \gamma\rangle$ or $\langle w\sqsubseteq R \rhd \gamma\rangle$, where ψ is of the form $a{:}C$, $(a, b){:}R$, $(a, b){:}\neg R$, $(a, v){:}T$ or $(a, v){:}\neg T$. If $\mathrm{KB}\vDash\zeta$, then $\#(\mathrm{KB})\vDash\#(\zeta)$.

The proof is similar to the proof of Theorem 3 of Ref. [24].

3 A crisp representation for L-SROIQ(D)

In this section we show how to reduce an L-SROIQ(D) fuzzy knowledge base KB into a crisp SROIQ(D) knowledge base. The procedure preserves reasoning, so existing SROIQ(D) reasoners such as Pellet[35], FaCT++[36] and RACER[37] could be applied to the resulting knowledge base.

The basic idea is to create some new crisp concepts and roles, representing the α-cuts of the fuzzy concepts and relations, and to rely on them. Next, some new axioms are added to preserve their semantics and finally every axiom in the ABox, the TBox and the RBox is represented, independently from other axioms, using these new crisp elements. In fact, the reduction presented in this section is an extension of that of f-SROIQ(D) under Zadeh semantics and Gödel semantics[4,9-10] and L-ALC under linear ordered lattices[20].

In Ref. [4,9-10,33,38] it has been shown that reasoning in fuzzy DLs can be reduced to reasoning in classical DLs and, thus, already existing reasoners can be applied directly. The first effort in this direction is due to Straccia, who showed a reasoning preserving procedure for fuzzy ALCH[38]. Recently, Ref. [4,10] provided a crisp representation for the fuzzy DLs f-SROIQ and f-SROIQ(D) respectively. It needs to be noted that in the rest of this paper we will restrict ourselves to the linear ordered lattices.

3.1 Adding new elements

Let AC be the set of atomic concepts, RA the set of atomic abstract roles and TC the set of concrete roles in a fuzzy knowledge base KB=⟨AB, TB, RB⟩. Ref. [20] showed that the set of the degrees of certainty, which must be considered for any reasoning task in L-ALC, is defined as follows.

Consider an L-ALC knowledge base KB. Define $X^{\mathrm{KB}}=\{f, t\} \cup \{\gamma\mid \langle\psi \rhd \gamma\rangle\in \mathrm{KB}\} \cup \{\neg\gamma\mid \langle\psi \lhd \gamma\rangle\in \mathrm{KB}\}$, from which we define $N^{\mathrm{KB}}=X^{\mathrm{KB}} \cup \{\neg\gamma\mid \gamma\in X^{\mathrm{KB}}\}$. If there is $\gamma'\in\mathsf{T}$ such that $\neg\gamma'=\gamma'$, then we add γ' to X^{KB}.

This also holds in L-SROIQ(D). Without loss of generality, it can be assumed that $N^{\mathrm{KB}}=\{\gamma_1, \gamma_2,\cdots, \gamma_{|N^{\mathrm{KB}}|}\}$ and $\gamma_i\prec\gamma_{i+1}$, for $1\leqslant i\leqslant|N^{\mathrm{KB}}|-1$. It is easy to see that $\gamma_1=f$ and $\gamma_{|N^{\mathrm{KB}}|}=t$.

For each $\alpha, \beta\in N^{\mathrm{KB}}$ with $\alpha\neq f$ and $\beta\neq t$, for each $A\in \mathrm{AC}$, two new atomic concepts $A_{\succcurlyeq\alpha}$, $A_{\succ\beta}$ are introduced. $A_{\succcurlyeq\alpha}$ represents the crisp set of individuals which are instance of A with degree of certainty higher or equal than α, i.e., the α-cut of A. $A_{\succ\beta}$ is defined in a similar way.

Similarly, for each $R_A \in$ RA and for each $T \in$ TC two new atomic abstract roles $R_{A \succcurlyeq \alpha}$, $R_{A \succ \beta}$ and two new concrete roles $T_{\succcurlyeq \alpha}$, $T_{\succ \beta}$ are introduced.

The atomic elements $A_{\succ t}$, $R_{A \succ t}$, $T_{\succ t}$, $A_{\succcurlyeq f}$, $R_{A \succcurlyeq f}$ and $T_{\succcurlyeq f}$ are not considered because they not necessary, due to the restrictions on the allowed degree of certainty of the axioms in the fuzzy knowledge base KB.

The semantics of these newly introduced atomic concepts and roles is preserved by some terminological and role axioms. For each $1 \leqslant i \leqslant |N^{\mathrm{KB}}|-1$, $2 \leqslant j \leqslant |N^{\mathrm{KB}}|-1$ and for each $A \in$ AC, TBX(N^{KB}) is the smallest TBox containing these two axioms:

$A_{\succcurlyeq \gamma_{i+1}} \sqsubseteq A_{\succ \gamma_i}$ and $A_{\succ \gamma_j} \sqsubseteq A_{\succcurlyeq \gamma_j}$.

Similarly, for each $R_A \in$ RA, RAX(N^{KB}) is the smallest RBox containing the following two axioms:

$R_{A \succcurlyeq \gamma_{i+1}} \sqsubseteq R_{A \succ \gamma_i}$ and $R_{A \succ \gamma_i} \sqsubseteq R_{A \succcurlyeq \gamma_i}$.

And for each $T \in$ TC, RCX(N^{KB}) is the smallest RBox containing the following two axioms:

$T_{\succcurlyeq \gamma_{i+1}} \sqsubseteq T_{\succ \gamma_i}$ and $T_{\succ \gamma_i} \sqsubseteq T_{\succcurlyeq \gamma_i}$.

Previous work[20] used two more atomic concepts $A_{\preccurlyeq \beta}$, $A_{\prec \alpha}$ and the following additional axioms (for $2 \leqslant k \leqslant |N^{\mathrm{KB}}|$):

$A_{\prec \gamma_k} \sqsubseteq A_{\preccurlyeq \gamma_k}$, $A_{\preccurlyeq \gamma_i} \sqsubseteq A_{\prec \gamma_{i+1}}$, $A_{\succcurlyeq \gamma_k} \sqcap A_{\prec \gamma_k} \sqsubseteq \bot$, $A_{\succ \gamma_i} \sqcap A_{\preccurlyeq \gamma_i} \sqsubseteq \bot$, $\top \sqsubseteq A_{\succcurlyeq \gamma_k} \sqcup A_{\prec \gamma_k}$, $\top \sqsubseteq A_{\succ \gamma_i} \sqcup A_{\preccurlyeq \gamma_i}$.

In contract to this, we use $\neg A_{\succ \gamma_k}$ rather than $A_{\preccurlyeq \gamma_k}$ and $\neg A_{\succcurlyeq \gamma_k}$ instead of $A_{\prec \gamma_k}$. This way is the same as that of f-SROIQ and f-SROIQ(D) proposed in Ref. [4,9-10,39]. The six axioms above follow immediately from the semantics of the crisp concepts as Proposition 5 shows:

Proposition 5 If $A_{\succcurlyeq \gamma_{i+1}} \sqsubseteq A_{\succ \gamma_i}$ and $A_{\succ \gamma_k} \sqsubseteq A_{\succcurlyeq \gamma_k}$ hold, then the following axioms are verified:

1) $\neg A_{\succcurlyeq \gamma_k} \sqsubseteq \neg A_{\succ \gamma_k}$, 2) $\neg A_{\succ \gamma_i} \sqsubseteq \neg A_{\succcurlyeq \gamma_{i+1}}$,

3) $A_{\succcurlyeq \gamma_k} \sqcap \neg A_{\succcurlyeq \gamma_k} \sqsubseteq \bot$, 4) $A_{\succ \gamma_i} \sqcap \neg A_{\succ \gamma_i} \sqsubseteq \bot$,

5) $\top \sqsubseteq A_{\succcurlyeq \gamma_k} \sqcup \neg A_{\succcurlyeq \gamma_k}$, 6) $\top \sqsubseteq A_{\succ \gamma_i} \sqcup \neg A_{\succ \gamma_i}$.

Proof To be omitted.

Obviously, Proposition 5 is similar to Proposition 2 of Ref. [10]. The aim is to optimize the size of $T(N^{\mathrm{KB}})$.

Similarly as f-SROIQ(D)[9-10], we do not introduce the axiom $A_{\succ f} \sqsubseteq A_{\succcurlyeq f}$, since $A_{\succcurlyeq f}$ is equivalent to $\top$ the axiom trivially holds. On the other hand, in the case of roles, we use $\neg R_{A \neg \lhd \gamma}$ instead of $R_{A \lhd \gamma}$, as we will see in the next subsection. This idea is essential in order to represent some of role constructors of SROIQ(D) (negated role assertions and self reflexivity concepts). Actually, it is not possible to use a role of the form $R_{A \preccurlyeq \gamma_k}$ rather than $\neg R_{A \succ \gamma_k}$ and $R_{A \prec \gamma_k}$ instead of $\neg R_{A \succcurlyeq \gamma_k}$. The reason is that the logic does not make possible to express the corresponding version of the axioms 3), 4), 5) and 6) of Proposition 5, which would be necessary to guarantee the correctness of the reduction, because the role conjunction and the bottom role are not allowed, and the universal role cannot appear in RIAs.

3.2 Mapping fuzzy concepts, roles and axioms

Fuzzy concept and role expressions are reduced using mapping ρ, as shown in Tables 2 and 3 respectively. Concrete predicates are reduced as in Table 4.

Table 2 Mapping of fuzzy concept expressions

x	y	$\rho(x, y)$
$\top$	$\rhd \gamma$	$\top$
$\top$	$\lhd \gamma$	$\bot$
$\bot$	$\rhd \gamma$	$\bot$
$\bot$	$\lhd \gamma$	$\top$
A	$\rhd \gamma$	$A_{\rhd \gamma}$
A	$\lhd \gamma$	$\neg A_{\neg \lhd \gamma}$
$\neg C$	$\bowtie \gamma$	$\rho(C, \bowtie^- \neg \gamma)$
$C \sqcap D$	$\rhd \gamma$	$\rho(C, \rhd \gamma) \sqcap \rho(D, \rhd \gamma)$
$C \sqcap D$	$\lhd \gamma$	$\rho(C, \lhd \gamma) \sqcup \rho(D, \lhd \gamma)$
$C \sqcup D$	$\rhd \gamma$	$\rho(C, \rhd \gamma) \sqcup \rho(D, \rhd \gamma)$
$C \sqcup D$	$\lhd \gamma$	$\rho(C, \lhd \gamma) \sqcap \rho(D, \lhd \gamma)$
$\exists R.C$	$\rhd \gamma$	$\exists \rho(R, \rhd \gamma).\rho(C, \rhd \gamma)$
$\exists R.C$	$\lhd \gamma$	$\exists \rho(R, \neg \lhd \gamma).\rho(C, \lhd \gamma)$
$\exists T.d$	$\rhd \gamma$	$\exists \rho(T, \rhd \gamma).\rho(d, \rhd \gamma)$
$\exists T.d$	$\lhd \gamma$	$\exists \rho(T, \neg \lhd \gamma).\rho(d, \lhd \gamma)$
$\forall R.C$	$\{\succcurlyeq, \succ\}\gamma$	$\forall \rho(R, \{\succ, \succcurlyeq\}\neg \gamma).\rho(C, \{\succcurlyeq, \succ\}\gamma)$
$\forall R.C$	$\lhd \gamma$	$\exists \rho(R, \lhd^- \neg \gamma).\rho(C, \lhd \gamma)$
$\forall T.d$	$\{\succcurlyeq, \succ\}\gamma$	$\forall \rho(T, \{\succ, \succcurlyeq\}\neg \gamma).\rho(d, \{\succcurlyeq, \succ\}\gamma)$
$\forall T.d$	$\lhd \gamma$	$\exists \rho(T, \lhd^- \neg \gamma).\rho(d, \lhd \gamma)$
$\{\alpha_1/o_1, \alpha_2/o_2, \cdots, \alpha_m/o_m\}$	$\bowtie \gamma$	$\{o_i \mid \alpha_i \bowtie \gamma, 1 \leqslant i \leqslant m\}$
$\geqslant m\ S.C$	$\rhd \gamma$	$\geqslant m\ \rho(S, \rhd \gamma).\rho(C, \rhd \gamma)$
$\geqslant m\ S.C$	$\lhd \gamma$	$\leqslant m-1\ \rho(S, \neg \lhd \gamma).\rho(C, \neg \lhd \gamma)$
$\geqslant m\ T.d$	$\rhd \gamma$	$\geqslant m\ \rho(T, \rhd \gamma).\rho(d, \rhd \gamma)$
$\geqslant m\ T.d$	$\lhd \gamma$	$\leqslant m-1\ \rho(T, \neg \lhd \gamma).\rho(d, \neg \lhd \gamma)$
$\leqslant n\ S.C$	$\{\succcurlyeq, \succ\}\gamma$	$\leqslant n\ \rho(S, \{\succ, \succcurlyeq\}\neg \gamma).\rho(C, \{\succ, \succcurlyeq\}\neg \gamma)$
$\leqslant n\ S.C$	$\lhd \gamma$	$\geqslant n+1\ \rho(S, \lhd^- \neg \gamma).\rho(C, \lhd^- \neg \gamma)$
$\leqslant n\ T.d$	$\{\succcurlyeq, \succ\}\gamma$	$\leqslant n\ \rho(T, \{\succ, \succcurlyeq\}\neg \gamma).\rho(d, \{\succ, \succcurlyeq\}\neg \gamma)$
$\leqslant n\ T.d$	$\lhd \gamma$	$\geqslant n+1\ \rho(T, \lhd^- \neg \gamma).\rho(d, \lhd^- \neg \gamma)$
$\exists S.\mathrm{Self}$	$\rhd \gamma$	$\exists \rho(S, \rhd \gamma).\mathrm{Self}$
$\exists S.\mathrm{Self}$	$\lhd \gamma$	$\neg \exists \rho(S, \neg \lhd \gamma).\mathrm{Self}$
$[C \succcurlyeq \alpha]$	$\rhd \gamma$	$\rho(C, \succcurlyeq \alpha)$
$[C \succcurlyeq \alpha]$	$\lhd \gamma$	$\rho(C, \prec \alpha)$
$[C \preccurlyeq \beta]$	$\rhd \gamma$	$\rho(C, \preccurlyeq \beta)$
$[C \preccurlyeq \beta]$	$\lhd \gamma$	$\rho(C, \succ \beta)$

Table 3　Mapping of fuzzy role expressions

x	y	$\rho(x, y)$
R_A	$\rhd\gamma$	$R_{A\rhd\gamma}$
R_A	$\lhd\gamma$	$\neg R_{A\neg\lhd\gamma}$
T	$\rhd\gamma$	$T_{\rhd\gamma}$
T	$\lhd\gamma$	$\neg T_{\neg\lhd\gamma}$
R^-	$\bowtie\gamma$	$\rho(R, \bowtie\gamma)^-$
U	$\rhd\gamma$	U
U	$\lhd\gamma$	$\neg U$
$[R\succcurlyeq\alpha]$	$\rhd\gamma$	$\rho(R, \succcurlyeq\alpha)$
$[R\succcurlyeq\alpha]$	$\lhd\gamma$	$\rho(R, \prec\alpha)$
$\neg R$	$\rhd\gamma$	$\rho(R, \preccurlyeq f)$
$\neg R$	$\lhd\gamma$	$\rho(R, \succ f)$
$\neg T$	$\rhd\gamma$	$\rho(T, \preccurlyeq f)$
$\neg T$	$\lhd\gamma$	$\rho(T, \succ f)$

Table 4　Mapping of fuzzy concrete predicates

x	y	$\rho(x, y)$
d	$\succcurlyeq\alpha$	real$[q_{\alpha\succcurlyeq-}, q_{\alpha\succcurlyeq+})$
d	$\succ\beta$	real$[q_{\beta\succ-}, q_{\beta\succ+})$
d	$\preccurlyeq\beta$	union-real$[k_1, q_{\beta\preccurlyeq+}, q_{\beta\preccurlyeq-}, k_2]$
d	$\prec\alpha$	union-real$[k_1, q_{\alpha\prec+}, q_{\alpha\prec-}, k_2]$

$q_{\alpha\succcurlyeq-}$, $q_{\alpha\succcurlyeq+}$, $q_{\beta\succ-}$, $q_{\beta\succ+}$, $q_{\beta\preccurlyeq+}$, $q_{\beta\preccurlyeq-}$, $q_{\alpha\prec+}$ and $q_{\alpha\prec-}$ are defined as follows.

Without loss of generality, it can be assumed that $N^{\mathrm{KB}}=\{\gamma_1, \gamma_2,\cdots, \gamma_{|N^{\mathrm{KB}}|}\}=\{\gamma_1, \gamma_2,\cdots, \gamma_e, \alpha, \gamma_f,\cdots, \gamma_g, \beta, \gamma_h,\cdots, \gamma_{|N^{\mathrm{KB}}|}\}$ and $\gamma_i\prec\gamma_{i+1}$, for $1\leqslant i\leqslant|N^{\mathrm{KB}}|-1$.

$q_{\alpha\succcurlyeq-}=\min\{x|\, f_{k_1,k_2}(x;\, q_1, q_2,\cdots, q_{k-1}, q_k)=\alpha\}$;

$q_{\alpha\succcurlyeq+}=\min\{x|\, f_{k_1,k_2}(x;\, q_1, q_2,\cdots, q_{k-1}, q_k)=\gamma_e \wedge \exists y, y\leqslant x \wedge f_{k_1,k_2}(y;\, q_1, q_2,\cdots, q_{k-1}, q_k)=t\}$;

$q_{\beta\succ-}=\min\{x|\, f_{k_1,k_2}(x;\, q_1, q_2,\cdots, q_{k-1}, q_k)=\gamma_h\}$;

$q_{\beta\succ+}=\min\{x|\, f_{k_1,k_2}(x;\, q_1, q_2,\cdots, q_{k-1}, q_k)=\beta \wedge \exists y, y\leqslant x \wedge f_{k_1,k_2}(y;\, q_1, q_2,\cdots, q_{k-1}, q_k)=t\}$;

$q_{\beta\preccurlyeq+}=\min\{x|\, f_{k_1,k_2}(x;\, q_1, q_2,\cdots, q_{k-1}, q_k)=\gamma_h\}$;

$q_{\beta\preccurlyeq-}=\min\{x|\, f_{k_1,k_2}(x;\, q_1, q_2,\cdots, q_{k-1}, q_k)=\beta \wedge \exists y, y\leqslant x \wedge f_{k_1,k_2}(y;\, q_1, q_2,\cdots, q_{k-1}, q_k)=t\}$;

$q_{\alpha\prec+}=\min\{x|\, f_{k_1,k_2}(x;\, q_1, q_2,\cdots, q_{k-1}, q_k)=\alpha\}$;

$q_{\alpha\prec-}=\min\{x|\, f_{k_1,k_2}(x;\, q_1, q_2,\cdots, q_{k-1}, q_k)=\gamma_e \wedge \exists y, y\leqslant x \wedge f_{k_1,k_2}(y;\, q_1, q_2,\cdots, q_{k-1}, q_k)=t\}$.

Given a fuzzy concept C, $\rho(C, \succcurlyeq\alpha)$ is the α-cut of C, a crisp set containing all the elements which belong to C with a degree of certainty greater or equal than α. The other cases $\rho(C, \bowtie\gamma)$ are similar.

Given a fuzzy role R, $\rho(R, \succcurlyeq\alpha)$ is a crisp set containing all the pair of elements which are related through R with a degree of certainty greater or equal than α. The other cases $\rho(R, \bowtie\gamma)$ and $\rho(T, \bowtie\gamma)$ are similar.

Finally, due to the restrictions in the definition of the fuzzy knowledge base KB, some expressions cannot appear during the process:

1) $\rho(R, \lhd\gamma)$, $\rho(U, \lhd\gamma)$ and $\rho(T, \lhd\gamma)$ can only appear in a (crisp) negated role assertion.

2) $\rho(A, \succcurlyeq f)$, $\rho(A, \succ t)$, $\rho(A, \preccurlyeq t)$ and $\rho(A, \prec f)$ cannot appear due to the existing restrictions on the degree of certainty of the axioms in the fuzzy knowledge base KB. The same also holds for $\top$, $\bot$, R_A, T and U.

Axioms are reduced as in Table 5, where $k(\tau)$ maps a fuzzy axiom τ in L-SROIQ(D) into a set of crisp axioms in SROIQ(D). We note k(AB) (resp. k(TB), k(RB)) the union of the reductions of all the fuzzy axioms in AB (resp. TB, RB).

Table 5　Reduction of the axioms

τ	$k(\tau)$
$\langle a{:}C \bowtie\gamma\rangle$	$\{a{:}\rho(C, \bowtie\gamma)\}$
$\langle(a, b){:}R \bowtie\gamma\rangle$	$\{(a, b){:}\rho(R, \bowtie\gamma)\}$
$\langle(a, v){:}T \bowtie\gamma\rangle$	$\{(a, v){:}\rho(T, \bowtie\gamma)\}$
$\langle(a, b){:}\neg R \bowtie\gamma\rangle$	$\{(a, b){:}\rho(\neg R, \bowtie\gamma)\}$
$\langle(a, v){:}\neg T \bowtie\gamma\rangle$	$\{(a, v){:}\rho(\neg T, \bowtie\gamma)\}$
$\langle a\neq b\rangle$	$\{a\neq b\}$
$\langle a=b\rangle$	$\{a=b\}$
$\langle C\sqsubseteq D \succcurlyeq \alpha\rangle$	$\rho(C, \succ\neg\alpha)\sqsubseteq\rho(D, \succcurlyeq\alpha)$
$\langle C\sqsubseteq D \succ \beta\rangle$	$\rho(C, \succcurlyeq\neg\beta)\sqsubseteq\rho(D, \succ\beta)$
$\langle R_1, R_2,\cdots, R_n\sqsubseteq R \succcurlyeq \alpha\rangle$	$\{\rho(R_1, \succ\neg\alpha), \rho(R_2, \succ\neg\alpha),\cdots, \rho(R_m, \succ\neg\alpha)\sqsubseteq\rho(R, \succcurlyeq\alpha)\}$
$\langle R_1, R_2,\cdots, R_n\sqsubseteq R \succ \beta\rangle$	$\{\rho(R_1, \succcurlyeq\neg\beta), \rho(R_1, \succcurlyeq\neg\beta),\cdots, \rho(R_m, \succcurlyeq\neg\beta)\sqsubseteq\rho(R, \succ\beta)\}$
$\langle T_1\sqsubseteq T_2 \succcurlyeq \alpha\rangle$	$\{\rho(T_1, \succ\neg\alpha)\sqsubseteq\rho(T_2, \succcurlyeq\alpha)\}$
$\langle T_1\sqsubseteq T_2 \succ \beta\rangle$	$\{\rho(T_1, \succcurlyeq\neg\beta)\sqsubseteq\rho(T_2, \succ\beta)\}$
trans(R)	$\bigcup_{\gamma\in N^{\mathrm{KB}}\backslash\{f\}} \{\mathrm{trans}(\rho(R, \succcurlyeq\gamma))\} \cup$ $_{\gamma\in N^{\mathrm{KB}}\backslash\{f\}} \{\mathrm{trans}(\rho(R, \succ\gamma))\}$
dis(S_1, S_2)	$\{\mathrm{dis}(\rho(S_1, \succ f), \rho(S_2, \succ f))\}$
dis(T_1, T_2)	$\{\mathrm{dis}(\rho(T_1, \succ f), \rho(T_2, \succ f))\}$
ref(R)	$\{\mathrm{ref}(\rho(R, \succcurlyeq t))\}$
irr(S)	$\{\mathrm{irr}(\rho(S, \succ f))\}$
sym(R)	$\bigcup_{\gamma\in N^{\mathrm{KB}}\backslash\{f\}} \{\mathrm{sym}(\rho(R, \succcurlyeq\gamma))\} \cup$ $_{\gamma\in N^{\mathrm{KB}}\backslash\{f\}} \{\mathrm{sym}(\rho(R, \succ\gamma))\}$
asy(S)	$\{\mathrm{asy}(\rho(S, \succ f))\}$

Obviously, the mappings ρ and k defined above are semantic extension of the mappings ρ and k defined in Ref. [9] for f-SROIQ(D) under Zadeh semantics. That is, the mappings ρ and k defined above are based on linear ordered lattices, but the mappings ρ and k defined in Ref. [9] are based on the real unit interval [0, 1].

3.3　Correctness of the reduction

As in the f-SROIQ(D) DL[9-10], the reduction presented in Section 3.2 preserves simplicity of the roles and regularity of the RIAs.

The following theorem shows that our L-SROIQ(D) over linear ordered lattices is decidable and that the reduction preserves reasoning.

Theorem 1 The satisfiability problem in L-SROIQ(D) over linear ordered lattices is decidable. Furthermore, an linear order L-SROIQ(D) fuzzy knowledge base KB=⟨AB, TB, RB⟩ is satisfiable iff its crisp representation k(KB)=⟨k(AB), TBX(N^{KB}) ∪ k(TB), RAX(N^{KB}) ∪ RCX(N^{KB}) ∪ k(RB)⟩ is satisfiable.

Proof To be omitted.

As in the f-SROIQ(D) DL[9-10], our procedure for L-SROIQ(D) also has the modularity property. That is, we have the following property.

Theorem 2 Let KB be an L-SROIQ(D) fuzzy knowledge base involving a set of fuzzy atomic concepts AC, a set of atomic roles R_a and a set of concrete roles R_c, let N^{KB} be the set of relevant certainty degrees to be considered and let τ be an L-SROIQ(D) axiom such that:

1) For every atomic concept A which appears in τ, $A \in$ AC;

2) For every atomic role R_A which appears in τ, $R_A \in R_a$;

3) for every concrete role T which appears in τ, $T \in R_c$;

4) If γ appears in τ, then $\gamma \in N^{\text{KB}}$.

Then, the reduction of the union of the KB and the axiom τ is equivalent to the union of the reduction of KB and the reduction of τ:

$k(\text{KB} \cup \tau) = k(\text{KB}) \cup k(\tau)$.

The proof of Theorem 2 is similar to that of Theorem 2 of Ref. [10].

Regarding the complexity, obviously, the complexity of the reduction of our L-SROIQ(D) is the same as that of the f-SROIQ(D) DL[9-10]. That is, the resulting knowledge base is quadratic. The ABox is actually linear while the TBox and the RBox are both quadratic (see Ref. [9-10] for more details).

4 Conclusions

Making applications capable of coping with vagueness (fuzziness) and imprecision will result in the creation of systems and applications which will provide us with high quality results and answers to complex user defined tasks. To this extent we have presented a very expressive fuzzy DL L-SROIQ(D) based on certainty lattice theory. Concretely, our work presents several contributions. Firstly, we augment the expressivity of fuzzy DLs by allowing the definition of fuzzy sets by extension and by allowing fuzzy GCIs and fuzzy RIAs to be verified up to some degree. Secondly, we present a very expressive fuzzy DL over uncertainty lattices L-SROIQ(D). Finally, we show the decidability of L-SROIQ(D) by providing a reasoning preserving procedure to obtain a crisp representation for it in case of linearly ordered lattices.

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