Image Inpainting Algorithm Based on Double Cross TV

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Abstract　The various total variation (TV) algorithms currently reconstruct the lost or deteriorated parts of images by related information of damaged pixel and its 4 neighborhood pixels. Their inpainting accuracy is low because of finite related information. The double cross TV algorithm proposed in this paper divides 8 neighborhood pixels into 2 groups, and computes the pixel value of damaged pixel by using 4 neighborhood pixels in each group respectively. Therefore, the weighted mean of these two pixel values is the final inpainting pixel value. The Experiments show an improvement in PSNR and less iteration number compared with the original TV.

Key words　double cross TV algorithm; image inpainting; inpainting accuracy; neighborhood information

基于双十字TV模型的图像修复算法

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【摘要】当前的各种TV(total variation)算法均只利用待修复点及其邻域的4个点的信息进行修复，由于所提供的参考信息有限，使得修复后的图像精确度欠佳。该文提出的双十字TV算法利用原始的TV算法，将待修复点邻域中的8个点分为两组，分别利用每组4个点的参考信息计算待修复点的像素值，然后将这两个像素值进行加权平均得到最终的修复值。实例验证结果表明，在不增加时间复杂度的情况下，双十字TV算法有效提高了修复后图像的精确度。

关　键　词　双十字TV算法;　图像修复;　修复精度;　邻域信息

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Inpainting is to reconstitute the missing or damaged portions of the artwork based upon the non-damaged image information under certain rules to make the image more legible and to restore its unity [1]. The current main inpainting algorithms are based on texture and higher order partial differential equations (PDE). The popular inpainting algorithm based on texture was proposed by Criminisi and based on samples block matching, finding a suitable match block in the intact parts to pad the inpainting region[2]-[3]. But this approach only takes the texture characteristic into account, and can only be applied to large damaged region. PDE-based inpainting algorithm mainly has two sects, one was proposed by Ref.[4], by directly using PDE, in which image information was diffused from intact region into the inpainting region by predicating information like the isophote of image edge, and achieved good results[5]. The other was proposed by Ref.[6-7], based on energy optimization and some assumptions about characters of structural image edge such as simplicity and small curvature, in which energy function was formed and PDE was transformed by variation to calculate the result. In 1992, Rudin, Osher and Fatemi proposed image smooth denoising model[8], which can denoise with keeping the image edge, and it overcame the

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Biofraphy：ZHAI Dong-hai, was born in 1974, Associate professor, his research interesting focus on digital image processing (inpainting).

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disadvantage that the linear filter denoises with smoothing image edge. After that, Chan and Shen improved the smooth denoising model by variation, and proposed the TV-based inpainting algorithm. This method is the most popular and the hottest research[9-11].

During the process of solution using difference, the TV model only takes the related information of 4 neighborhood pixels {N, S, W, E} of damaged pixel into account[12-13], but the inpainting accuracy is not enough in the whole damaged region, because the effective related information from the 4 neighborhood pixels is not adequate. In fact, the other 4 neighborhood pixels {NE, NW, SE, SW} of damaged pixel can also provide related information, and, if these 4 pixels are taken into account in inpainting process, the inpainting accuracy would be greatly improved. Based on this idea, double cross TV model is proposed in this paper, and this method can achieve better inpainting results with less iteration number.

1　Introduction of TV model

The main idea of the TV inpainting algorithm is to find a minimum-energy functional, and the less energy, the smoother images will be. The algorithm creates a model with noise constraint condition, and then, the model is turned into an unconstrained minimization problem by using the Lagrange multiplier and it is solved by difference equation [6].

In the original TV algorithm, the final unconstrained cost function is given by Ref. [6]

$$J_\lambda(u)=\int_{E\cup D}|\nabla u|\,\mathrm{d}x\mathrm{d}y+\frac{\lambda}{2}\int_E|u-u^0|^2\,\mathrm{d}x\mathrm{d}y \tag{1}$$

Let D be an inpainting domain, E be an extended domain, which is usually ringlike, so that the pixels in $E\cup D$ is u, and u_0 is the original pixel before inpainting. $|\nabla u|$ is a gradient modulus, and λ plays a role in the constrained variational problem. The Euler-Lagrange equation for the energy functional $J_\lambda(u)$ is:

$$-\mathrm{div}\left(\frac{\nabla u}{|\nabla u|}\right)+\lambda_e(u-u^0)=0 \tag{2}$$

where, the extended Lagrange multiplier λ_e is given by

$$\lambda_e=\begin{cases}\lambda & (x,y)\in E\\ 0 & (x,y)\in D\end{cases} \tag{3}$$

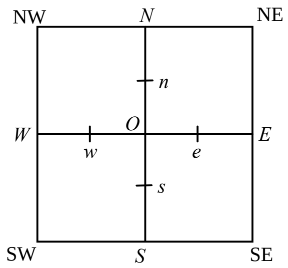


Fig. 1　Damaged pixel O and its neighborhood

As in Figure 1, at a given damaged pixel O, let B = (N, S, W, E) denote its four neighborhood pixels, and B'=(n, s, w, e) be the corresponding four midway points. Therefore, the pixel O is discretized and obtained to (4)

$$\sum_{\substack{X\in B\\ x\in B'}}\frac{1}{|\nabla u_x|}(u_O-u_X)+\lambda_e(O)(u_O-u_O^0)=0 \tag{4}$$

Then, final value u_O is as follows

$$u_O=\frac{\displaystyle\sum_{\substack{X\in B\\ x\in B'}}\frac{u_X}{|\nabla u_x|}+\lambda_e(O)u_O^0}{\displaystyle\sum_{\substack{X\in B\\ x\in B'}}\frac{1}{|\nabla u_x|}+\lambda_e(O)} \tag{5}$$

In Eq. (5), the original TV algorithm just used the damaged pixel O and its neighborhood-pixels B = (N, S, W, E), so that the inpainting accuracy is low because of finite related information. That is, the u_O is the weighted mean of its neighborhood-pixels, therefore, the more terms in the sum formula, the more accurate the final weighted mean is. In fact, if its neighborhood-pixels C = (NW, NE, SW, SE) is added during the process of the weighted mean, the inpainting accuracy would be improved effectively.

2　Double cross TV algorithm

To improve the inpainting accuracy, the proposed double cross TV algorithm divides 8 neighborhood pixels into 2 groups B and C, and computes pixel value of damaged pixel using 4 neighborhood pixels in each group u_{O_1} and u_{O_2} respectively. Then, the weighted mean of these two pixel values is the final inpainting pixel value.

2.1　Calculation of u_{O_1}

As shown in Fig. 2, Fig. 2a can be divided into two subgraphs: Fig. 2b and Fig. 2c.

In Fig. 2b, the neighborhood of damaged pixel

O_1(i.e., mirror point of O) are B = (N, S, W, E), and its corresponding four midway points are B' = (n, s, w, e) and h denotes the grid size. The pixel u_{O_1} can be solved by the original TV model easily, such as:

$$u_{O1}=\frac{\sum\limits_{\substack{X\in B\\ x\in B'}}\frac{u_X}{|\nabla u_x|}+\lambda_e(O)u_O^0}{\sum\limits_{\substack{X\in B\\ x\in B'}}\frac{1}{|\nabla u_x|}+\lambda_e(O)} \tag{6}$$

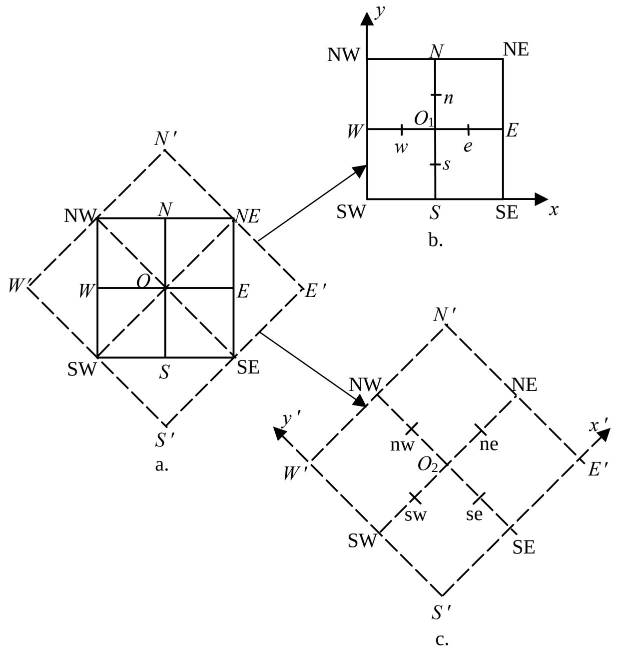


Fig. 2 Decomposition of neighborhood

2.2 Calculation of u_{O_2}

As shown in Fig. 2c, let C = (NW, NE, SW, SE) denotes four neighborhood pixels of damaged pixel O_2(i.e., mirror point of O), and its corresponding four midway points are C' = (nw, ne, sw, se), and h denotes the grid size. As a calculation method of Fig. 2b, we can calculate the pixel O_2 in Fig. 2c:

Let $\bar{v}=(v^1,v^2)=\dfrac{\nabla u}{|\nabla u|}$. Therefore, the divergence is discretized by central differencing:

$$\mathrm{div}(\bar{v})=\frac{\partial\bar{v}^1}{\partial x}+\frac{\partial\bar{v}^2}{\partial y}\approx\frac{v^1_{\mathrm{ne}}-v^1_{\mathrm{sw}}}{\sqrt{2}h}+\frac{v^2_{\mathrm{nw}}-v^2_{\mathrm{se}}}{\sqrt{2}h} \tag{7}$$

where h is always taken to be 1. Next, further approximations of O_2 can be obtained from the midway points. Take the midway point ne for example,

$$v^1_{\mathrm{ne}}=\frac{1}{|\nabla u_{\mathrm{ne}}|}\left[\frac{\partial u}{\partial x}\right]_{\mathrm{ne}}\approx\frac{1}{|\nabla u_{\mathrm{ne}}|}\frac{u_{\mathrm{NE}}-u_{O_2}}{\sqrt{2}h} \tag{8}$$

$$|\nabla u_{\mathrm{ne}}|=\frac{1}{\sqrt{2}h}\times\sqrt{(u_{\mathrm{NE}}-u_{O_2})^2+[(u_{N'}+u_{\mathrm{NW}}-u_{\mathrm{SE}}-u_{E'})/4]^2} \tag{9}$$

So, Eq.(7)-Eq.(9) are introduced into Eq.(2), and Eq. (2) is presented by the following equation:

$$\frac{u_{O_2}-u_{\mathrm{NE}}}{2h^2|\nabla u_{\mathrm{ne}}|}+\frac{u_{O_2}-u_{\mathrm{SW}}}{2h^2|\nabla u_{\mathrm{sw}}|}+\frac{u_{O_2}-u_{\mathrm{NW}}}{2h^2|\nabla u_{\mathrm{nw}}|}+\frac{u_{O_2}-u_{\mathrm{SE}}}{2h^2|\nabla u_{\mathrm{se}}|}+\lambda_e(O)(u_{O_2}-u_O^0)=0 \tag{10}$$

that is:

$$\frac{1}{2h^2}\sum_{\substack{X\in C\\ x\in C'}}\frac{1}{|\nabla u_x|}(u_{O_2}-u_X)+\lambda_e(O)(u_{O_2}-u_O^0)=0 \tag{11}$$

Then, the grid size h=1 is introduced into Eq.(11), and Eq.(11) is simplified as following:

$$u_{O_2}=\frac{\frac{1}{2}\sum\limits_{\substack{X\in C\\ x\in C'}}\frac{u_X}{|\nabla u_x|}+\lambda_e(O)u_O^0}{\frac{1}{2}\sum\limits_{\substack{X\in C\\ x\in C'}}\frac{1}{|\nabla u_x|}+\lambda_e(O)} \tag{12}$$

2.3 Weighted mean of u_{O_1} and u_{O_2}

Taking into account the different distance among O and B, C, the final pixel value is the weighted mean of u_{O_1} and u_{O_2}

$$u_O=\alpha u_{O_1}+(1-\alpha)u_{O_2} \tag{13}$$

As shown in Fig. 2a, u_{O_1} is calculated from B = (N, S, W, E), and u_{O_2} is calculated form C = (NW, NE, SW, SE), which are on diagonal. Therefore, the distance ratio is 1:$\sqrt{2}$, and $\alpha=\dfrac{\sqrt{2}}{1+\sqrt{2}}$, so the final pixel value u_O is given by:

$$u_O=\frac{\sqrt{2}}{1+\sqrt{2}}u_{O_1}+\frac{1}{1+\sqrt{2}}u_{O_2} \tag{14}$$

After adopting the Gauss-Jacobi iteration scheme for linear systems, Eq. (14) can be updated by:

$$u_O^{(n)}=\frac{\sqrt{2}}{1+\sqrt{2}}u_{O_1}^{(n)}+\frac{1}{1+\sqrt{2}}u_{O_2}^{(n)} \tag{15}$$

3 Experiments and results analysis

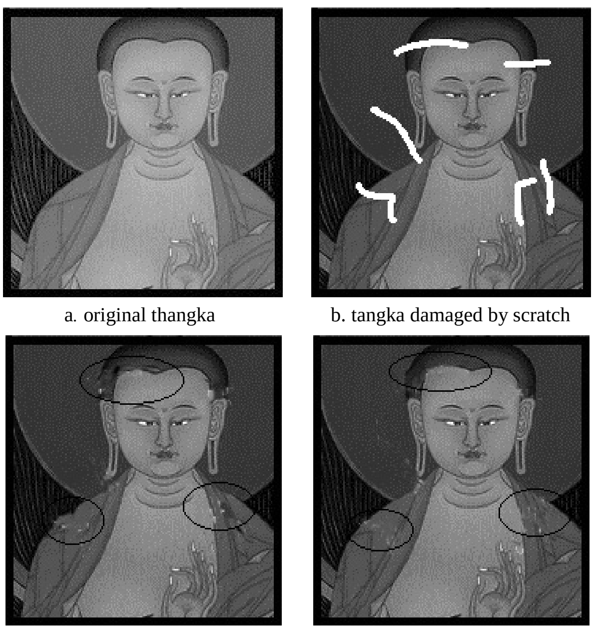
In the experiments herein reported, we use a lot of pictures such as Thangkas and murals to show that the proposed approach can inpaint the damaged region without extra cost. The inpainting experiments by using the TV model and the proposed algorithm were carried out in Matlab 7.0 on PC. Most image inpainting algorithms based on PDE always use a peak signal-to-noise ratio to evaluate the accuracy of the inpainting results, which is given by:

$$\mathrm{PSNR} = 10\lg\left(\frac{255^2}{\frac{1}{M \times N}\sum_{i=0}^{M-1}\sum_{j=0}^{N-1}(U(i,j)-U_1(i,j))^2}\right) \quad (16)$$

where, U is the original image, U_1 is the inpainted image, and M and N are the number of rows and columns of the image, respectively. From Eq.(16), we can see that the larger the PSNR value is, the smaller the difference, between original image and inpainted image would be. In addition, we can compare the iteration number to evaluate the performance of above mentioned.

3.1 Inpainting Results Comparison between TV Model and the Proposed Algorithm

Fig. 3 is the inpainting results comparison on scratch, and Figure 4 is the comparison of PSNR and iterations. As shown in Fig. 3, from the subjective aspect, the proposed algorithm can achieve better inpainting effect compared with the original TV algorithm. At the same time, in the case of similar iterations, the image inpainted by double cross TV algorithm is closer to original image than the one by original TV algorithm as shown in Fig. 4.



a. original thangka　b. tangka damaged by scratch

c. tangka inpainted by original TV algorithm　d. tangka inpainted by double cross TV algorithm

Fig. 3　The inpainting results comparison on scratch

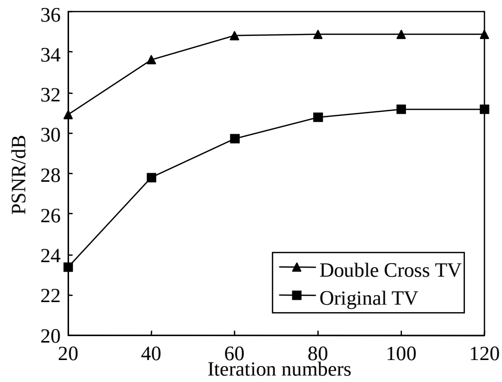
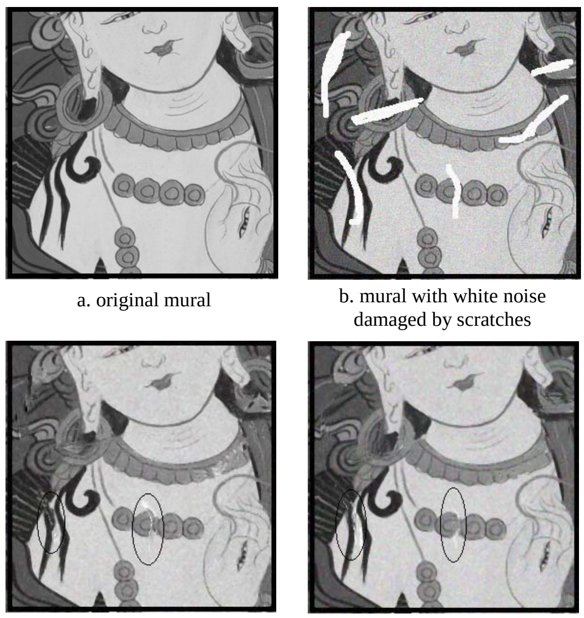


Fig. 4　Diagram of PSNR and iterations

3.2 Removal Effect of Noise and Scratches

The proposed algorithm can remove noise and scratches at the same time, and the removal results are shown in Fig. 5 and Fig 6. Fig. 5a is the original mural, Fig.5b is the damaged mural with white noise, Fig.5c is the mural inpainted by the original TV algorithm, and Fig.5d is the mural inpainted by the double cross TV algorithm. According to Fig. 6, the proposed algorithm can effectively inpaint damaged image by noise and scratches.



a. original mural　b. mural with white noise damaged by scratches

c. mural inpainted by original TV algorithm　d. mural inpainted by double cross TV algorithm

Fig. 5　The inpainting results comparison on scratch and white noise

3.3 Inpainting Effect of Color Image with the Proposed Algorithm

The proposed algorithm can be easily extended into color image inpainting.

To color image, the inpainting process is carried out in different color channels, such as R, G, B channel. Then, the inpainting results in different color channels are composed into final inpainting result. Fig. 7a is the original color mural, Fig.7b is the color mural damaged by scratch, Fig.7c is the color mural inpainted by the original TV algorithm, and Fig.7d is the color mural inpainted by the double cross TV algorithm.

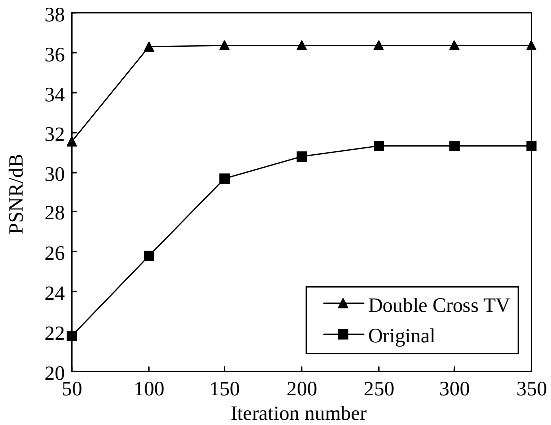


Fig. 6 Diagram of PSNR and iterations

a. original color mural

b. color mural damaged by scratch

c. color mural inpainted by original TV algorithm

d. color mural inpainted by double cross TV algorithm

Fig. 7 The color image inpainting results comparison on scratch

It can be seen from the illustration that the proposed algorithm can effectively inpaint damaged color image.

4 Concluding remarks

The proposed double cross TV algorithm divides 8 neighborhood pixels into 2 groups, and computes pixel value of damaged pixel by using 4 neighborhood pixels in each group respectively. Then, the final inpainting pixel value is the weighted mean of these two pixel values. The experimental results show that better inpainting results can be achieved with much less computation time.

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