

Approximate Trace and Singleton Failures Equivalences for Event Structures

LI Shao-rong¹ and WANG Chao²

(1. School of Optoelectronic Information, University of Electronic Science and Technology of China Chengdu 610054;

2. The 6th Research Institute of China Electronics Corporation Haidian Beijing 100083)

Abstract Establishing system equivalences for event structures such as trace and testing equivalences, require the observed actions to be exactly identical. However, an accurate measurement is impossible when interacting with the physical world, hence exact equivalence is restrictive and not robust. Using Baire metric, this paper proposes a generalized framework of event structures approximation by developing the notions of approximate interleaving (trace and singleton failures) equivalence and approximate step (trace and singleton failures) equivalence. The proposed framework captures the traditional exact equivalence as a special case. These approximate equivalences satisfy the transitive property, consequently, they can be successively used in event structures approximations.

Key words approximate equivalence; event structures; singleton failures; step; trace

事件结构的近似迹与单元失败等价

李绍荣¹, 王超²

(1. 电子科技大学光电信息学院 成都 610054; 2. 中国电子信息产业集团有限公司第六研究所 北京 海淀区 100083)

【摘要】迹等价和测试等价等事件结构上的系统等价关系都立足于观测集上的精确相等,然而精确相等需要的精确度量在现实物理世界里是无法实现的,因此精确等价的应用是非常有限的,并且精确等价关系不具有鲁棒性。为了克服以上缺陷,该文通过使用Baire度量,提出了包含近似交织迹等价、近似交织单元失败等价、近似步骤迹等价和近似步骤单元失败等价的事件系统近似等价框架。这些近似等价具有以下几种良好性质: 1) 近似等价涵盖了精确等价,即传统精确等价是近似等价的特例。2) 近似等价关系具有传递性,可以连续使用。

关键词 近似等价; 事件结构; 单元失败语义; 步进语义; 迹语义

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Event structures^[1]are nowadays widely recognized models of true concurrency^[2-3]. They are closely related to Petri Nets, but mathematically more tractable.

Interleaving^[4-5] and step^[6-7] are two discriminating views of event structures, which respectively use single actions and steps (i.e., multi-sets of simultaneously occurring actions) as the basic execution unit. In Ref.[7], step is applied to give a non-interleaving semantics to Communicating Sequential Processes (CSP), called step failure semantics. Step gives a more precise account of concurrency than interleaving, e.g., the systems $a|b$

and $ab+ba$ are distinguished.

Van Glabbeek studied various semantic equivalences in the linear time-branching time spectrum^[8], this paper only focuses on two canonical semantics, trace semantics and singleton failures (SF) semantics, which are respectively representatives of linear time semantics and branching time semantics.

Currently, established system equivalences for event structures, such as trace and testing equivalences, require the observed actions to be exactly identical. However, accurate measurement is impossible in the case of physical world, therefore, exact equivalence is restrictive and not robust.

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Biography: LI Shao-rong was born in 1964, and his research interests include signal processing, photoelectric detection.

作者简介: 李绍荣(1964-), 男, 教授, 主要从事信号处理和光电测控方面的研究。

The notion of distance between the original event structure and the target event structure is much more adequate in this context^[9]. Instead of requiring the observed actions of the two event structures to be equal, the distance between them is only required to remain bounded by some parameter called precision of the approximation.

In our earlier work^[10], we present a generalized framework of transition system approximation by developing the notions of approximate language equivalence and approximate SF equivalence. Now, we extend the obtained results to event structures. In this paper, using Baire metric^[11], we propose a generalized framework of event structures approximation including approximate interleaving (trace and SF) equivalence and approximate step (trace and SF) equivalence. Note that the proposed framework captures the traditional exact equivalence as a special case.

The unique feature of these approximate equivalences is that they satisfy the transitive property. Hence, for an original complex event structure, we can successively use these approximate equivalences to obtain an equivalent event structure with lesser complexity.

1 Preliminaries

The following notations are used throughout the paper. N denotes the set of all natural numbers (including zero), R denotes the set of all real numbers, and \emptyset denotes the empty set. For $l, l' \in R$, $\max(l, l')$ denotes the larger one between l and l' . For a set π , $|\pi|$ denotes the number of elements of π , π^∞ denotes both finite and infinite sequences over π . A closed interval is denoted with square brackets. For example $[0, 1]$ means greater than or equal to 0 and less than or equal to 1. The infimum (or supremum) of a subset S of real numbers is denoted by $\inf(S)$ (or $\sup(S)$) and is defined to be the largest (or least) real number which is smaller (or greater) than or equal to all numbers in S .

Definition 1 A metric on a set π is a function (called the distance function) $d: \pi \times \pi \rightarrow [0, +\infty]$ such that $\forall x, y, z \in \pi$,

- 1) $d(x, y) = 0$ iff (if and only if) $x = y$ (identity of

indiscernibles);

- 2) $d(x, y) = d(y, x)$ (symmetry);

- 3) $d(x, z) \leq d(x, y) + d(y, z)$ (triangular inequality).

Definition 2 A pseudo-metric on a set π is a function $d: \pi \times \pi \rightarrow [0, +\infty]$ such that:

$$\forall x, y, z \in \pi, d(x, x) = 0, d(x, y) = d(y, x), d(x, z) \leq d(x, y) + d(y, z)$$

A metric space is an ordered pair (π, d) where π is a set and d is a metric on π . A pseudo-metric space is an ordered pair (π, d) where π is a set and d is a pseudo-metric on π . Note that, for pseudo-metric space, the distance between two distinct points can be zero, in other words, $d(x, y) = 0$ does not necessarily imply that $x = y$. And this is critical to Hausdorff distance (see below).

Definition 3 Baire metric $d_B: \pi^\infty \times \pi^\infty \rightarrow [0, 1]$ is defined as follows:

$$d_B(\rho_1, \rho_2) = \begin{cases} 0, & \rho_1 = \rho_2 \\ 2^{-l}, & \text{otherwise} \end{cases}$$

where $l \in N$ is the length of the longest common prefix of ρ_1 and ρ_2 .

For brevity, if $\rho_1 = \rho_2$ we define $l = +\infty$ no matter what the length of ρ_1 .

It is easy to verify that d_B satisfies all conditions of metric. Hence (π^∞, d_B) is a metric space and $d_B(\rho_1, \rho_2)$ measures the similarity between ρ_1 and ρ_2 , the lower the value, the more similar the two sequences.

Definition 4 π_1 and π_2 are two non-empty subsets of π^∞ while (π^∞, d_B) is a metric space.

Then their Hausdorff distance is defined as follows:

$$d_H(\pi_1, \pi_2) = \max(\sup \inf d_B(\rho_1, \rho_2), \sup \inf d_B(\rho_1, \rho_2))$$

Informally, for metric space, two non-empty subsets are close in their Hausdorff distance, if every point of either subset is close to some point of the other subset. In other words, Hausdorff distance d_H is the greatest of all the distances from a point in one subset to the closest point in the other subset.

It is easy to prove that $d_H(\pi_1, \pi_2) = 0$ iff $cl(\pi_1) = cl(\pi_2)$ where $cl(\pi_1)$ and $cl(\pi_2)$ denote the topological closures of π_1 and π_2 respectively. Moreover, d_H is a pseudo-metric.

2 Event Structures

We use event structures equipped with a

labelling function l to obtain labelled event structures. An event E such that $l(e)=a$ should be thought of as an occurrence of an action a that the system may perform. In fact, an action identifies the external aspect of an event. Since different event occurrences can ‘look the same’ from an external observer, the labelling function may associate the same action with different events. Moreover, since some events may be not externally observable, they associate to a distinguished symbol τ which is used to denote the invisible or internal actions. A denotes the set of observable actions.

Definition 5 A labelled event structure is a quadruple $E=(E, \preceq, \#, l)$, where E is a countable set of events; \preceq is a partial ordering relation on E called the causality relation, such that $\forall e \in E, \downarrow e = \{e' \in E \mid e' \preceq e\}$ ($\downarrow e$ is the left closure of E) is finite (every event in E is finitely preceded); $\#$ is a symmetric, irreflexive relation on E called the conflict relation, such that $\forall e, e', e'' \in E, e \preceq e'$ and $e \# e''$ implies $e' \# e''$ (conflict is hereditary); $l: E \rightarrow A \cup \tau$ is a labelling function.

If no confusion arises, event structure will be referred to as labelled event structure.

The concurrency relation is defined by $\sim_E = (E \times E) - (\preceq \cup \preceq^{-1} \cup \#)$, where symbol $-$ is the set minus operator and \preceq^{-1} is the inverse relation of \preceq . The subscript restricts the domain of concurrency relation to E . The following example shows that the subscript of concurrency relation is necessary.

Example 1

Consider two event structures E_1 and E_2 in Fig.1, where partial ordering relation is depicted by arrow (omitting those derivable by transitivity) and conflict relation is depicted by dashed line (omitting those derivable by conflict heredity). The events of E_1 are e_1, e_2, e_3 where $e_1 \preceq e_2$ and $e_1 \# e_3$. E_2 is the restriction of E_1 to $\{e_2, e_3\}$. Then, we have $e_2, e_3 \in \sim_{E_2}$. Since conflict is hereditary, $e_2, e_3 \notin \sim_{E_1}$.

An execution of an event structure is a configuration. The notion of a configuration captures the intuition that an event can occur only after all the events that lie in its past have occurred. Formally, the definition is as follows.

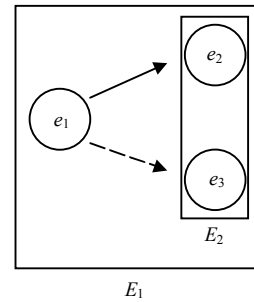


Fig.1 The subscript of concurrency relation

Definition 6 A set $C \subseteq E$ is called a configuration of E iff

- 1) C is conflict-free in E , i.e., $\forall e, e' \in C$ not $(e \# e')$;
- 2) C is left-closed in E , i.e., $\forall e \in C, \forall e' \in E, e' \preceq e$ implies $e' \in C$;
- 3) $|C|$ is finite.

The set of all configurations of E is denoted by $C(E)$. Since $\emptyset \subseteq E$ is conflict-free and left-closed, $|\emptyset|$ is finite, $\emptyset \in C(E)$.

The semantics of event structures is typically given as partially ordered sets^[1], or labelled partially ordered sets^[12], an alternative semantical view on event structures is to consider configurations and the transitions between them.

Definition 7 A labelled transition system is a tuple $(Q, Q^0, \Sigma, \rightarrow)$, where Q is a (possibly infinite) set of states; $Q^0 \subseteq Q$ is a (possibly infinite) set of initial states; Σ is a (possibly infinite) set of action labels; $\rightarrow \subseteq Q \times \Sigma \times Q$ is a set of transitions.

A configuration C represents a state of E , namely, the state in which the events in C have occurred. Hence $C(E)$ is the set of states. The initial state of E is the state where no action has been occurred yet. Hence \emptyset is the set of initial states.

Example 2

A trivial event structure E_3 is shown in Fig.2, and the labelling function satisfies that $l(e_1)=a, l(e_2)=b$ and $l(e_3)=c$. $C(E_3) = \{\emptyset, \{e_1\}, \{e_1, e_2\}, \{e_1, e_3\}, \{e_1, e_2, e_3\}\}$.

Interleaving and step are respectively used in sequential nondeterministic event structures and concurrent deterministic event structures, where sequential means that every two events in the event structure are either causally related or conflicting ($\sim_E = \emptyset$), deterministic means that the event structure does not contain any pair of conflicting events ($\# = \emptyset$).

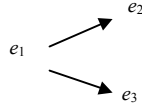


Fig.2 Event structure E_3

3 Interleaving Equivalences

In this section, we define approximate trace equivalence and approximate SF equivalence based on interleaving semantics. In the interleaving semantics, an event structure progresses through a sequence of single actions. An occurrence of a single action $a \in A \cup \tau$ leads to the state changing from configuration C_1 to configuration C_2 , denoted by $C_1 \xrightarrow{a} C_2$, iff $C_1 \subset C_2$ and $\exists e \in E$ such that $C_2 - C_1 = \{e\}$, $l(e) = a$. Let \rightarrow_I denote the set of all the leading relations. Hence:

$$\rightarrow_I = \{(C_1, a, C_2) \in C(E) \times A \cup \tau \times C(E) \mid C_1 \xrightarrow{a} C_2\}$$

Write ε for the empty sequences of actions, a, δ for the concatenation of $a \in A \cup \tau$ and $\delta \in (A \cup \tau)^\infty$. The generalized transition $\xrightarrow{\delta}_I$ is defined recursively by:

$$\begin{cases} C \xrightarrow{\varepsilon}_I C \\ C \xrightarrow{a\delta}_I C'' \text{ is equal to by definition } C \xrightarrow{a}_I C' \xrightarrow{\delta}_I C'' \end{cases}$$

It is straightforward to show that $(C(E), \phi, A \cup \tau, \rightarrow_I)$ forms a labelled transition system.

Definition 8 $TS_I(E) = (C(E), \phi, A \cup \tau, \rightarrow_I)$ is called the interleaving view of E .

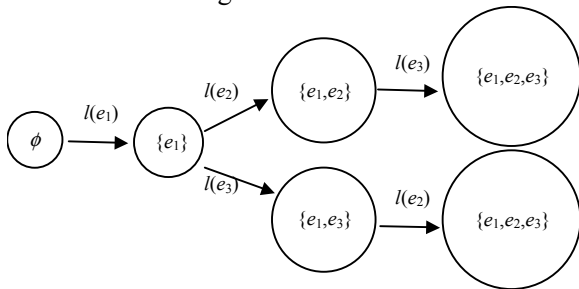


Fig.3 The interleaving view of E_3

It is convenient to describe interleaving view by directed graphs. Each state is represented by a circle, the corresponding configuration is shown inside the circle, and a transition from one state to another is indicated by an arrow with a label.

Example 3

Consider again E_3 from Example 2, we denote the interleaving view of E_3 by Fig.3.

3.1 Approximate Interleaving Trace Equivalence

$\delta \in (A \cup \tau)^\infty$ is an interleaving trace of E if $\exists C \in C(E)$ such that $\phi \xrightarrow{\delta}_I C$. Let $T_I(E)$ denote the set of all interleaving traces of E . Note that repeated elements are counted only once in $T_I(E)$.

The traditional exact interleaving trace equivalence is given as follows.

Definition 9 Two event structures E_1 and E_2 are interleaving trace equivalent, in other words, there is an interleaving trace equivalence relation between E_1 and E_2 , notation $E_1 =_{it} E_2$, iff $T_I(E_1) = T_I(E_2)$.

Definition 10 The distance between interleaving traces δ_1 and δ_2 is naturally defined as their Baire distance, $d_{it}(\delta_1, \delta_2) = d_B(\delta_1, \delta_2)$.

Proposition 1

$$\forall \delta_j \in T_I(E_j), j=1,2,3 \\ d_{it}(\delta_1, \delta_3) \leq \max(d_{it}(\delta_1, \delta_2), d_{it}(\delta_2, \delta_3))$$

Proof If $\delta_1 = \delta_2$ or $\delta_2 = \delta_3$, then the right-hand side of the inequality is equal to the left-hand side. Hence the inequality trivially holds.

If $\delta_1 = \delta_3$, then the left-hand side of the inequality is equal to 0. Hence the inequality holds.

Otherwise $\delta_1 \neq \delta_2, \delta_2 \neq \delta_3$ and $\delta_1 \neq \delta_3$. Let $l \in N$ be the length of the longest common prefix of δ_1 and $\delta_2, l' \in N$ be the length of the longest common prefix of δ_2 and δ_3 . Without loss of generality, assume $l \leq l'$, then the length of the longest common prefix of δ_1 and δ_3 is no less than l .

$$d_{it}(\delta_1, \delta_3) \leq 2^{-l} = \max(2^{-l}, 2^{-l'}) = \max(d_{it}(\delta_1, \delta_2), d_{it}(\delta_2, \delta_3))$$

Hence $d_{it}(\delta_1, \delta_3) \leq \max(d_{it}(\delta_1, \delta_2), d_{it}(\delta_2, \delta_3))$ holds in all cases.

The following theorem shows that the definition of d_{it} is reasonable, i.e., the distance is a pseudo-metric.

Theorem 1 d_{it} is a pseudo-metric on the set of interleaving traces.

Proof $\forall \delta_j \in T_I(E_j), j=1,2,3$.

1) We prove that $d_{it}(\delta_1, \delta_2) = 0$ iff $\delta_1 = \delta_2$ (identity of indiscernibles).

$$d_{it}(\delta_1, \delta_2) = d_B(\delta_1, \delta_2) \geq 0, d_{it}(\delta_1, \delta_1) = d_B(\delta_1, \delta_1) = 0$$

2) We prove that $d_{it}(\delta_1, \delta_2) = d_{it}(\delta_2, \delta_1)$ (symmetry).

$d_{it}(\delta_1, \delta_2) = 2^{-l} = d_{it}(\delta_2, \delta_1)$, where $l = +\infty$ or $l \in N$ is the length of the longest common prefix of δ_1 and δ_2 .

3) We prove that $d_{it}(\delta_1, \delta_3) \leq d_{it}(\delta_1, \delta_2) + d_{it}(\delta_2, \delta_3)$

(triangular inequality).

$$d_{ii}(\delta_1, \delta_3) \leq \max(d_{ii}(\delta_1, \delta_2), d_{ii}(\delta_2, \delta_3)) \leq d_{ii}(\delta_1, \delta_2) + d_{ii}(\delta_2, \delta_3) \quad (\text{Proposition 1})$$

Furthermore, the pseudo-metric d_{ii} on the set of interleaving traces naturally induces an interleaving trace distance between two event structures.

Definition 11 The interleaving trace distance between E_1 and E_2 is defined as follows:

$$d_{ii}(E_1, E_2) = d_H(T_I(E_1), T_I(E_2))$$

Since Hausdorff distance d_H is a pseudo-metric, the definition of d_{ii} is reasonable. Event structures together with the interleaving trace distance form a

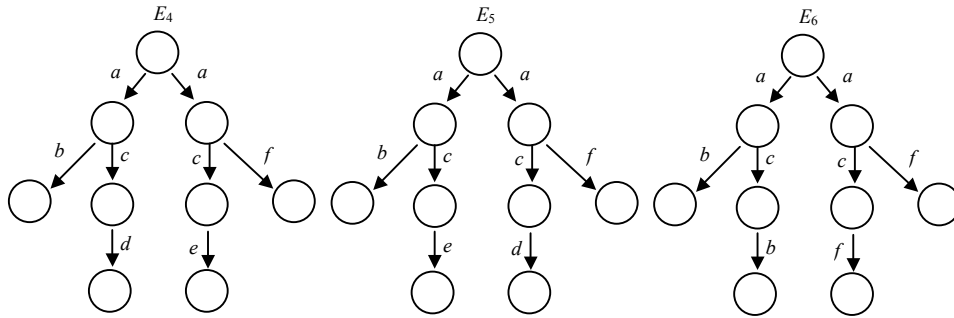


Fig.4 A complex case

$$T_I(E_4) = T_I(E_5) = \{a, ab, ac, acd, ace, af\}$$

$$T_I(E_6) = \{a, ab, ac, acb, acf, af\}$$

$d_{ii}(E_4, E_5) = 0$ intuitively means that for any execution of E_4 (or E_5), we can find an execution of E_5 (or E_4) such that the two executions are indistinguishable to an external observer.

$d_{ii}(E_5, E_6) = 2^{-2}$ intuitively means that for any execution of E_5 (or E_6), we can find an execution of E_6 (or E_5) such that from the view of an external observer, the two executions share a common prefix whose length is at least 2, in other words, they are indistinguishable from the first 2 actions.

One of the great advantages of having metric structure on event structures is that the distance enables us to use quantitative approximation.

Definition 12 Two event structures E_1 and E_2 are approximate interleaving trace equivalent with the precision $\xi \geq 0$, in other words, there is an approximate interleaving trace relation \approx_{ii}^ξ between E_1 and E_2 , notation $E_1 \approx_{ii}^\xi E_2$, iff $d_{ii}(E_1, E_2) \leq \xi$.

It is intuitive that the lower the value of ξ , the higher the degree to which \approx_{ii}^ξ is an interleaving trace relation. It should be emphasized that two event structures are approximate interleaving trace

pseudo-metric space.

The intuitive meaning of the interleaving trace distance is as follows. For any interleaving trace of E_1 (or E_2), we can find an interleaving trace of E_2 (or E_1) such that the distance between them remains bounded by $d_{ii}(E_1, E_2)$.

For clarity, we omit the configuration inside the circle in the following directed graphs.

Example 4

Consider the interleaving views of E_4 , E_5 and E_6 in Fig.4, where the first two directed graphs are taken from counterexample 5 in Ref.[8].

equivalent with the precision $\xi=0$ is also known as they are exact interleaving trace equivalent. In other words, the traditional exact interleaving trace equivalence is a special case of approximate interleaving trace equivalence.

Theorem 2 \approx_{ii}^ξ ($\xi \geq 0$) is an equivalence relation on the set of event structures.

Proof Let E_1, E_2, E_3 be arbitrary event structures.

1) We prove that \approx_{ii}^ξ is a reflexive relation.

$$d_{ii}(E_1, E_1) = d_H(T_I(E_1), T_I(E_1)) = 0 \leq \xi, \text{ hence } E_1 \approx_{ii}^\xi E_1$$

2) We prove that \approx_{ii}^ξ is a symmetric relation.

$$d_{ii}(E_1, E_2) = d_H(T_I(E_1), T_I(E_2)) = d_H(T_I(E_2), T_I(E_1)) = d_{ii}(E_2, E_1), \text{ hence } E_1 \approx_{ii}^\xi E_2 \text{ implies that } E_2 \approx_{ii}^\xi E_1.$$

3) We prove that \approx_{ii}^ξ is a transitive relation.

Assume $E_1 \approx_{ii}^\xi E_2$ and $E_2 \approx_{ii}^\xi E_3$.

$$\xi \geq d_{ii}(E_1, E_2) = d_H(T_I(E_1), T_I(E_2)) =$$

$$\max \left(\sup_{\delta_1 \in T_I(E_1)} \inf_{\delta_2 \in T_I(E_2)} d_{ii}(\delta_1, \delta_2), \sup_{\delta_2 \in T_I(E_2)} \inf_{\delta_1 \in T_I(E_1)} d_{ii}(\delta_1, \delta_2) \right) \quad (1)$$

$$\xi \geq d_{ii}(E_2, E_3) = d_H(T_I(E_2), T_I(E_3)) =$$

$$\max \left(\sup_{\delta_2 \in T_I(E_2)} \inf_{\delta_3 \in T_I(E_3)} d_{ii}(\delta_2, \delta_3), \sup_{\delta_3 \in T_I(E_3)} \inf_{\delta_2 \in T_I(E_2)} d_{ii}(\delta_2, \delta_3) \right) \quad (2)$$

$\forall \delta_1 \in T_l(E_1)$, by Eq.(1), $\exists \delta_2 \in T_l(E_2)$ such that $d_{ii}(\delta_1, \delta_2) \leq \zeta$. By Eq.(2), $\exists \delta_3 \in T_l(E_3)$ such that $d_{ii}(\delta_2, \delta_3) \leq \zeta$. By Proposition 1, $d_{ii}(\delta_1, \delta_3) \leq \max(d_{ii}(\delta_1, \delta_2), d_{ii}(\delta_2, \delta_3)) \leq \zeta$, hence:

$$\inf_{\delta_3 \in T_l(E_3)} d_{ii}(\delta_1, \delta_3) \leq \zeta$$

Since this formula holds for all $\delta_1 \in T_l(E_1)$,

$$\sup_{\delta_1 \in T_l(E_1)} \inf_{\delta_3 \in T_l(E_3)} d_{ii}(\delta_1, \delta_3) \leq \zeta$$

Similarly,

$$\begin{aligned} & \sup_{\delta_3 \in T_l(E_3)} \inf_{\delta_1 \in T_l(E_1)} d_{ii}(\delta_1, \delta_3) \leq \zeta \\ & d_{ii}(E_1, E_3) = d_H(T_l(E_1), T_l(E_3)) \\ & \max \left(\sup_{\delta_1 \in T_l(E_1)} \inf_{\delta_3 \in T_l(E_3)} d_{ii}(\delta_1, \delta_3), \right. \\ & \left. \sup_{\delta_3 \in T_l(E_3)} \inf_{\delta_1 \in T_l(E_1)} d_{ii}(\delta_1, \delta_3) \right) \leq \zeta \end{aligned}$$

Example 5

Consider again the event structures in Example 4. Since $d_{ii}(E_4, E_5) < 0.25$, $E_4 \approx_{ii}^{0.25} E_5$. Since $d_{ii}(E_5, E_6) = 0.25$, $E_5 \approx_{ii}^{0.25} E_6$. By Theorem 2, $E_4 \approx_{ii}^{0.25} E_6$.

3.2 Approximate Interleaving Singleton Failures Equivalence

The definition of singleton failure pair presented here is that of Ref.[8]. It differs from that of Ref.[13] in one important respect, the refusal set of singleton failure pair must contain one element, instead of at most one element. In our earlier work^[14], a revised version of singleton failure pair is proposed to simplify SF equivalence checking. And the RSF algorithm gains in efficiency in terms of speed than the traditional SF algorithm by reducing redundancy elements, while retaining the same information content.

The set of next interleaving actions of a configuration C is defined as follows:

$$N_l(C) = \{a \in A \cup \tau \mid \exists C' \in C(E) \text{ such that } \xrightarrow{a}_l C'\}$$

The set of possible continuations of an interleaving trace δ of E is defined as follows:

$$C_l(E, \delta) = \{a \in A \cup \tau \mid \exists C, C' \in C(E) \text{ such that}$$

$$\phi \xrightarrow{a}_l C, C \xrightarrow{a}_l C'\}$$

$|\delta|$ denotes the length of δ . If δ has infinite length, then $|\delta| = +\infty$. In order to let interleaving trace and interleaving singleton failure pair have the same form, δ is identified with a pair (δ, ε) in this paper.

Definition 13 $(\delta, \pi) \in (A \cup \tau)^\infty \times (A \cup \tau \cup \varepsilon)$ is an interleaving RSF pair of E if $\exists C \in C(E)$ such that

$$\phi \xrightarrow{\delta}_l C,$$

$$\begin{cases} \pi = \varepsilon \wedge C_l(E, \delta) = \phi \\ \pi = \varepsilon \wedge |\delta| = +\infty \\ \pi \in C_l(E, \delta) \wedge \pi \notin N_l(C) \end{cases}$$

Let $F_l(E)$ denote the set of all interleaving RSF pairs of E .

The traditional exact interleaving SF equivalence is given as follows.

Definition 14 Two event structures E_1 and E_2 are interleaving SF equivalent, in other words, there is an interleaving SF equivalence relation between E_1 and E_2 , notation $E_1 =_{fi} E_2$, iff $F_l(E_1) = F_l(E_2)$.

Definition 15 The distance $d_{fi}: F_l(E_1) \times F_l(E_2) \rightarrow [0, 1]$ between interleaving RSF pairs (δ_1, π_1) and (δ_2, π_2) is defined as follows:

$$d_{fi}((\delta_1, \pi_1), (\delta_2, \pi_2)) = \begin{cases} 0, & |\delta_1| = |\delta_2| \text{ and } \pi_1 = \pi_2 \\ 2^{-l}, & |\delta_1| = |\delta_2| \text{ and } \pi_1 \neq \pi_2 \\ 2^{-l}, & \text{otherwise} \end{cases}$$

where l is the length of the longest common prefix of δ_1 and δ_2 , L is the maximum length of all interleaving traces of E_1 and E_2 . $l = \max(|\delta_1|, |\delta_2|)$,

$$L = \max(\max_{\delta_1 \in T_l(E_1)} |\delta_1|, \max_{\delta_2 \in T_l(E_2)} |\delta_2|)$$

Hence, the definition insures that if $\delta_1 \neq \delta_2$, then $d_{fi}((\delta_1, \pi_1), (\delta_2, \pi_2)) = 2^{-l} > 2^{-L} = d_{fi}((\delta_1, \pi_1), (\delta_2, \pi_1))$.

Proposition 2

$$\forall (\delta_j, \pi_j) \in F_l(E_j) \quad j=1, 2, 3$$

$$d_{fi}((\delta_1, \pi_1), (\delta_3, \pi_3)) \leq$$

$$\max(d_{fi}((\delta_1, \pi_1), (\delta_2, \pi_2)), d_{fi}((\delta_2, \pi_2), (\delta_3, \pi_3)))$$

The proof is omitted since it is similar to the proof of Proposition 1.

Theorem 3 d_{fi} is a pseudo-metric on the set of interleaving RSF pairs.

The proof is omitted since it is similar to the proof of Theorem 1.

Furthermore, the pseudo-metric d_{fi} on the set of interleaving RSF pairs naturally induces an interleaving SF distance between two event structures.

Definition 16 The interleaving SF distance between E_1 and E_2 is defined as follows:

$$d_{fi}(E_1, E_2) = d_H(F_l(E_1), F_l(E_2))$$

Since Hausdorff distance d_H is a pseudo-metric, the definition of d_{fi} is reasonable. Event structures together with the interleaving SF distance form a pseudo-metric space.

Definition 17 Two event structures E_1 and E_2 are approximate interleaving SF equivalent with the precision $\xi \geq 0$, in other words, there is an approximate interleaving SF relation \approx_{fi}^ξ between E_1 and E_2 , notation $E_1 \approx_{fi}^\xi E_2$, iff $d_{fi}(E_1, E_2) \leq \xi$.

It is intuitive that the lower the value of ξ , the higher the degree to which \approx_{fi}^ξ is an interleaving SF relation. It should be emphasized that two event structures are approximate interleaving SF equivalent with the precision $\xi=0$ is also known as they are exact interleaving SF equivalent.

Theorem 4 \approx_{fi}^ξ ($\xi \geq 0$) is an equivalence relation on the set of event structures.

The proof is omitted since it is similar to the proof of Theorem 2.

4 Step Equivalences

In this section, we define approximate trace equivalence and approximate SF equivalence based on step semantics. In the step semantics, an event structure progresses through a sequence of steps (i.e., multi-sets of simultaneously occurring actions). The multi-set is needed, since concurrent events may have the same label.

A multi-set $f:A \rightarrow N$ is a function, $\forall a \in A, f(a)$ is the multiplicity of a in the multi-set f .

Similarly to finitely branching labelled transition systems, which in each state has only finitely many possible ways to proceed, a multi-set f is finite iff for $X = \{a \in A \mid f(a) \neq 0\}$, $|X|$ is finite.

μA denotes the set of all finite multi-sets over A . Since the multi-set of simultaneously occurring actions which are all invisible (called invisible multi-set) is finite, invisible multi-set belongs to μA .

The operations on finite multi-sets are the usual operations on functions.

Definition 18 The equality operator for finite multi-sets is defined as follows:

$$f, g \in \mu A, f = g \text{ iff } \forall a \in A, f(a) = g(a)$$

An occurrence of a finite multi-set f leads to the state changing from configuration C_1 to configuration C_2 , denoted by $C_1 \xrightarrow{f} C_2$, iff $C_1 \subset C_2$ and $\exists X \subseteq E$ such that: X is conflict-free in E ; $\forall e, e' \in X, e \neq e'$, we have $l(e), l(e') \in A$ implies $(e, e') \in \sim_E$; X defines a multi-set of

actions equal to f , i.e., $\forall a \in A, f(a) = |\{e \in X \mid l(e) = a\}|$; $X = C_2 - C_1$.

Let \rightarrow_s denote the set of all the leading relations. Hence $\rightarrow_s = \{(C_1, f, C_2) \in C(E) \times \mu A \times C(E) \mid C_1 \xrightarrow{f} C_2\}$.

The generalized transition $\xrightarrow{\theta}_s$ for $\theta \in \mu A^\infty$ is defined recursively by:

$$\begin{cases} C \xrightarrow{\varepsilon}_s C \\ C \xrightarrow{f, \theta}_s C'' \text{ is equal to by definition } C \xrightarrow{f}_s C' \xrightarrow{\theta}_s C'' \end{cases}$$

It is straightforward to show that $(C(E), \emptyset, \mu A, \rightarrow_s)$ forms a labelled transition system.

Definition 19 $T_S(E) = (C(E), \emptyset, \mu A, \rightarrow_s)$ is called the step view of E .

Example 6

Consider again E_3 from Example 2, we denote the step view of E_3 by Fig.5.

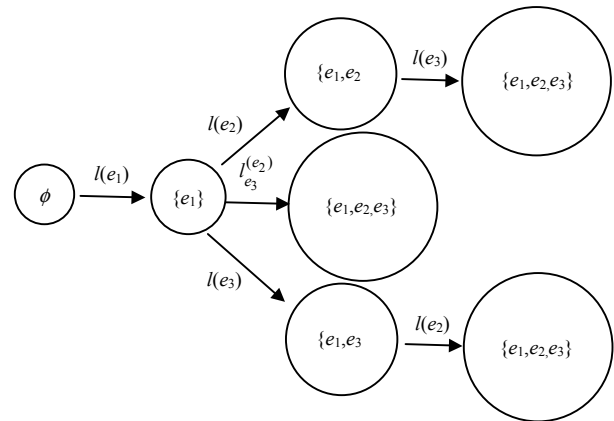


Fig.5 The step view of E_3

4.1 Approximate Step Trace Equivalence

$\theta \in \mu A^\infty$ is a step trace of E if $\exists C \in C(E)$ such that $\phi \xrightarrow{\theta}_s C$. Let $T_S(E)$ denote the set of all step traces of E .

Definition 20 Two event structures E_1 and E_2 are step trace equivalent, in other words, there is a step trace equivalence relation between E_1 and E_2 , notation $E_1 \approx_{st} E_2$, iff $T_S(E_1) = T_S(E_2)$.

With the help of the equality operator for finite multi-sets, we can define the distance between two step traces.

Definition 21 The distance between step traces θ_1 and θ_2 is naturally defined as their Baire distance, $d_{st}(\theta_1, \theta_2) = d_B(\theta_1, \theta_2)$.

Theorem 5 d_{st} is a pseudo-metric on the set of step traces.

Definition 22 The step trace distance between

E_1 and E_2 is defined as follows:

$$d_{ts}(E_1, E_2) = d_H(T_S(E_1), T_S(E_2))$$

Definition 23 Two event structures E_1 and E_2 are approximate step trace equivalent with the precision $\xi \geq 0$, in other words, there is an approximate step trace relation \simeq_{ts}^ξ between E_1 and E_2 , notation $E_1 \simeq_{ts}^\xi E_2$, iff $d_{ts}(E_1, E_2) \leq \xi$.

Theorem 6 \simeq_{ts}^ξ ($\xi \geq 0$) is an equivalence relation on the set of event structures.

4.2 Approximate step singleton failures equivalence

The set of next step actions of a configuration C is defined as follows:

$$N_S(C) = \{f \in \mu A \mid \exists C' \in C(E) \text{ such that } C \xrightarrow{f} C'\}$$

The set of possible continuations of a step trace θ of E is defined as follows:

$$C_S(E, \theta) =$$

$$\{f \in \mu A \mid \exists C, C' \in C(E) \text{ such that } \theta \xrightarrow{f} C, C \xrightarrow{f} C'\}$$

Definition 24 $(\theta, v) \in \mu A \times \mu A$ is a step RSF pair of E if $\exists C \in C(E)$ such that $\theta \xrightarrow{f} C$,

$$\begin{cases} v = \varepsilon \wedge C_S(E, \theta) = \emptyset \\ v = \varepsilon \wedge |\theta| = +\infty \\ v \in C_S(E, \theta) \wedge v \notin N_S(C) \end{cases}$$

Let $F_S(E)$ denote the set of all step RSF pairs of E .

Definition 25 Two event structures E_1 and E_2 are step SF equivalent, notation $E_1 =_{fs} E_2$, iff $F_S(E_1) = F_S(E_2)$.

Definition 26 The distance $d_{fs}: F_S(E_1) \times F_S(E_2) \rightarrow [0, 1]$ between step RSF pairs (θ_1, v_1) and (θ_2, v_2) is defined as follows:

$$d_{fs}(\theta_1, v_1), (\theta_2, v_2) = \begin{cases} 0, & \theta_1 = \theta_2 \text{ and } v_1 = v_2 \\ 2^{-L}, & \theta_1 = \theta_2 \text{ and } v_1 \neq v_2 \\ 2^{-l}, & \text{otherwise} \end{cases}$$

where $l \in \mathbb{N}$ is the length of the longest common prefix of θ_1 and θ_2 , L is the maximum length of all step traces of E_1 and E_2 .

Theorem 7 d_{fs} is a pseudo-metric on the set of step RSF pairs.

Definition 27 The step SF distance between E_1 and E_2 is defined as follows:

$$d_{fs}(E_1, E_2) = d_H(F_S(E_1), F_S(E_2))$$

Definition 28 Two event structures E_1 and E_2 are approximate step SF equivalent with the precision $\xi \geq 0$, in other words, there is an approximate step SF relation \simeq_{fs}^ξ between E_1 and E_2 , notation $E_1 \simeq_{fs}^\xi E_2$,

iff $d_{fs}(E_1, E_2) \leq \xi$.

Theorem 8 \simeq_{fs}^ξ ($\xi \geq 0$) is an equivalence relation on the set of event structures.

5 Related Work

Our version of Baire metric (Definition 3) is described in Ref.[11], which presented an introduction to metric semantics for programming and specification languages. In this paper, we extend the definition of Baire metric to suit the specific needs of RSF pairs (Definition 15 and Definition 26).

A metric very similar to this definition was defined by Baire^[15]. The alternative version of Baire metric was typically used to measure the similarity between strings^[16-18], but it does not support Proposition 1 and Proposition 2, which are the essential preconditions for the derivation of the transitive property of approximate equivalences.

In the past several decades, there have been many efforts to develop system approximation. Approximate trace and approximate (bi)simulation relations^[19] have been studied for transition systems^[20] and hybrid systems^[21-22] by Girard and Pappas. Their approximation is based on a metric on the set of observations, which can be any metric. However, none of their approximate relations is an equivalence relation, as they do not satisfy the transitive property. Hence their approximate relations can not be successively used, while our approximate equivalences do not have this limit. For example, $E_1 \simeq_{ts}^\xi E_2$, $E_2 \simeq_{ts}^\xi E_3, \dots, E_{n-1} \simeq_{ts}^\xi E_n$ imply that $E_1 \simeq_{ts}^\xi E_n$.

6 Conclusions

This paper proposes a generalized framework of event structures approximation by developing the notions of approximate interleaving (trace and SF) equivalence and approximate step (trace and SF) equivalence. Where interleaving and step are two discriminating views of event structures, trace semantics and SF semantics are respectively representatives of linear time semantics and branching time semantics. Note that each of these traditional exact equivalences is a special case of the corresponding approximate equivalence.

The motivation of these approximate equivalences

is to define more robust relations between event structures, and to achieve a considerable complexity reduction in the approximation process.

These approximate equivalences are deduced as following. Firstly, we use Baire metric in interleaving so as to give the definition of the distance between interleaving traces (or interleaving RSF pairs). On the other hand, we regard multi-sets as functions, and define an equality operator for finite multi-sets. Therefore, we can use Baire metric in step. Then, we give the definition of the distance between step traces (or step RSF pairs). Secondly, we use Hausdorff distance d_H which is defined in Definition 4 to measure set (i.e., the set of all interleaving or step, traces or RSF pairs) distance. In other words, we define four kinds of distances on event structures. Each distance together with the event structures form a pseudo-metric space. Thirdly, these distances enable us to use quantitative approximation. We define that two event structures are approximate equivalent with the precision ξ , iff the distance between them is not greater than ξ .

The main conclusion of this paper is that these approximate equivalences satisfy the transitive property, hence they can be successively used in event structures approximation. That is to say, we can successively use these approximate equivalences to obtain a much more simplified equivalent system than an original complex system. These approximate equivalences guarantee some quality and efficiency in the specification and verification.

In our future work, we plan to develop approximate trace and SF equivalences based on partial orders.

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李绍荣(1964—), 男, 研究员, 现为电子科技大学光电技术工程中心主任、成都市重大产业技术平台(光电显示与高效照明)主任。教育部、科技部、四川省等科技项目评审专家库专家。国家科技奖励评审专家、四川省科技奖励评审专家。

长期从事光电技术、图像处理、导航技术、高可信复杂系统的科研与应用等方面的工作, 研究成果已在国内广东、江苏、浙江及四川等省市以及航

空工业、航天科工、中电科技、中国工程物理研究院、中国节能环保集团等行业大集团得到广泛应用。主持研制的“机载无线电导航罗盘WL-5C”是中国民航首架具有自主知识产权航电系统飞机“海鸥-300型”的主要组成部分; 是“中国民用航空技术标准(CTSO)——机载自动定向(ADF)设备”的起草撰写人之一。获国家科技进步二等奖一项; 获教育部科技进步二等奖一项。申请发明专利10项, 已获授权4项。在Latin American Applied Research, 光学学报等国内外刊物发表文章30余篇。